Sample Paper

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MA: Mathematics

Q.No. 1 The Cauchy problem $uu_x + yu_y = x$ with $u(x, 1) = 2x$, when solved using its characteristic equations with an independent variable t, is found to admit of a solution in the form $x = \frac{3}{2}se^t - \frac{1}{2}se^{-t}, \quad y = e^t, \quad u = f(s, t)$. Then f(s, t) =

(A) $\frac{3}{2}se^t + \frac{1}{2}se^{-t}$

(B) $\frac{1}{2}se^t + \frac{3}{2}se^{-t}$

(C) $\frac{1}{2}se^t - \frac{3}{2}se^{-t}$

(D) $\frac{3}{2}se^t - \frac{1}{2}se^{-t}$

Answer: (A)

Q.No. 2 Let $X_1, X_2, \ldots, X_n (n \geq 2)$ be a random sample from a N(θ,θ) population, where θ>0, and let $W = \frac{1}{n}\sum_{i=1}^{n} X_i^2$. Then the maximum likelihood of θ is

(A) $\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4W}$

(B) $\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4W}$

(C) $\frac{-1}{2} + \frac{1}{2}\sqrt{1 - 4W}$

(D) $\frac{-1}{2} + \frac{1}{2}\sqrt{1 + 4W}$

Answer: (D)

Q.No. 3 For a linear programming problem, which one of the following statements is FALSE?

(A) If a constraint is an equality, then the corresponding dual variable is unrestricted in sign

(B) Both primal and its dual can be infeasible

(C) If primal is unbounded, then its dual is infeasible

(D) Even if both primal and dual are feasible, the optimal values of the primal and the dual can differ

Answer: (D)

Q.No. 4 Let f : X → Y be a continuous map from a Hausdorff topological space X to a metric space Y . Consider the following two statements: P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact foreach y ∈ Y. Q: For every compact subset K ⊂ Y, the inverse image $f^{-1}(K)$ is a compact subset of X. Which one of the following is true?

(A) Q implies P but P does NOT imply Q

(B) P implies Q but Q does NOT imply P

(C) P and Q are equivalent

(D) neither P implies Q nor Q implies P

Answer: (C)

Q.No. 5 Which one of the following statements is true?

(A) Every group of order 12 has a non-trivial proper normal subgroup

(B) Some group of order 12 does not have a non-trivial proper normal subgroup

(C) Every group of order 12 has a subgroup of order 6

(D) Every group of order 12 has an element of order 12

Answer: (A)

Q.No. 6 For a linear programming problem (LPP) and its dual, which one of the following is NOT TRUE?

(A) The dual of the dual is primal

(B) If the primal LPP has an unbounded objective function. then the dual LPP is infeasible

(C) If the primal LPP is infeasible. then the dual LPP must have unbounded objective function

(D) If the primal LPP has a finite optimal solution. then the dual I-PP also has a finite optimal Solution

Answer: (C)

Q.No. 7 If u(x,y) = 1 + x + y + f(xy), where f:$R^2 \rightarrow$ R is a differentiable function, then u satisfies

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x^2 - y^2$

(B) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$

(C) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x - y$

(D) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = x - y$

Answer: (C)

Q.No. 8 For a balanced transportation problem with three sources and three destinations where costs, availabilities and demands are all finite and positive, which one of the following statements is FALSE?

(A) The transportation problem does not have unbounded solution

(B) The number of non-basic variables of the transportation problem is 4

(C) The dual variables of the transportation problem are unrestricted in sign

(D) The transportation problem has at most 5 basic feasible solutions

Answer: (D)

Q.No. 9 Let X1, X2, . . . , Xn be independent and identically distributed random variables with proba-bility density function given by $f_X(x;\theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$ Also, let X = 1nPni=1 Xi. Then the maximum likelihood estimator of θ is

(A) $1/\bar{X}$

(B) $(1/\bar{X}) - 1$

(C) $1/(\bar{X} - 1)$

(D) $\bar{X}$

Answer: (C)

Q.No. 10 Suppose that M is a 5 x 5 matrix with real entries and p(x) = det(xl — M). Then

(A) p(0) = det(M)

(B) every eigenvalue of M is real if p(1) + p(2) = 0 = p(2) + p(3)

(C) m-1 is necessarily a polynomial in M of degree 4 if M is invertible

(D) M is not invertible if m2 - 2M = 0

Answer: (C)

Q.No. 11 Let C[0, 1] denote the space of all real-valued continuous functions on [0, 1] equipped with the supremum norm $\|\cdot\|_\infty$. Let $T: C[0,1] \to C[0,1]$ be the linear operator defined by $T(f)(x) = \int_0^x e^{-y} f(y)\,dy$ Then

(A) $\|T\| = 1$

(B) I — T is not invertible

(C) T is surjective

(D) $\|I + T\| = 1 + \|T\|$

Answer: (D)

Q.No. 12 Let R be a commutative ring with 1 (unity) which is not a field. Let I $\subset$ R be a proper ideal such that every element of R not in I is invertible in R. Then the number of maximal ideals of R is

(A) 1

(B) 2

(C) 3

(D) infinite

Answer: (A)

Q.No. 13 The initial value problem $y' = y^{\frac{3}{5}}, \quad y(0) = b$ has

(A) a unique solution if b = 0

(B) no solution if b = 1

(C) infinitely many solutions if b = 2

(D) a unique solution if b = 1

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Answer: (D)

Q.No. 14 Consider the iterative scheme $x_n = \frac{x_{n-1}}{2} + \frac{3}{x_{n-1}}, \quad n \geq 1$ with initial point x0 > 0. Then the sequence {xn}

(A) converges only if x0 > 1

(B) converges only if x0 < 3

(C) converges for any x0

(D) does not converge for any x0

Answer: (C)

Q.No. 15 Let $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ for x∈R. Consider the following statement: (P): y1(x) and y2(x) are linearly independent solutions of $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$ on R. (Q): The Wronskian $y_1(x)\frac{dy_2}{dx}(x) - y_2(x)\frac{dy_1}{dx}(x) = 0$ for all x∈R. Which of the above statements hold TRUE?

(A) Both P and Q

(B) Only P

(C) Only Q

(D) Neither P nor Q

Answer: (A)

Q.No. 16 If f: C \ {0}→C is a function such that $f(z) = f\left(\frac{z}{|z|}\right)$ and its restriction to the unit circle is continuous, then

(A) f is continuous but not necessarily analytic

(B) f is analytic but not necessarily a constant function

(C) f is a constant function

(D) $\lim_{z\to 0} f(z)$ exists

Answer: (A)

Q.No. 17 Let D = [-1,1] x [-1,1], If the function f: D→R is defined by f(x, y)=\left\{

$$\begin{array}{ll} \frac{x^2-y^2}{\left(x^2+y^2\right)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{array}$$

\right, then

(A) f is continuous at (0, 0)

(B) both the first order partial derivatives of f exist at (0, 0)

(C) $\iint_D |f(x,y)|^{\frac{1}{2}}\,dx\,dy$

(D) $\iint_D |f(x,y)|\,dx\,dy$

Answer: (C)

Q.No. 18 Let C[0,1] denote the space of all real-valued continuous functions on [0, 1] equipped with the supremum norm $\|\cdot\|_\infty$. Let $f \in C[0,1]$ be such that
$|f(x) - f(y)| \leq M|x-y|, \$ for all \$ x, y \in [0,1] \$ and for some \$ M > 0 \$$ For n ∈ ℕ, Let
$f_n(x) = f\left(x^{1+\frac{1}{n}}\right)$. If $S = \{f_n : n \in \mathbb{N}\}$, then

(A) the closure of S is compact

(B) S is closed and bounded

(C) S is bounded but not totally bounded

(D) S is compact

Answer: (A)

Q.No. 19 Let f: C → C be non-zero and analytic at all points in Z . If F(z) =πf(z) cot(πz) for z∈ C\Z then the residue of F at n∈ Z is _____ . (C is the set of all complex numbers, Z is the set of all integers and C \ Z denotes the set of all complex numbers excluding integers)

(A) x f(n)

(B) f(n)

(C) f(n)/π

(D) $(df/dz)_{z=n}$

Answer: (B)

Q.No. 20 Consider the subspace Y={(x, x): x∈ C} of the normed linear space (C^2,|| ||) . If Φ is a bounded linear functional on ,Y defined by Φ(x,x) , then which one of the following sets is equal to {Ψ(1,0) : Ψ is a norm preserving extension of Φ , (C^2,|| ||)} (C is the set of all complex numbers, C^2 = {(x ,y,): x ,y∈ C} and || (x_1,x_2) || sup {|,x_1|,|x_2||

(A) {1}

(B) [½ ,3/2]

(C) [1,∞)

(D) [0,1]

Answer: (D)

Q.No. 21 Let T_1 be the co-countable topology on R (the set of real numbers) and T_2 be the co-finite topology on . Consider the following statements: I. In (R ,T_1) the sequence $\left\{\frac{1}{n}\right\}_{-1}^{2}$ converges to 0. II. In (R ,T_2), the sequence $\left\{\frac{1}{n}\right\}_{-1}^{2}$ converges to 0. III. In (R ,T_1) there is no sequence of rational numbers which converges to √3. IV. In (R ,T_2) there is no sequence of rational numbers which converges to √3. Which of the above statements are TRUE?

(A) I and II only

(B) II and III only

(C) III and IV only

(D) I and IV only

Answer: (B)

Q.No. 22 Consider the following statements: I : log(lzl) is harmonic on C \ {0} II : log(lzl) has a harmonic conjugate on C \ {O} Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Answer: (B)

Q.No. 23 Suppose that $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are topologies on X induced by metrics d1 and d2, respectively, such that $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$. Then which of the following statements is TRUE?

(A) If a sequence converges in (X, d2) then it converges in (X, d1)

(B) If a sequence converges in (X, d1) then it converges in (X, d2)

(C) Every open ball in (X, d1) is an open ball in (X, d2)

(D) The map x↦x from (X,d1) to (X,d2) is continuous

Answer: (A)

Q.No. 24 Suppose that $U = \mathbb{R}^2 \setminus \{(x, y) \in \mathbb{R}^2 : x, y \in \mathbb{Q}\}$, $V = \mathbb{R}^2 \setminus \{(x, y) \in \mathbb{R}^2 : x > 0, y = \frac{1}{x}\}$ Then, with respect to the Euclidean metric on $\mathbb{R}2$,

(A) both U and V are disconnected

(B) U is disconnected but V is connected

(C) U is connected but V is disconnected

(D) both U and V are connected

Answer: (C)

Q.No. 25 If $y = 3e^{2x} + e^{-2x} - \alpha x$ is the solution of the initial value problem $\frac{d^2y}{dx^2} + \beta y = 4\alpha x, y(0) = 4$ and $\frac{dy}{dx}(0) = 1$, where **α**,B∈ R, then

(A) **α** = 3 and B = 4

(B) **α** = 1 and B = 2

(C) **α** = 3 and B = -4

(D) **α** = 1 and B = -2

Answer: (C)

Q.No. 26 An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

(A) 2/7

(B) 3/8

(C) 1/2

(D) 5/7

Answer: (B)

Q.No. 27 For an odd prime p, consider the ring $\mathbb{Z}[\sqrt{-p}] = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}[\sqrt{-p}]$ is

(A) a unit

(B) a square

(C) a prime

(D) irreducible

Answer: (D)

Q.No. 28 Let T_u and T_d denote the usual topology and the discrete topology on R, respectively. Consider the following three topologies: T1 = Usual topology on $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, T2 = Topology generated by the basis $\{U \times V : U \in \mathcal{T}_d, V \in \mathcal{T}_u\}$ on R x R, T3 = Dictionary order topology on R x R. Then

(A) $\mathcal{T}_3 \subsetneqq \mathcal{T}_1 \subseteq \mathcal{T}_2$

(B) $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 \subsetneqq \mathcal{T}_3$

(C) $\mathcal{T}_3 \subseteq \mathcal{T}_2 \subsetneqq \mathcal{T}_1$

(D) $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 = \mathcal{T}_3$

Answer: (D)

Q.No. 29 Let P ε $M_{m \times n}$ .Consider the following statements: I : If X P Y = 0 for all X ε $M_{1 \times m}$(R) and Y ε $M_{n \times 1}$(R), then P= 0. II : If m = n, P is symmetric and P^2 = 0, then P = 0. Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Answer: (A)

Q.No. 30 Consider the following statements: I. The set QxZ is uncountable. II. The set {f : f is a function from to {0, 1}} is uncountable. III. The set {√ p : is a prime number} is uncountable. IV. For any infinite set, there exists a bijection from the set to one of its proper subsets. (Q is the set of all rational numbers, Z is the set of all integers and is the set of all natural numbers) Which of the above statements are TRUE?

(A) I and IV only

(B) II and IV only

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(C) II and III only

(D) I, II and IV only

Answer: (B)

Q.No. 31 Let f : R → R be a twice continuously differentiable function. The order of convergence of the secant method for finding root of the equation f(x) = 0 is

(A) $\frac{1+\sqrt{5}}{2}$

(B) $\frac{2}{1+\sqrt{5}}$

(C) $\frac{1+\sqrt{5}}{3}$

(D) $\frac{3}{1+\sqrt{5}}$

Answer: (A)

Q.No. 32 Let the general integral of the partial differential equation $(2xy-1)\frac{\partial z}{\partial x}+\left(z-2x^2\right)\frac{\partial z}{\partial y}=2(x-yz)$ be given by F (u,v)=0, where F=R² is a continuously differentiable function. (R is the set of all real numbers and R²={ (x ,y) : x, y ∈ R}) Then which one of the following is TRUE?

(A) u= x² +y² +z, v=xz +y

(B) u= x² +y² -z, v=xz -y

(C) u= x² -y² +z, v=yz +x

(D) u= x² +y² -z, v=yz -x

Answer: (A)

Q.No. 33 Let X and Y be normed linear spaces, and let T: X→ Y be any bijective linear map with closed graph. Then which one of the following statements is TRUE?

(A) The graph of T is equal to X xY

(B) T^{-1} is continuous

(C) The graph of T^{-1} is closed

(D) T is continuous

Answer: (C)

Q.No. 34 Consider the Linear Programming Problem (LPP):

$$Maximize\$\alpha x_1+x_2 \quad \text{Subject to } \begin{matrix} 2x_1+x_2\le 6 \\ -x_1+x_2\le 1 \\ x_1+x_2\le 4 \\ x_1\ge 0, x_2\ge 0 \end{matrix}$$

where α is a constant. If (3, 0) is the only optimal solution, then

(A) α < −2

(B) −2 < α < 1

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(C) 1 < α < 2

(D) α > 2

Answer: (D)

Q.No. 35 Let $X_1, X_2, \ldots, X_n \quad (n \geq 2)$ be independent and identically distributed random variables with finite variance $\boldsymbol{\sigma}^2$ and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the covariance between $\bar{X}$ and $X_1 - \bar{X}$ is

(A) 0

(B) $-\boldsymbol{\sigma}^2$

(C) $\frac{-\sigma^2}{\eta}$

(D) $\frac{\sigma^2}{\eta}$

Answer: (A)

Q.No. 36 Let $M = \begin{bmatrix} \alpha & 3 & 0 \\ \beta & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ Consider the following statements: l: There exists a lower triangular matrix L such that $M = LL^t,$ where L^t denotes transpose of L. II: Gauss-Seidel method for Mx = b (b ∈ R^3) converges for any initial choice x_0 ∈ R^3 Then,

(A) I is not true when $\alpha > \frac{9}{2}, \beta = 3$

(B) II is not true when $\alpha > \frac{9}{2}, \beta = -1$

(C) II is not true when $\alpha = 4, \beta = \frac{3}{2}$

(D) I is true when $\alpha = 5, \beta = 3$

Answer: (D)

Q.No. 37 Consider the ordered square I_0^2, the set [0,1] x[0,1] with the dictionary order topology. Let the general element of I_0^2 be denoted by x * y, where x y ∈ [0,1]. Then the closure of the subset S= { x * ¾ : 0

(A) S U ((a,b] x{0}) U ([a,b) x{1})

(B) S U ((a,b] x{0}) U ((a,b] x{1})

(C) S U ((a,b) x{0}) U ((a,b) x{1})

(D) S U ((a,b] x{0})

Answer: (A)

Q.No. 38 Let f: R^2 → R be defined by $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{x^2+y^2}\right), & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$ Consider the following statements: I. The partial derivatives , f f exist at (0, 0) but are unbounded in any neighbourhood of (0, 0). II. f is continuous but not differentiable at (0, 0). III. f is not continuous at (0, 0). IV. f is differentiable at (0, 0). (R is the set of all real numbers and R^2 ={(x ,y) : x ,y∈ R) Which of the above statements is/are TRUE?

(A) I and II only

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(B) I and IV only

(C) IV only

(D) III only

Answer: (B)

Q.No. 39 Suppose V is a finite dimensional vector space over R. If W_1, W_2 and W_3 are subspaces of V , then which of the following statements is TRUE?

(A) If $W_1 + W_2 + W_3 = V$ then span($W_1 \cup W_2$) U span($W_2 \cup W_3$) U span($W_3 \cup W_1$) = V

(B) If $W_1 \cap W_2 = \{0\}$ and $W_1 \cap W_3 = \{0\}$, then $W_1 \cap (W_2 + W_3) = \{0\}$

(C) If $W_1 + W_2 = W_1 + W_3$, then $W_2 = W_3$

(D) If $W_1 \neq V$, then span(V\W_1) = V

Answer: (D)

Q.No. 40 A solution of the Dirichlet problem $\nabla^2 u(r,\boldsymbol{\theta}) = 0$, 0

(A) $u(r,\boldsymbol{\theta}) = \pi/2 + \sum_{n=1}^{\infty} \left[\frac{(-1)^n - 1}{n^2}\right] r^n \cos(n\theta)$

(B) $u(r,\boldsymbol{\theta}) = \pi/2 + \sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1} - 1}{n^2}\right] r^n \cos(n\theta)$

(C) $u(r,\boldsymbol{\theta}) = \pi/2 + \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\frac{(-1)^n - 1}{n^2}\right] r^n \cos(n\theta)$

(D) $u(r,\boldsymbol{\theta}) = \pi/2 - \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\frac{(-1)^n - 1}{n^2}\right] r^n \cos(n\theta)$

Answer: (C)

Q.No. 41 The general solution of the differential equation $xy' = y + \sqrt{x^2 + y^2}$ for $x > 0$ is given by (with an arbitrary positive constant k)

(A) $ky^2 = x + \sqrt{x^2 + y^2}$

(B) $kx^2 = x + \sqrt{x^2 + y^2}$

(C) $kx^2 = y + \sqrt{x^2 + y^2}$

(D) $ky^2 = y + \sqrt{x^2 + y^2}$

Answer: (C)

Q.No. 42 Let $X_1, X_2, \ldots, X_m (m \geq 2)$ be a random sample from a binomial distribution with parameters n=1 and p, p∈(0,1) and let $\bar{X} = \frac{1}{m} \sum_{i=1}^{m} X_i$. Then a uniformly minimum variance unbiased estimator for p(1-p) is

(A) $\frac{m}{m-1} \bar{X}(1 - \bar{X})$

(B) $\bar{X}(1 - \bar{X})$

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(C) $\frac{m-1}{m}\bar{X}(1-\bar{X})$

(D) $\frac{1}{m}\bar{X}(1-m\bar{X})$

Answer: (A)

Q.No. 43 Let $f(z)=\left(x^2+y^2\right)+i2xy$ and $g(z)=2xy+i\left(y^2-x^2\right)$ for $z=x+iy\in\mathbb{C}$ Then, in the complex plane C.

(A) f is analytic and g is NOT analytic

(B) f is NOT analytic and g is analytic

(C) neither f nor g is analytic

(D) both f and g are analytic

Answer: (B)

Q.No. 44 For the linear programming problem (LPP): Maximize Z = 2 x_1 + 4 x_2 subject to -x_1 + 2 x_2 <4, 3x_1 + $\boldsymbol{\beta}$x$_2$<6, x_1,x_2 ≥ 0 $\boldsymbol{\beta} \in$ R (R is the set of all real numbers) consider the following statements: I. The LPP always has a finite optimal value for any $\boldsymbol{\beta}$ ≥ 0. II. The dual of the LPP may be infeasible for some $\boldsymbol{\beta}$ ≥ 0. III. If for some ,$\boldsymbol{\beta}$ the point (1,2) is feasible to the dual of the LPP, then Z ≤ 16 for any feasible solution (x_1 , xsub>2) of the LPP. IV. If for some ,$\boldsymbol{\beta}$,x_1 and x_2 are the basic variables in the optimal table of the LPP with x_1= 1/2 then the optimal value of dual of the LPP is 10. Then which of the above statements are TRUE?

(A) I and III only

(B) I, III and IV only

(C) III and IV only

(D) II and IV only

Answer: (B)

Q.No. 45 Let $f_n:[0,1]\to\mathbb{R}$ be given by $f_n(x)=\frac{2x^2}{x^2+(1-2nx)^2}$, $n=1,2,\ldots$ Then the sequence (fₙ)

(A) converges uniformly on [0, 1]

(B) does NOT converge uniformly on [0 , 1] but has a subsequence that converges uniformly on [0, 1]

(C) does NOT converge pointwise on [0, 1]

(D) converges pointwise on [0, 1] but does NOT have a subsequence that converges uniformly on [0, 1]

Answer: (D)

Q.No. 46 Let {Xi} be a sequence of independent Poisson(λ) variables and let t $W_n=\frac{1}{n}\sum_{i=1}^{n}X_i$. Thenthe limiting distribution of $\sqrt{n}(W_n-\lambda)$ is the normal distribution with zero mean and variance given by

(A) 1

(B) √λ

(C) λ

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(D) λ2

Answer: (C)

Q.No. 47 Consider the polynomial $p(X) = X^4 + 4$ in the ring $\mathbb{Q}[X]$ of polynomials in the variable X with coefficients in the field Q of rational numbers. Then

(A) the set of zeros of p(X) in C forms a group under multiplication

(B) p(X) is reducible in the ring Q[X]

(C) the splitting field of p(X) has degree 3 over Q

(D) the splitting field of p(X) has degree 4 over Q

Answer: (B)

Q.No. 48 Let $f(z) = e^{1/z}, z \in \mathbb{C}\setminus\{0\}$ and let, for n∈N
$R_n = \left\{z = x + iy \in \mathbb{C} : |x| < \frac{1}{n}, |y| < \frac{1}{n}\right\}\setminus\{0\}$. If for a subset S of $\mathbb{C}$, denotes the closure of S in $\mathbb{C}$, then

(A) $\overline{f(R_{n+1})} \neq f(R_n)$

(B) $\overline{f(R_n)}\setminus\overline{f(R_{n+1})} = \overline{f(R_n\setminus R_{n+1})}$

(C) $\overline{f\left(\bigcap_{n=1}^{\infty} R_n\right)} = \bigcap_{n=1}^{\infty}\overline{f(R_n)}$

(D) $\overline{f(R_n)} = \overline{f(R_{n+1})}$

Answer: (A)

Q.No. 49 If , $u_n = \int_1^n e^{-r^2}\,dt$, n=1,2,3…… then which one of the following statements is TRUE?

(A) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are convergent

(B) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are divergent

(C) The sequence $\{u_n\}_{n=1}^{\infty}$ is convergent but the series $\sum_{n=1}^{\infty} u_n$ is divergent

(D) $\lim_{x\to\infty} u_x = \frac{2}{e}$

Answer: (C)

Q.No. 50 In the Laurent series expansion of f(z) = 1 z(z − 1) valid for |z − 1| > 1, the coefficient of 1 z − 1 is

(A) −2

(B) −1

(C) 0

(D) 1

Answer: (C)

Q.No. 51 Let X denote R 2 endowed with the usual topology. Let Y denote R endowed with the co-finite topology. If Z is the product topological space Y × Y, then

(A) the topology of X is the same as the topology of Z

(B) the topology of X is strictly coarser (weaker) than that of Z

(C) the topology of Z is strictly coarser (weaker) than that of X

(D) the topology of X cannot be compared with that of Z

Answer: (C)

Q.No. 52 Let f : C → C be an entire function with f(0) = 1, f(1) = 2 and f'(0) = 0. If there exists M > 0 such that $|f''(z)| \leq M$ for all $z \in \mathbb{C}$, then $f(2) =$

(A) 2

(B) 5

(C) 2 + 5i

(D) 5 + 2i

Answer: (B)

Q.No. 53 Consider the subspace $V = \left\{(x_n) \in \ell^2 : \sum_{n=1}^{\infty} |x_n| < \infty\right\}$ of the Hilbert space l^2 of all square summable real sequences. For n∈N, define $T_n : V \to \mathbb{R}$ by $T_n((x_k)) = \sum_{i=1}^{n} x_i$. Consider the following statements: (P): $\{T_n : n \in \mathbb{N}\}$ is pointwise bounded on V. (Q): $\{T_n : n \in \mathbb{N}\}$ is uniformly bounded on $\{x \in V : \|x\|_2 = 1\}$. Which of the above statements hold TRUE?

(A) Both P and Q

(B) Only P

(C) Only Q

(D) Neither P nor Q

Answer: (B)

Q.No. 54 For n ε N, let $T_n : \left(l^1, \|\cdot\|_1\right) \to (l^\infty, \|\cdot\|_\infty)$ and $T : \left(l^1, \|\cdot\|_1\right) \to (l^\infty, \|\cdot\|_\infty)$ be the bounded linear operators defined by $T_n(x_1, x_2, \ldots) = (y_1, y_2, \ldots)$, where $y_j = y_j = \begin{cases} x_j, & j \leq n \\ x_n, & j > n \end{cases}$ and $T(x_1, x_2, \ldots) = (x_1, x_2, \ldots)$. Then

(A) $\|T_n\|$ does not converge to $\|T_n\|$ as n → ∞

(B) $\|T_n - T\|$ converges to zero as n → ∞

(C) for all x ε l_1, $\|T_n(x) - T(x)\|$ converges to zero as n → ∞

(D) for each non-zero x ε l_1 there exists a continuous linear functional g on l^∞ that g (T_n (x)) does not converge to g(T(x)) as n → ∞

Answer: (C)

Q.No. 55 Let f: R → R be defined by $f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \chi_{(n,n+1]}(x)$ where $\chi_{(n,n+1]}$ is the characteristic function of the interval (n, n + 1]. For α ε R, let S_α = {x ε R : f (x) > α). Then

(A) $S_{1/2}$ is open

(B) $S_{\sqrt{5}/2}$ is not measurable

(C) S_0 is closed

(D) $S_{1/\sqrt{2}}$ is measurable

Answer: (D)

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