Using factor theorem, factorize each of the following polynomials :

Q1. $x^3 + 6x^2 + 11x + 6$

Sol:

Given polynomial, f(x) = $x^3 + 6x^2 + 11x + 6$

The constant term in f(x) is 6

The factors of 6 are ±1, ±2, ±3, ±6

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $)(-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 12 – 12

= 0

So, (x + 1) is the factor of f(x)

Similarly, (x + 2) and (x + 3) are also the factors of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x + 1)(x + 2)(x + 3)

=> $x^3 + 6x^2 + 11x + 6$ = k(x + 1)(x + 2)(x + 3)

Substitute x = 0 on both the sides

=> 0 + 0 + 0 + 6 = k(0 +1)(0 + 2)(0 + 3)

=> 6 = k(1*2*3)

=> 6 = 6k

=> k = 1

Substitute k value in f(x) = k(x + 1)(x + 2)(x + 3)

=> f(x) = (1)(x + 1)(x + 2)(x + 3)

=> f(x) = (x + 1)(x + 2)(x + 3)

∴ $x^3 + 6x^2 + 11x + 6$ = (x + 1)(x + 2)(x + 3)

Q2. $x^3 + 2x^2 - x - 2$

Sol:

Given, f(x) = $x^3 + 2x^2 - x - 2$

The constant term in f(x) is -2

The factors of (-2) are ±1, ±2

Let , x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $(1)^3 + 2(1)^2 - 1 - 2$

= 1 + 2 – 1 – 2

= 0

Similarly , the other factors (x + 1) and (x + 2) of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x – 1)(x + 2)(x + 1)

$x^3 + 2x^2 - x - 2$ = k(x – 1)(x + 2)(x + 1)

Substitute x = 0 on both the sides

0 + 0 – 0 – 2 = k(-1)(1)(2)

=> – 2 = -2k

=> k = 1

Substitute k value in f(x) = k(x – 1)(x + 2)(x + 1)

f(x) = (1)(x – 1)(x + 2)(x + 1)

=> f(x) = (x – 1)(x + 2)(x + 1)

So, $x^3 + 2x^2 - x - 2$ = (x – 1)(x + 2)(x + 1)

Q3. $x^3 - 6x^2 + 3x + 10$

Sol:

Let, f(x) = $x^3 - 6x^2 + 3x + 10$

The constant term in f(x) is 10

The factors of 10 are ±1, ±2, ±5, ±10

Let , x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^3 - 6(-1)^2 + 3(-1) + 10$

= -1 – 6 – 3 + 10

= 0

Similarly , the other factors (x – 2) and (x – 5) of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x + 1)(x – 2)(x – 5)

Substitute x = 0 on both sides

=> $x^3 - 6x^2 + 3x + 10$ = k(x + 1)(x – 2)(x – 5)

=> 0 – 0 + 0 + 10 = k(1)(-2)(-5)

=> 10 = k(10)

=> k = 1

Substitute k = 1 in f(x) = k(x + 1)(x – 2)(x – 5)

f(x) = (1)(x + 1)(x – 2)(x – 5)

so, $x^3-6x^2+3x+10$ = (x + 1)(x – 2)(x – 5)

Q4. $x^4-7x^3+9x^2+7x-10$

Sol:

Given, f(x) = $x^4-7x^3+9x^2+7x-10$

The constant term in f(x) is 10

The factors of 10 are ±1, ±2, ±5, ±10

Let , x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(x) = $1^4-7(1)^3+9(1)^2+7(1)-10$

= 1 – 7 + 9 + 7 – 10

= 10 – 10

= 0

(x – 1) is the factor of f(x)

Simarly, the other factors are (x + 1) ,(x – 2) , (x – 5)

Since, f(x) is a polynomial of degree 4. So, it cannot have more than four linear factor.

So, f(x) = k(x – 1)(x + 1)(x – 2)(x – 5)

=> $x^4-7x^3+9x^2+7x-10$ = k(x – 1)(x + 1)(x – 2)(x – 5)

Put x = 0 on both sides

0 – 0 + 0 – 10 = k(-1)(1)(-2)(-5)

– 10 = k(-10)

=> k = 1

Substitute k = 1 in f(x) = k(x – 1)(x + 1)(x – 2)(x – 5)

f(x) = (1)(x – 1)(x + 1)(x – 2)(x – 5)

= (x – 1)(x + 1)(x – 2)(x – 5)

So, $x^4-7x^3+9x^2+7x-10$ = (x – 1)(x + 1)(x – 2)(x – 5)

Q5. $x^4-2x^3-7x^2+8x+12$

Sol:

Given , f(x) = $x^4 - 2x^3 - 7x^2 + 8x + 12$

The constant term f(x) is equal is 12

Tha factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 - 2(-1)^3 - 7(-1)^2 + 8(-1) + 12$

= 1 + 2 – 7 – 8 + 12

= 0

So, x + 1 is factor of f(x)

Similarly, (x + 2), (x – 2), (x – 3) are also the factors of f(x)

Since, f(x) is a polynomial of degree 4 , it cannot have more than four linear factors.

=> f(x) = k(x + 1)(x + 2)(x – 3)(x – 2)

=> $x^4 - 2x^3 - 7x^2 + 8x + 12$ = k(x + 1)(x + 2)(x – 3)(x – 2)

Substitute x = 0 on both sides,

=> 0 – 0 – 0 + 12 = k(1)(2)(-2)(-3)

=> 12 = k12

=> k = 1

Substitute k = 1 in f(x) = k(x – 2)(x + 1)(x + 2)(x – 3)

f(x) = (x – 2)(x + 1)(x + 2)(x – 3)

so, $x^4 - 2x^3 - 7x^2 + 8x + 12$ = (x – 2)(x + 1)(x + 2)(x – 3)

Q6. $x^4 + 10x^3 + 35x^2 + 50x + 24$

Sol:

Given, f(x) = $x^4 + 10x^3 + 35x^2 + 50x + 24$

The constant term in f(x) is equal to 24

The factors of 24 are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Let, x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24$

= 1 – 10 + 35 – 50 + 24

= 0

=> (x + 1) is the factor of f(x)

Similarly, (x + 2),(x + 3),(x + 4) are also the factors of f(x)

Since, f(x) is a polynomial of degree 4, it cannot have more than four linear factors.

=> f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)

=> $x^4 + 10x^3 + 35x^2 + 50x + 24$ = k(x + 1)(x + 2)(x + 3)(x + 4)

Substitute x = 0 on both sides

=> 0 + 0 + 0 + 0 + 24 = k(1)(2)(3)(4)

=> 24 = k(24)

=> k = 1

Substitute k = 1 in f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)

f(x) = (1)(x + 1)(x + 2)(x + 3)(x + 4)

f(x) = (x + 1)(x + 2)(x + 3)(x + 4)

hence, $x^4 + 10x^3 + 35x^2 + 50x + 24$ = (x + 1)(x + 2)(x + 3)(x + 4)

Q7. $2x^4 - 7x^3 - 13x^2 + 63x - 45$

Sol :

Given, f(x) = $2x^4 - 7x^3 - 13x^2 + 63x - 45$

The factors of constant term -45 are ±1, ±3, ±5, ±9, ±15, ±45

The factors of the coefficient of x^{4} is 2. Hence possible rational roots of f(x) are

±1, ±3, ±5, ±9, ±15, ±45, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Let, x − 1 = 0

=> x= 1

f(1) = $2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$

= 2 − 7 − 13 + 63 − 45

= 0

Let, x − 3 = 0

=> x = 3

f(3) = $2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

= 162 − 189 − 117 + 189 − 45

= 0

So, (x − 1) and (x − 3) are the roots of f(x)

=> x^2 − 4x + 3 is the factor of f(x)

Divide f(x) with x^2 − 4x + 3 to get other three factors

By long division,

$2x^2$ + x − 15

$x^2 - 4x + 3$ $2x^4 - 7x^3 - 13x^2 + 63x - 45$

$2x^4 - 8x^3 + 6x^2$

(-) (+) (-)

$x^3 - 19x^2 + 63x$

$x^3 - 4x^2 + 3x$

(-) (+) (-)

$-15x^2 + 60x - 45$

$-15x^2 + 60x - 45$

(+) (-) (+)

0

=> $2x^4-7x^3-13x^2+63x-45$ = (x^{2} - 4x + 3)(2x^{2} + x - 15)

=> $2x^4-7x^3-13x^2+63x-45$ = (x - 1) (x - 3)(2x^{2} + x - 15)

Now,

$2x^2$ + x - 15 = 2x^{2} + 6x - 5x - 15

= 2x(x + 3) - 5 (x + 3)

= (2x - 5) (x + 3)

So, $2x^4-7x^3-13x^2+63x-45$ = (x - 1)(x - 3)(x + 3)(2x - 5)

Q8. $3x^3-x^2-3x+1$

Sol :

Given , f(x) = $3x^3-x^2-3x+1$

The factors of constant term 1 is ±1

The factors of the coefficient of x^2 = 3

The possible rational roots are ±1 , $\frac{1}{3}$

Let, x - 1 = 0

=> x = 1

f(1) = $3(1)^3-(1)^2-3(1)+1$

= 3 - 1 - 3 + 1

= 0

So, x - 1 is tha factor of f(x)

Now, divide f(x) with (x - 1) to get other factors

By long division method,

$3x^2 + 2x - 1$

$x - 1$ $3x^3 - x^2 - 3x + 1$

$3x^3 - x^2$

(-) (+)

$2x^2 - 3x$

$2x^2 - 2x$

(-) (+)

-x + 1

-x + 1

(+) (-)

0

=> $3x^3 - x^2 - 3x + 1$ = (x – 1)($3x^2$ + 2x – 1)

Now,

$3x^2$ + 2x – 1 = $3x^2$ + 3x – x – 1

= 3x(x + 1) -1(x + 1)

= (3x – 1)(x + 1)

Hence , $3x^3 - x^2 - 3x + 1$ = (x – 1) (3x – 1)(x + 1)

Q9. $x^3 - 23x^2 + 142x - 120$

Sol :

Let, f(x) = $x^3 - 23x^2 + 142x - 120$

The constant term in f(x) is -120

The factors of -120 are ±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, ±20, ±24, ±30, ±40, ±60, ±120

Let, x – 1 = 0

=> x = 1

f(1) = $(1)^3 - 23(1)^2 + 142(1) - 120$

= 1 – 23 + 142 – 120

= 0

So, (x – 1) is the factor of f(x)

Now, divide f(x) with (x – 1) to get other factors

By long division,

x^2 – 22x + 120

x – 1 x^3 – 23x^2 + 142x – 120

x^3 – x^2

(-) (+)

-22x^2 + 142x

$-22x^2 + 22x$

(+) (-)

120x – 120

120x – 120

(-) (+)

0

=> $x^3 – 23x^2 + 142x – 120 = (x – 1) (x^2 – 22x + 120)$

Now,

$x^2 – 22x + 120 = x^2 – 10x – 12x + 120$

= x(x – 10) – 12(x – 10)

= (x – 10) (x – 12)

Hence, $x^3 – 23x^2 + 142x – 120 = (x – 1) (x – 10) (x – 12)$

Q10. $y^3 – 7y + 6$

Sol :

Given, f(y) = $y^3 – 7y + 6$

The constant term in f(y) is 6

The factors are ±1, ±2, ±3, ±6

Let , y – 1 = 0

=> y = 1

f(1) = $(1)^3 – 7(1) + 6$

= 1 – 7 + 6

= 0

So, (y – 1) is the factor of f(y)

Similarly, (y – 2) and (y + 3) are also the factors

Since, f(y) is a polynomial which has degree 3, it cannot have more than 3 linear factors

=> f(y) = k(y – 1)(y – 2)(y + 3)

=> $y^3 – 7y + 6$ = k(y – 1)(y – 2)(y + 3) ———– 1

Substitute k = 0 in eq 1

=> 0 – 0 + 6 = k(-1)(-2)(3)

=> 6 = 6k

=> k = 1

$y^3 – 7y + 6$ = (1)(y – 1)(y – 2)(y + 3)

$y^3 – 7y + 6$ = (y – 1)(y – 2)(y + 3)

Hence, y^3-7y+6 = (y – 1)(y – 2)(y + 3)

Q11. $x^3-10x^2-53x-42$

Sol :

Given , f(x) = $x^3-10x^2-53x-42$

The constant in f(x) is -42

The factors of -42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3-10(-1)^2-53(-1)-42$

= -1 – 10 + 53 – 42

= 0

So., (x + 1) is the factor of f(x)

Now, divide f(x) with (x + 1) to get other factors

By long division,

x^2 – 11x – 42

x + 1 x^3 – $10x^2$ – 53x – 42

x^3 + x^2

(-) (-)

-$11x^2$ – 53x

-$11x^2$ – 11x

(+) (+)

-42x – 42

-42x – 42

(+) (+)

0

=> x^3 – $10x^2$ – 53x – 42 = (x + 1) (x^2 – 11x – 42)

Now,

x^2 – 11x – 42 = x^2 – 14x + 3x – 42

= x(x – 14) + 3(x – 14)

= (x + 3)(x – 14)

Hence, x^3 – $10x^2$ – 53x – 42 = (x + 1) (x + 3)(x – 14)

Q12. $y^3-2y^2-29y-42$

Sol :

Given, f(x) = $y^3-2y^2-29y-42$

The constant in f(x) is -42

The factors of -42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, y + 2 = 0

=> y = -2

f(-2) = $(-2)^3-2(-2)^2-29(-2)-42$

= -8 -8 + 58 – 42

= 0

So, (y + 2) is the factor of f(y)

Now, divide f(y) with (y + 2) to get other factors

By, long division

$y^2 – 4y – 21$

y + 2 $y^3 – 2y^2 – 29y – 42$

$y^3 + 2y^2$

(-) (-)

$-4y^2 – 29y$

$-4y^2 – 8y$

(+) (+)

-21y – 42

-21y – 42

(+) (+)

0

=> $y^3 – 2y^2 – 29y – 42 = (y + 2) (y^2 – 4y – 21)$

Now,

$y^2 – 4y – 21 = y^2 – 7y + 3y – 21$

= y(y – 7) +3(y – 7)

= (y – 7)(y + 3)

Hence, $y^3 – 2y^2 – 29y – 42 = (y + 2) (y – 7)(y + 3)$

Q13. $2y^3-5y^2-19y+42$

Sol :

Given, f(x) = $2y^3-5y^2-19y+42$

The constant in f(x) is +42

The factors of 42 are ±1, ±2, ±3, ±6, ±7, ±14, ±21,±42

Let, y – 2 = 0

=> y = 2

f(2) = $2(2)^3-5(2)^2-19(2)+42$

= 16 – 20 – 38 + 42

= 0

So, (y – 2) is the factor of f(y)

Now, divide f(y) with (y – 2) to get other factors

By, long division method

$2y^2 – y – 21$

y – 2 $2y^3 – 5y^2$ -19y + 42

$2y^3 – 4y^2$

(-) (+)

$-y^2 – 19y$

$-y^2 + 2y$

(+) (-)

-21y + 42

-21y + 42

(+) (-)

0

=> $2y^3 – 5y^2$ -19y + 42 = (y – 2) ($2y^2 – y – 21$)

Now,

$2y^2 – y – 21$

The factors are (y + 3) (2y – 7)

Hence, $2y^3 – 5y^2$ -19y + 42 = (y – 2) (y + 3) (2y – 7)

Q14. $x^3+13x^2+32x+20$

Sol:

Given, f(x) = $x^3+13x^2+32x+20$

The constant in f(x) is 20

The factors of 20 are ±1, ±2, ±4, ±5, ±10, ±20

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

= -1 + 13 – 32 + 20

= 0

So, (x + 1) is the factor of f(x)

Divide f(x) with (x + 1) to get other factors

By, long division

$x^2 + 12x + 20$

$x + 1 \;\; x^3 + 13x^2 + 32x + 20$

$x^3 \;\; + x^2$

(-) (-)

$12x^2 + 32x$

$12x^2 + 12x$

(-) (-)

20x – 20

20x – 20

(-) (-)

0

=> $x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

Now,

$x^2 + 12x + 20 = x^2 + 10x + 2x + 20$

= x(x + 10) + 2(x + 10)

The factors are (x + 10) and (x + 2)

Hence, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 10)(x + 2)$

Q15. x^3-3x^2-9x-5

Sol :

Given, f(x) = x^3-3x^2-9x-5

The constant in f(x) is -5

The factors of -5 are ±1, ±5

Let, x + 1 = 0

=> x = -1

f(-1) = $(-1)^3-3(-1)^2-9(-1)-5$

= -1 – 3 + 9 – 5

= 0

So, (x + 1) is the factor of f(x)

Divide f(x) with (x + 1) to get other factors

By, long division

$x^2 - 4x - 5$

$x + 1 \quad x^3 - 3x^2 - 9x - 5$

$x^3 + x^2$

(-) (-)

$-4x^2 - 9x$

$-4x^2 - 4x$

(+) (+)

$-5x - 5$

$-5x - 5$

(+) (+)

0

$\Rightarrow x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$

Now,

$x^2 - 4x - 5 = x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

The factors are $(x - 5)$ and $(x + 1)$

Hence, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

Q16. $2y^3 + y^2 - 2y - 1$

Sol :

Given , $f(y) = 2y^3 + y^2 - 2y - 1$

The constant term is 2

The factors of 2 are $\pm 1, \pm \frac{1}{2}$

Let, $y - 1 = 0$

$\Rightarrow y = 1$

$f(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1$

$= 0$

So, $(y - 1)$ is the factor of $f(y)$

Divide $f(y)$ with $(y - 1)$ to get other factors

By, long division

$2y^2 + 3y + 1$

$y - 1 \quad 2y^3 + y^2 - 2y - 1$

$2y^3 - 2y^2$

(-) (+)

$3y^2 - 2y$

$3y^2 - 3y$

(-) (+)

y – 1

y – 1

(-) (+)

0

=> $2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1)$

Now,

$2y^2 + 3y + 1 = 2y^2 + 2y + y + 1$

$= 2y(y + 1) + 1(y + 1)$

$= (2y + 1)(y + 1)$ are the factors

Hence, $2y^3 + y^2 - 2y - 1 = (y - 1)(2y + 1)(y + 1)$

Q17. $x^3 - 2x^2 - x + 2$

Sol :

Let, f(x) = $x^3 - 2x^2 - x + 2$

The constant term is 2

The factors of 2 are ±1, $\pm\frac{1}{2}$

Let, x – 1= 0

=> x = 1

f(1) = $(1)^3 - 2(1)^2 - (1) + 2$

= 1 – 2 – 1 + 2

= 0

So, (x – 1) is the factor of f(x)

Divide f(x) with (x – 1) to get other factors

By, long division

$x^2 - x - 2$

x – 1 $x^3 - 2x^2 - y + 2$

$x^3 - x^2$

(-) (+)

$-x^2 - x$

$-x^2 + x$

(+) (-)

$-2x + 2$

$-2x + 2$

(+) (-)

0

$\Rightarrow x^3 - 2x^2 - y + 2 = (x - 1)(x^2 - x - 2)$

Now,

$x^2 - x - 2 = x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$=(x - 2)(x + 1)$ are the factors

Hence, $x^3 - 2x^2 - y + 2 = (x - 1)(x + 1)(x - 2)$

Q18. Factorize each of the following polynomials :

1. $x^3 + 13x^2 + 31x - 45$ **given that x + 9 is a factor**
2. $4x^3 + 20x^2 + 33x + 18$ **given that 2x + 3 is a factor**

Sol :

1. $x^3 + 13x^2 + 31x - 45$ given that x + 9 is a factor

let, f(x) = $x^3 + 13x^2 + 31x - 45$

given that (x + 9) is the factor

divide f(x) with (x + 9) to get other factors

by , long division

$x^2 + 4x - 5$

$x + 9 \quad x^3 + 13x^2 + 31x - 45$

$x^3 + 9x^2$

(-) (-)

$4x^2 + 31x$

$4x^2 + 36x$

(-) (-)

$-5x - 45$

$-5x - 45$

(+) (+)

0

$\Rightarrow x^3 + 13x^2 + 31x - 45 = (x + 9)(x^2 + 4x - 5)$

Now,

$x^2 + 4x - 5 = x^2 + 5x - x - 5$

$= x(x + 5) -1(x + 5)$

$= (x + 5)(x - 1)$ are the factors

Hence, $x^3 + 13x^2 + 31x - 45 = (x + 9)(x + 5)(x - 1)$

2. $4x^3 + 20x^2 + 33x + 18$ given that 2x + 3 is a factor

let, f(x) = $4x^3 + 20x^2 + 33x + 18$

given that 2x + 3 is a factor

divide f(x) with (2x + 3) to get other factors

by, long division

$2x^2 + 7x + 6$

$2x + 3 \quad 4x^3 + 20x^2 + 33x + 18$

$4x^3 + 6x^2$

(-) (-)

$14x^2 - 33x$

$14x^2 - 21x$

(-) (+)

$12x + 18$

$12x + 18$

(-) (-)

0

$\Rightarrow 4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x^2 + 7x + 6)$

Now,

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$

$= 2x(x + 2) + 3(x + 2)$

$= (2x + 3)(x + 2)$ are the factors

Hence, $4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x + 3)(x + 2)$