

Q1. If $f(x) = 2x^3 - 13x^2 + 17x + 12$, Find

1. $f(2)$
2. $f(-3)$
3. $f(0)$

Sol :

The given polynomial is $f(x) = 2x^3 - 13x^2 + 17x + 12$

1. $f(2)$

we need to substitute the '2' in $f(x)$

$$f(2) = 2(2)^3 - 13(2)^2 + 17(2) + 12$$

$$= (2 * 8) - (13 * 4) + (17 * 2) + 12$$

$$= 16 - 52 + 34 + 12$$

$$= 10$$

therefore $f(2) = 10$

2. $f(-3)$

we need to substitute the '(-3)' in $f(x)$

$$f(-3) = 2(-3)^3 - 13(-3)^2 + 17(-3) + 12$$

$$= (2 * -27) - (13 * 9) - (17 * 3) + 12$$

$$= -54 - 117 - 51 + 12$$

$$= -210$$

therefore $f(-3) = -210$

3. $f(0)$

we need to substitute the '(0)' in $f(x)$

$$f(0) = 2(0)^3 - 13(0)^2 + 17(0) + 12$$

$$= (2 * 0) - (13 * 0) + (17 * 0) + 12$$

$$= 0 - 0 + 0 + 12$$

$$= 12$$

therefore $f(0) = 12$

Q2. Verify whether the indicated numbers are zeros of the polynomial corresponding to them in the following cases :

1. $f(x) = 3x + 1, x = \frac{-1}{3}$

2. $f(x) = x^2 - 1, x = (1, -1)$

3. $g(x) = 3x^2 - 2, x = (\frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}})$

4. $p(x) = x^3 - 6x^2 + 11x - 6, x = 1, 2, 3$

5. $f(x) = 5x - \pi, x = \frac{4}{5}$

$$6. f(x) = x^2, x = 0$$

$$7. f(x) = lx + m, x = \frac{-m}{l}$$

$$8. f(x) = 2x + 1, x = \frac{1}{2}$$

Sol :

$$(1) f(x) = 3x + 1, x = \frac{-1}{3}$$

we know that ,

$$f(x) = 3x + 1$$

substitute $x = \frac{-1}{3}$ in $f(x)$

$$f\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1$$

$$= -1 + 1$$

$$= 0$$

Since, the result is 0 $x = \frac{-1}{3}$ is the root of $3x + 1$

$$(2) f(x) = x^2 - 1, x = (1, -1)$$

we know that,

$$f(x) = x^2 - 1$$

Given that $x = (1, -1)$

substitute $x = 1$ in $f(x)$

$$f(1) = 1^2 - 1$$

$$= 1 - 1$$

$$= 0$$

Now , substitute $x = (-1)$ in $f(x)$

$$f(-1) = (-1)^2 - 1$$

$$= 1 - 1$$

$$= 0$$

Since , the results when $x = (1, -1)$ are 0 they are the roots of the polynomial $f(x) = x^2 - 1$

$$(3) g(x) = 3x^2 - 2, x = \left(\frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$$

Sol :

We know that

$$g(x) = 3x^2 - 2$$

Given that , $x = \left(\frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$

Substitute $x = \frac{2}{\sqrt{3}}$ in $g(x)$

$$g\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2$$

$$= 3\left(\frac{4}{3}\right) - 2$$

$$= 4 - 2$$

$$= 2 \neq 0$$

Now, Substitute $x = \frac{-2}{\sqrt{3}}$ in $g(x)$

$$g\left(\frac{-2}{\sqrt{3}}\right) = 3\left(\frac{-2}{\sqrt{3}}\right)^2 - 2$$

$$= 3\left(\frac{4}{3}\right) - 2$$

$$= 4 - 2$$

$$= 2 \neq 0$$

Since, the results when $x = \left(\frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$ are not 0, they are roots of $3x^2 - 2$

$$(4) p(x) = x^3 - 6x^2 + 11x - 6, x = 1, 2, 3$$

Sol :

We know that ,

$$p(x) = x^3 - 6x^2 + 11x - 6$$

given that the values of x are 1, 2, 3

substitute $x = 1$ in $p(x)$

$$p(1) = 1^3 - 6(1)^2 + 11(1) - 6$$

$$= 1 - (6 * 1) + 11 - 6$$

$$= 1 - 6 + 11 - 6$$

$$= 0$$

Now, substitute $x = 2$ in $p(x)$

$$P(2) = 2^3 - 6(2)^2 + 11(2) - 6$$

$$= (2 * 3) - (6 * 4) + (11 * 2) - 6$$

$$= 8 - 24 - 22 - 6$$

$$= 0$$

Now, substitute $x = 3$ in $p(x)$

$$P(3) = 3^3 - 6(3)^2 + 11(3) - 6$$

$$= (3 * 3) - (6 * 9) + (11 * 3) - 6$$

$$= 27 - 54 + 33 - 6$$

$$= 0$$

Since , the result is 0 for $x = 1, 2, 3$ these are the roots of $x^3 - 6x^2 + 11x - 6$

$$(5) f(x) = 5x - \pi, x = \frac{4}{5}$$

we know that ,

$$f(x) = 5x - \pi$$

$$\text{Given that , } x = \frac{4}{5}$$

Substitute the value of x in $f(x)$

$$f\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi$$

$$= 4 - \pi$$

$$\neq 0$$

Since , the result is not equal to zero , $x = \frac{4}{5}$ is not the root of the polynomial $5x - \pi$

$$(6) f(x) = x^2, x = 0$$

Sol :

$$\text{we know that , } f(x) = x^2$$

Given that value of x is ' 0 '

Substitute the value of x in $f(x)$

$$f(0) = 0^2$$

$$= 0$$

Since, the result is zero , $x = 0$ is the root of x^2

$$(7) f(x) = lx + m, x = \frac{-m}{l}$$

Sol :

We know that,

$$f(x) = lx + m$$

$$\text{Given , that } x = \frac{-m}{l}$$

Substitute the value of x in $f(x)$

$$f\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m$$

$$= -m + m$$

$$= 0$$

Since, the result is 0 , $x = \frac{-m}{l}$ is the root of $lx + m$

$$(8) f(x) = 2x + 1, x = \frac{1}{2}$$

Sol :

We know that ,

$$f(x) = 2x + 1$$

$$\text{Given that } x = \frac{1}{2}$$

Substitute the value of x and f(x)

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1$$

$$= 1 + 1$$

$$= 2 \neq 0$$

Since , the result is not equal to zero

$$x = \frac{1}{2} \text{ is the root of } 2x + 1$$

Q3. If $x = 2$ is a root of the polynomial $f(x) = 2x^2 - 3x + 7a$, Find the value of a

Sol :

$$\text{We know that , } f(x) = 2x^2 - 3x + 7a$$

Given that $x = 2$ is the root of $f(x)$

Substitute the value of x in $f(x)$

$$f(2) = 2(2)^2 - 3(2) + 7a$$

$$= (2 * 4) - 6 + 7a$$

$$= 8 - 6 + 7a$$

$$= 7a + 2$$

Now, equate $7a + 2$ to zero

$$\Rightarrow 7a + 2 = 0$$

$$\Rightarrow 7a = -2$$

$$\Rightarrow a = \frac{-2}{7}$$

$$\text{The value of } a = \frac{-2}{7}$$

Q4. If $x = \frac{-1}{2}$ is zero of the polynomial $p(x) = 8x^3 - ax^2 - x + 2$, Find the value of a

Sol :

$$\text{We know that , } p(x) = 8x^3 - ax^2 - x + 2$$

$$\text{Given that the value of } x = \frac{-1}{2}$$

Substitute the value of x in $f(x)$

$$p\left(\frac{-1}{2}\right) = 8\left(\frac{-1}{2}\right)^3 - a\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) + 2$$

$$= -8\left(\frac{1}{8}\right) - a\left(\frac{1}{4}\right) + \frac{1}{2} + 2$$

$$= -1 - \left(\frac{a}{4} + \frac{1}{2} + 2\right)$$

$$= 1 - \left(\frac{a}{4} + \frac{1}{2}\right)$$

$$= \frac{3}{2} - \frac{a}{4}$$

To , find the value of a , equate $p\left(\frac{-1}{2}\right)$ to zero

$$p\left(\frac{-1}{2}\right) = 0$$

$$\frac{3}{2} - \frac{a}{4} = 0$$

On taking L.C.M

$$\frac{6-a}{4} = 0$$

$$\Rightarrow 6 - a = 0$$

$$\Rightarrow a = 6$$

Q5. If $x = 0$ and $x = -1$ are the roots of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$, Find the of a and b.

Sol :

We know that , $f(x) = 2x^3 - 3x^2 + ax + b$

Given , the values of x are 0 and -1

Substitute $x = 0$ in $f(x)$

$$f(0) = 2(0)^3 - 3(0)^2 + a(0) + b$$

$$= 0 - 0 + 0 + b$$

$$= b \quad \text{---- 1}$$

Substitute $x = (-1)$ in $f(x)$

$$f(-1) = 2(-1)^3 - 3(-1)^2 + a(-1) + b$$

$$= -2 - 3 - a + b$$

$$= -5 - a + b \quad \text{----- 2}$$

We need to equate equations 1 and 2 to zero

$$b = 0 \text{ and } -5 - a + b = 0$$

since, the value of b is zero

substitute $b = 0$ in equation 2

$$\Rightarrow -5 - a = -b$$

$$\Rightarrow -5 - a = 0$$

$$a = -5$$

the values of a and b are -5 and 0 respectively

Q6. Find the integral roots of the polynomial $f(x) = x^3 + 6x^2 + 11x + 6$

Sol :

Given , that $f(x) = x^3 + 6x^2 + 11x + 6$

Clearly we can say that, the polynomial $f(x)$ with an integer coefficient and the highest degree term coefficient which is known as leading factor is 1.

So, the roots of $f(x)$ are limited to integer factor of 6, they are

$\pm 1, \pm 2, \pm 3, \pm 6$

Let $x = -1$

$$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$$

$$= -1 + 6 - 11 + 6$$

$$= 0$$

Let $x = -2$

$$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6$$

$$= -8 - (6 * 4) - 22 + 6$$

$$= -8 + 24 - 22 + 6$$

$$= 0$$

Let $x = -3$

$$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6$$

$$= -27 - (6 * 9) - 33 + 6$$

$$= -27 + 54 - 33 + 6$$

$$= 0$$

But from all the given factors only -1 , -2 , -3 gives the result as zero .

So, the integral multiples of $x^3 + 6x^2 + 11x + 6$ are -1 , -2 , -3

Q7. Find the rational roots of the polynomial $f(x) = 2x^3 + x^2 - 7x - 6$

Sol :

Given that $f(x) = 2x^3 + x^2 - 7x - 6$

$f(x)$ is a cubic polynomial with an integer coefficient . If the rational root in the form of $\frac{p}{q}$, the values of p are limited to factors of 6 which are $\pm 1, \pm 2, \pm 3, \pm 6$

and the values of q are limited to the highest degree coefficient i.e 2 which are $\pm 1, \pm 2$

here, the possible rational roots are

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Let , $x = -1$

$$f(-1) = 2(-1)^3 + (-1)^2 - 7(-1) - 6$$

$$= -2 + 1 + 7 - 6$$

$$= -8 + 8$$

$$= 0$$

$$\text{Let, } x = 2$$

$$f(-2) = 2(2)^3 + (2)^2 - 7(2) - 6$$

$$= (2 * 8) + 4 - 14 - 6$$

$$= 16 + 4 - 14 - 6$$

$$= 20 - 20$$

$$= 0$$

$$\text{Let, } x = \frac{-3}{2}$$

$$f\left(\frac{-3}{2}\right) = 2\left(\frac{-3}{2}\right)^3 + \left(\frac{-3}{2}\right)^2 - 7\left(\frac{-3}{2}\right) - 6$$

$$= 2\left(\frac{-27}{8}\right) + \frac{9}{4} - 7\left(\frac{-3}{2}\right) - 6$$

$$= \left(\frac{-27}{4}\right) + \frac{9}{4} - \left(\frac{-21}{2}\right) - 6$$

$$= -6.75 + 2.25 + 10.5 - 6$$

$$= 12.75 - 12.75$$

$$= 0$$

But from all the factors only -1, 2 and $\frac{-3}{2}$ gives the result as zero

So, the rational roots of $2x^3 + x^2 - 7x - 6$ are -1, 2 and $\frac{-3}{2}$