$x_1 - 3x_2 - x_3 \leq 7$

$x_1 + x_3 - 3x_4 = -2$

$x_1, x_4 \geq 0$, x_2, x_3 are unrestricted. (9)

6. (a) Use dominance to solve the following game:

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$ (9)

(b) Transform the matrix game

$$\begin{bmatrix} 3 & 5 & 2 & 8 \\ 2 & 6 & 5 & 1 \\ 4 & 7 & 3 & 9 \end{bmatrix}$$

into its corresponding primal and dual linear programming problem. (9)

(c) Find the range of values $p\&q$ which will render (2,2) a saddle point of the game:

$$\begin{bmatrix} 0 & 2 & 3 \\ 8 & 5 & q \\ 2 & p & 4 \end{bmatrix}$$ (9)

[This question paper contains 12 printed pages.]

Your Roll No. ....

1406

B.A. Prog./III E-I

MATHEMATICS–Paper III

(Selected Topics in Mathematics)

(NC–Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all selecting two parts from each question.

Unit I and Unit II are compulsory and contain four questions.

In Unit III choose any of the options and attempt two questions from the same.

Marks are indicated against each question.

P.T.O.

Unit-I
(Real Analysis)

1. (a) Prove that the intersection of a finite number of open sets is open. What happens if the family consists of infinite number of open sets? Justify your answer. (7)

(b) Define limit point of a set S of real numbers. Determine the limit points of the following sets:

(i) The set Q of all rational numbers

(ii) The set Z of all integers. (7)

(c) Prove that a function continuous on a closed interval is bounded on the interval. (7)

2. (a) Prove that every convergent sequence is bounded. Is every bounded sequence convergent? Justify. (9)

(b) Test the convergence of any two of the following series:

(i) $\sum(\sqrt{n+1}-\sqrt{n})$

(ii) $\sum_{n=2}^{\infty}\frac{1}{(\log n)^n}$

(iii) $\sum\frac{1.3.5..........(2n-1)}{2.4.6.......2n}\frac{1}{n}$. (9)

(c) Define an alternating series and state the Leibnitz test for their convergence. Hence check for convergence of the following series:

$$\sum\frac{(-1)^{n+1}}{n^p} \quad for \quad p>0.$$ (9)

3. (a) Show that the sequence $\{f_n\}$, where

$$f_n=\frac{n}{x+n}$$

is uniformly convergent in [0,k] whatever k may be, but not uniformly convergent in $[0,\infty]$. (7)

(b) Show that the integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

converges if and only if $m > 0, n > 0$. (7)

(c) Prove that

$$\int_0^{\pi/2} \sin^p x\cos^q x\,dx=\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right),\ p>-1, q>-1$$

and hence deduce that

$$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$$ (7)

Unit-II
(Computer Programming)

4. (a) Write a program to read and check the equality of two matrices. (9)

(b) Write a program to calculate the factorial of an integer. (9)

(c) (i) How do the computer detects whether an identifier used in a program is a constant or a variable or an array? Support your answer with example, for all three.

(ii) How do the break statement differs with the continue statement. Show the difference with one simple example. (9)

Unit III(1)
(Numerical Analysis)
(Use of scientific calculator is allowed)

5. (a) Find the interval in which the smallest positive root of the equation

$x^3 - x - 4 = 0$ lies.

Determine the root correct to two decimal places, using the bisection method. (9)

(b) Find the NewtonRaphson iterative Method, for finding $N^{1/2}$, where

N is a positive real number and apply the method to N = 18, to obtain the result correct to two decimal places. (9)

(c) Solve the system

$$\begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix}$$

by Gauss Elimination Method. (9)

6. (a) Find the values of y when x = 3.2 and when x = 2.9 from the following data using Gauss's forward formula:

x	2.0	2.5	3.0	3.5	4.0
y	246.2	409.3	537.2	636.3	715.9

(9)

(b) Evaluate the integral

$$\int_0^1 \frac{2x}{1+x^4}dx$$

Using Gauss- Legendre 3-point formula. (9)

(c) Derive the basic Newton-Cotes formula:

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}(f_0 + f_1) - \frac{1}{12}h^3 f''(\xi)$$

where $f_0 = f(x_0), f_1 = f(x_1)$ *and* $x_0 < \xi < x_1$. (9)

Unit-III(2)

(Discrete Mathematics)

5. (a) Show that in a graph with n-vertices, if there is a path from vertex v_1 to vertex v_2, then there is a path of no more than n-l edges from vertex v_1 *to* vertex v_2. (9)

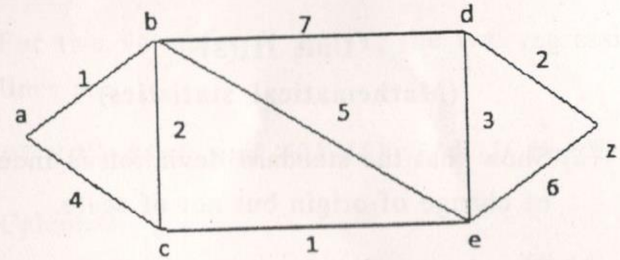
(b) Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?

(i) K_5

(ii) $K_{3,3}$

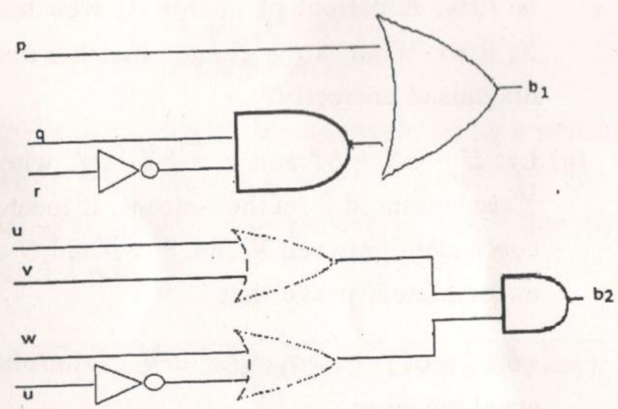
(iii) K_6 (9)

(c) Obtain a shortest path from the vertex a to vertex z in the weighted graph.



(9)

6. (a) Write Boolean expressions b_1 and b_2 represented by the following electronic circuit:



(9)

(b) Prove the equation:

$$(a \vee \bar{b} \vee d) \wedge (a \vee b \vee \bar{c}) \wedge (a \vee c \vee d) = a \vee (b \wedge d) \vee (\bar{c} \wedge d)$$

(9)

(c) Show that the following statement is a tautology:

$$(A \to B) \to [A \to (A \wedge B)]$$

(9)

Unit III(3)
(Mathematical statistics)

5. (a) Show that the standard deviation is independent of change of origin but not of scale. (9)

(b) The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? (9)

(c) Let $U = aX + bY$ and $V = bX - aY$, where X and Y are measured from their means. If the correlation coefficient between X and Y is ρ and U and V are uncorrelated, prove that

$ab(\sigma_X^2 - \sigma_Y^2) = \rho\sigma_X\sigma_Y(a^2 - b^2)$, symbols having usual meaning.

(9)

6. (a) Find the first four moments about origin of binomial distribution.

(9)

(b) Prove that for a normal distribution, the quartile deviation, mean deviation and the standard deviation are approximately in the ratio 10: 12: !5.

(9)

(c) For two variables X and Y, the two regression lines are

$8X-10Y+66=0$ and $40X-18Y=214$. If $Var(X)=9$

Calculate

(i) the mean values of X and Y

(ii) the correlation coefficient between X and Y.

(9)

Unit-III (4)
(Mechanics)

5. (a) Three forces P, Q, R in one plane act on a particle, the angles between Q and R, R and P, P and Q being α, β and γ respectively. Show that their resultant is

$$\sqrt{P^2 + Q^2 + R^2 - 2QR\cos\alpha - 2RP\cos\beta - 2PQ\cos\gamma}$$

(9)

(b) A particle of weight W rest on a rough horizontal plane. If the angle of friction be μ, prove that the least force which will just make it move along the plane is

$P = W\sin\mu$.

(9)

(c) Find the centre of gravity of a solid uniform hemisphere of radius 'a'.

(9)

6. (a) A particle is moving with S.H.M of amplitude a and periodic time T. Prove that

$$\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}.$$

(9)

(b) A particle moves under influence of the centre which attracts with a force:

$\left(\frac{b}{r^2}+\frac{c}{r^2}\right)$, b and c being positive constants and r the distance from the centre. The particle moves in a circular orbit of radius a. Prove that the motion is stable if and only if, $a^2b > c$.

(9)

(c) A gun is fired from a moving platform, and the ranges of the shot are observed to be R and S, when the platform is moving backward and forward respectively, with velocity v.

Prove that the elevation of the gun is :

$$\tan^{-1}\left[\frac{g(R-S)^2}{4v^2(R+S)}\right]$$

(9)

Unit III(5)

(Theory of Games)

5. (a) What is meant by a feasiblesolution of a linear programming problem?

Write down the following LPP in standard form:

Maximize: $Z = 3x_1 + 2x_2 + 5x_3$

subject to :

$2x_1 - 3x_2 \leq 3$

$x_1 + 2x_2 + 3x_3 \geq 5$

$3x_1 + 2x_3 \leq 2$

$x_1, x_2, x_2 \geq 0.$

(9)

(b) Use Big- M method to solve following problem:

Minimize: $Z = 4x_1 + x_2$

subject to :

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

(9)

(c) Find the dual of the LPP:

Maximize: $Z = x_1 - x_2 + 3x_3 + 2x_4$

subject to : $x_1 + x_2 \geq -1$