

$$Q1. x^3 + x - 3x^2 - 3$$

SOLUTION :

Taking x common in $x^3 + x$

$$=x(x^2 + 1) - 3x^2 - 3$$

Taking -3 common in $-3x^2 - 3$

$$=x(x^2 + 1) - 3(x^2 + 1)$$

Now , we take $(x^2 + 1)$ common

$$=(x^2 + 1)(x - 3)$$

$$\therefore x^3 + x - 3x^2 - 3 = (x^2 + 1)(x - 3)$$

$$Q2. a(a + b)^3 - 3a^2b(a + b)$$

SOLUTION :

Taking $(a + b)$ common in the two terms

$$= (a + b) \{a(a + b)^2 - 3a^2b\}$$

Now, using $(a + b)^2 = a^2 + b^2 + 2ab$

$$= (a + b) \{a(a^2 + b^2 + 2ab) - 3a^2b\}$$

$$= (a + b) \{a^3 + ab^2 + 2a^2b - 3a^2b\}$$

$$= (a + b) \{a^3 + ab^2 - a^2b\}$$

$$= (a + b) \{a^2 + b^2 - ab\}$$

$$= (a + b)(a^2 + b^2 - ab)$$

$$\therefore a(a + b)^3 - 3a^2b(a + b) = a(a + b)(a^2 + b^2 - ab)$$

$$Q3. x(x^3 - y^3) + 3xy(x - y)$$

SOLUTION :

Elaborating $x^3 - y^3$ using the identity $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$=x(x - y)(x^2 + xy + y^2) + 3xy(x - y)$$

Taking common $x(x - y)$ in both the terms

$$=x(x - y)(x^2 + xy + y^2 + 3y)$$

$$\therefore x(x^3 - y^3) + 3xy(x - y) = x(x - y)(x^2 + xy + y^2 + 3y)$$

$$Q4. a^2x^2 + (ax^2 + 1)x + a$$

SOLUTION :

$$\text{We multiply } x(ax^2 + 1) = ax^3 + x$$

$$= a^2x^2 + ax^3 + x + a$$

$$\text{Taking common } ax^2 \text{ in } (a^2x^2 + ax^3) \text{ and } 1 \text{ in } (x + a)$$

$$= ax^2(a + x) + 1(x + a)$$

$$= ax^2(a + x) + 1(a + x)$$

$$\text{Taking } (a + x) \text{ common in both the terms}$$

$$= (a + x)(ax^2 + 1)$$

$$\therefore a^2x^2 + (ax^2 + 1)x + a = (a + x)(ax^2 + 1)$$

$$Q5. x^2 + y - xy - x$$

SOLUTION :

On rearranging

$$x^2 - xy - x + y$$

$$\text{Taking } x \text{ common in the } (x^2 - xy) \text{ and } -1 \text{ in } (-x + y)$$

$$= x(x - y) - 1(x - y)$$

$$\text{Taking } (x - y) \text{ common in the terms}$$

$$= (x - y)(x - 1)$$

$$\text{latex}::[\text{latex}]x^2 + y - xy - x = (x - y)(x - 1)$$

$$Q6. x^3 - 2x^2y + 3xy^2 - 6y^3$$

SOLUTION :

$$\text{Taking } x^2 \text{ common in } (x^3 - 2x^2y) \text{ and } +3y^2 \text{ common in } (3xy^2 - 6y^3)$$

$$= x^2(x - 2y) + 3y^2(x - 2y)$$

$$\text{Taking } (x - 2y) \text{ common in the terms}$$

$$= (x - 2y)(x^2 + 3y^2)$$

$$\text{latex}::[\text{latex}]x^3 - 2x^2y + 3xy^2 - 6y^3 = (x - 2y)(x^2 + 3y^2)$$

$$Q7. 6ab - b^2 + 12ac - 2bc$$

SOLUTION :

Taking b common in $(6ab - b^2)$ and $2c$ in $(12ac - 2bc)$

$$=b(6a - b) + 2c(6a - b)$$

Taking $(6a - b)$ common in the terms

$$=(6a - b)(b + 2c)$$

$$\therefore 6ab - b^2 + 12ac - 2bc = (6a - b)(b + 2c)$$

$$Q8. \left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right] + 6$$

SOLUTION :

$$=x^2 + \frac{1}{x^2} - 4x - \frac{4}{x} + 4 + 2$$

$$=x^2 + \frac{1}{x^2} + 4 + 2 - \frac{4}{x} - 4x$$

$$=(x^2) + \left(\frac{1}{x}\right)^2 + (-2)^2 + 2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-2) + 2(-2)x$$

Using identity

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$$

We get,

$$=\left[x + \frac{1}{x} + (-2)\right]^2$$

$$=\left[x + \frac{1}{x} - 2\right]^2$$

$$=\left[x + \frac{1}{x} - 2\right]\left[x + \frac{1}{x} - 2\right]$$

$$\therefore \left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right] + 6 = \left[x + \frac{1}{x} - 2\right]\left[x + \frac{1}{x} - 2\right]$$

$$Q9. x(x-2)(x-4) + 4x - 8$$

SOLUTION :

$$=x(x-2)(x-4) + 4(x-2)$$

Taking $(x-2)$ common in both the terms

$$=(x-2)\{x(x-4) + 4\}$$

$$=(x-2)\{x^2 - 4x + 4\}$$

Now splitting the middle term of $x^2 - 4x + 4$

$$=(x-2)\{x^2 - 2x - 2x + 4\}$$

$$=(x-2)\{x(x-2) - 2(x-2)\}$$

$$=(x-2)\{(x-2)(x-2)\}$$

$$=(x-2)(x-2)(x-2)$$

$$=(x-2)^3$$

$$\therefore x(x-2)(x-4)+4x-8=(x-2)^3$$

$$Q10. (x+2)(x^2+25)-10x^2-20x$$

SOLUTION :

$$(x+2)(x^2+25)-10x(x+2)$$

Taking $(x+2)$ common in both the terms

$$=(x+2)(x^2+25-10x)$$

$$=(x+2)(x^2-10x+25)$$

Splitting the middle term of $(x^2-10x+25)$

$$=(x+2)(x^2-5x-5x+25)$$

$$=(x+2)\{x(x-5)-5(x-5)\}$$

$$=(x+2)(x-5)(x-5)$$

$$\therefore (x+2)(x^2+25)-10x^2-20x=(x+2)(x-5)(x-5)$$

$$Q11. 2a^2+2\sqrt{6}ab+3b^2$$

SOLUTION :

$$=(\sqrt{2}a)^2+2\times\sqrt{2}a\times\sqrt{3}b+(\sqrt{3}b)^2$$

Using the identity $(p+q)^2=p^2+q^2+2pq$

$$=(\sqrt{2}a+\sqrt{3}b)^2$$

$$=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$$

$$\therefore 2a^2+2\sqrt{6}ab+3b^2=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$$

$$Q12. (a-b+c)^2+(b-c+a)^2+2(a-b+c)\times(b-c+a)$$

SOLUTION :

Let $(a-b+c)=x$ and $(b-c+a)=y$

$$= x^2 + y^2 + 2xy$$

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

$$= (x + y)^2$$

Now, substituting x and y

$$(a - b + c + b - c + a)^2$$

Cancelling -b, +b & +c, -c

$$= (2a)^2$$

$$= 4a^2$$

$$\therefore (a - b + c)^2 + (b - c + a)^2 + 2(a - b + c) \times (b - c + a) = 4a^2$$

$$Q13. a^2 + b^2 + 2(ab + bc + ca)$$

SOLUTION :

$$= a^2 + b^2 + 2ab + 2bc + 2ca$$

Using the identity $(p + q)^2 = p^2 + q^2 + 2pq$

We get,

$$= (a + b)^2 + 2bc + 2ca$$

$$= (a + b)^2 + 2c(b + a)$$

$$\text{Or } (a + b)^2 + 2c(a + b)$$

Taking (a + b) common

$$= (a + b)(a + b + 2c)$$

$$\therefore a^2 + b^2 + 2(ab + bc + ca) = (a + b)(a + b + 2c)$$

$$Q14. 4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$$

SOLUTION :

$$\text{Let } (x - y) = x, (x + y) = y$$

$$= 4x^2 - 12xy + 9y^2$$

Splitting the middle term $-12 = -6 - 6$ also $4 \times 9 = -6 \times -6$

$$= 4x^2 - 6xy - 6xy + 9y^2$$

$$= 2x(2x - 3y) - 3y(2x - 3y)$$

$$= (2x - 3y)(2x - 3y)$$

$$= (2x - 3y)^2$$

Substituting $x = x - y$ & $y = x + y$

$$=[2(x - y) - 3(x + y)]^2 = [2x - 2y - 3x - 3y]^2$$

$$=(2x - 3x - 2y - 3y)^2$$

$$=[-x - 5y]^2$$

$$=[(-1)(x + 5y)]^2$$

$$=(x + 5y)^2 \quad [\because (-1)^2 = 1]$$

$$\therefore 4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2 = (x + 5y)^2$$

$$Q 15. a^2 - b^2 + 2bc - c^2$$

SOLUTION :

$$a^2 - (b^2 - 2bc + c^2)$$

$$\text{Using the identity } (a - b)^2 = a^2 + b^2 - 2ab$$

$$= a^2 - (b - c)^2$$

$$\text{Using the identity } a^2 - b^2 = (a + b)(a - b)$$

$$= (a + b - c)(a - (b - c))$$

$$= (a + b - c)(a - b + c)$$

$$\therefore a^2 - b^2 + 2bc - c^2 = (a + b - c)(a - b + c)$$

$$Q 16. a^2 + 2ab + b^2 - c^2$$

SOLUTION :

$$\text{Using the identity } (p + q)^2 = p^2 + q^2 + 2pq$$

$$= (a + b)^2 - c^2$$

$$\text{Using the identity } p^2 - q^2 = (p + q)(p - q)$$

$$= (a + b + c)(a + b - c)$$

$$\therefore a^2 + 2ab + b^2 - c^2 = (a + b + c)(a + b - c)$$

$$Q 17. a^2 + 4b^2 - 4ab - 4c^2$$

SOLUTION :

On rearranging

$$= a^2 - 4ab + 4b^2 - 4c^2$$

$$= (a)^2 - 2 \times a \times 2b + (2b)^2 - 4c^2$$

$$\text{Using the identity } (a - b)^2 = a^2 + b^2 - 2ab$$

$$= (a - 2b)^2 - 4c^2$$

$$= (a - 2b)^2 - (2c)^2$$

$$\text{Using the identity } a^2 - b^2 = (a + b)(a - b)$$

$$= (a - 2b - 2c)(a - 2b + 2c)$$

$$\therefore a^2 + 4b^2 - 4ab - 4c^2 = (a - 2b - 2c)(a - 2b + 2c)$$

$$Q18. xy^9 - yx^9$$

SOLUTION :

$$= xy(y^8 - x^8)$$

$$= xy((y^4)^2 - (x^4)^2)$$

$$\text{Using the identity } p^2 - q^2 = (p + q)(p - q)$$

$$= xy(y^4 + x^4)(y^4 - x^4)$$

$$= xy(y^4 + x^4)((y^2)^2 - (x^2)^2)$$

$$\text{Using the identity } p^2 - q^2 = (p + q)(p - q)$$

$$= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$$

$$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$$

$$= xy(x^4 + y^4)(x^2 + y^2)(x + y)(-1)(x - y)$$

$$\therefore (y - x) = -1(x - y)$$

$$= -xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

$$\therefore xy^9 - yx^9 = -xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

$$Q19. x^4 + x^2y^2 + y^4$$

SOLUTION :

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$$= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$$

$$= x^4 + 2x^2y^2 + y^4 - x^2y^2$$

$$= (x^2)^2 + 2 \times x^2 \times y^2 + (y^2)^2 - (xy)^2$$

Using the identity $(p + q)^2 = p^2 + q^2 + 2pq$

$$= (x^2 + y^2)^2 - (xy)^2$$

Using the identity $p^2 - q^2 = (p + q)(p - q)$

$$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$$

$$\therefore x^4 + x^2y^2 + y^4 = (x^2 + y^2 + xy)(x^2 + y^2 - xy)$$

$$Q20. x^2 - y^2 - 4xz + 4z^2$$

SOLUTION :

On rearranging the terms

$$= x^2 - 4xz + 4z^2 - y^2$$

$$= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$$

Using the identity $x^2 - 2xy + y^2 = (x - y)^2$

$$= (x - 2z)^2 - y^2$$

Using the identity $p^2 - q^2 = (p + q)(p - q)$

$$= (x - 2z + y)(x - 2z - y)$$

$$\therefore x^2 - y^2 - 4xz + 4z^2 = (x - 2z + y)(x - 2z - y)$$

$$Q21. x^2 + 6\sqrt{2}x + 10$$

SOLUTION :

Splitting the middle term ,

$$= x^2 + 5\sqrt{2}x + \sqrt{2}x + 10 \quad [\because 6\sqrt{2} = 5\sqrt{2} + \sqrt{2} \text{ and } 5\sqrt{2} \times \sqrt{2} = 10]$$

$$= x(x + 5\sqrt{2}) + \sqrt{2}(x + 5\sqrt{2})$$

$$= (x + 5\sqrt{2})(x + \sqrt{2})$$

$$\therefore x^2 + 6\sqrt{2}x + 10 = (x + 5\sqrt{2})(x + \sqrt{2})$$

$$Q22. x^2 - 2\sqrt{2}x - 30$$

SOLUTION :

Splitting the middle term,

$$= x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$$

$$[\because -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2} \text{ also } -5\sqrt{2} \times 3\sqrt{2} = -30]$$

$$= x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$$

$$= (x - 5\sqrt{2})(x + 3\sqrt{2})$$

$$\therefore x^2 - 2\sqrt{2}x - 30 = (x - 5\sqrt{2})(x + 3\sqrt{2})$$

$$Q23. x^2 - \sqrt{3}x - 6$$

SOLUTION :

Splitting the middle term,

$$= x^2 - 2\sqrt{3}x + \sqrt{3}x - 6 \quad [\because -\sqrt{3} = -2\sqrt{3} + \sqrt{3} \text{ also } -2\sqrt{3} \times \sqrt{3} = -6]$$

$$= x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$$

$$= (x - 2\sqrt{3})(x + \sqrt{3})$$

$$\therefore x^2 - \sqrt{3}x - 6 = (x - 2\sqrt{3})(x + \sqrt{3})$$

$$Q24. x^2 + 5\sqrt{5}x + 30$$

SOLUTION :

Splitting the middle term,

$$= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30 \quad [\because 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} \text{ also } 2\sqrt{5} \times 3\sqrt{5} = 30]$$

$$= x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$$

$$= (x + 2\sqrt{5})(x + 3\sqrt{5})$$

$$\therefore x^2 + 5\sqrt{5}x + 30 = (x + 2\sqrt{5})(x + 3\sqrt{5})$$

$$Q25. x^2 + 2\sqrt{3}x - 24$$

SOLUTION :

Splitting the middle term,

$$=x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$$

$$[\because 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} \text{ also } 4\sqrt{3}(-2\sqrt{3}) = -24]$$

$$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$$

$$= (x + 4\sqrt{3})(x - 2\sqrt{3})$$

$$\therefore x^2 + 2\sqrt{3}x - 24 = (x + 4\sqrt{3})(x - 2\sqrt{3})$$

$$Q26. 2x^2 - \frac{5}{6}x + \frac{1}{12}$$

SOLUTION :

Splitting the middle term,

$$= 2x^2 - \frac{x}{2} - \frac{x}{3} + \frac{1}{12} \quad [\because -\frac{5}{6} = -\frac{1}{2} - \frac{1}{3} \text{ also } -\frac{1}{2} \times -\frac{1}{3} = 2 \times \frac{1}{12}]$$

$$= x(2x - \frac{1}{2}) - \frac{1}{6}(2x - \frac{1}{2})$$

$$= (2x - \frac{1}{2})(x - \frac{1}{6})$$

$$\therefore 2x^2 - \frac{5}{6}x + \frac{1}{12} = (2x - \frac{1}{2})(x - \frac{1}{6})$$

$$Q27. x^2 + \frac{12}{35}x + \frac{1}{35}$$

SOLUTION :

Splitting the middle term,

$$= x^2 + \frac{5}{35}x + \frac{7}{35}x + \frac{1}{35} \quad [\because \frac{12}{35} = \frac{5}{35} + \frac{7}{35} \text{ and } \frac{5}{35} \times \frac{7}{35} = \frac{1}{35}]$$

$$= x^2 + \frac{x}{7} + \frac{x}{5} + \frac{1}{35}$$

$$= x(x + \frac{1}{7}) + \frac{1}{5}(x + \frac{1}{7})$$

$$= (x + \frac{1}{7})(x + \frac{1}{5})$$

$$\therefore x^2 + \frac{12}{35}x + \frac{1}{35} = (x + \frac{1}{7})(x + \frac{1}{5})$$

$$Q28. 21x^2 - 2x + \frac{1}{21}$$

SOLUTION :

$$= (\sqrt{21}x)^2 - 2\sqrt{21}x \times \frac{1}{\sqrt{21}} + \left(\frac{1}{\sqrt{21}}\right)^2$$

Using the identity $(x - y)^2 = x^2 + y^2 - 2xy$

$$= \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$$

$$\therefore 21x^2 - 2x + \frac{1}{21} = \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$$

Q29. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

SOLUTION :

Splitting the middle term,

$$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5} \quad [\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}]$$

$$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$$

$$= (\sqrt{5}x + 3)(5x + \sqrt{5})$$

$$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$$

Q30. $2x^2 + 3\sqrt{5}x + 5$

SOLUTION :

Splitting the middle term,

$$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$$

$$= 2x(x + \sqrt{5}) + \sqrt{5}(x + \sqrt{5})$$

$$= (x + \sqrt{5})(2x + \sqrt{5})$$

$$\therefore 2x^2 + 3\sqrt{5}x + 5 = (x + \sqrt{5})(2x + \sqrt{5})$$

Q31. $9(2a - b)^2 - 4(2a - b) - 13$

SOLUTION :

Let $2a - b = x$

$$= 9x^2 - 4x - 13$$

Splitting the middle term,

$$= 9x^2 - 13x + 9x - 13$$

$$= x(9x - 13) + 1(9x - 13)$$

$$= (9x - 13)(x + 1)$$

Substituting $x = 2a - b$

$$= [9(2a - b) - 13](2a - b + 1)$$

$$= (18a - 9b - 13)(2a - b + 1)$$

$$\therefore 9(2a - b)^2 - 4(2a - b) - 13 = (18a - 9b - 13)(2a - b + 1)$$

$$Q\ 32. \ 7(x - 2y)^2 - 25(x - 2y) + 12$$

SOLUTION :

Let $x - 2y = P$

$$= 7P^2 - 25P + 12$$

Splitting the middle term,

$$= 7P^2 - 21P - 4P + 12$$

$$= 7P(P - 3) - 4(P - 3)$$

$$= (P - 3)(7P - 4)$$

Substituting $P = x - 2y$

$$= (x - 2y - 3)(7(x - 2y) - 4)$$

$$= (x - 2y - 3)(7x - 14y - 4)$$

$$\therefore 7(x - 2y)^2 - 25(x - 2y) + 12 = (x - 2y - 3)(7x - 14y - 4)$$

$$Q\ 33. \ 2(x + y)^2 - 9(x + y) - 5$$

SOLUTION :

Let $x + y = z$

$$= 2z^2 - 9z - 5$$

Splitting the middle term,

$$= 2z^2 - 10z + z - 5$$

$$= 2z(z - 5) + 1(z - 5)$$

$$= (z - 5)(2z + 1)$$

Substituting $z = x + y$

$$= (x + y - 5)(2(x + y) + 1)$$

$$= (x + y - 5)(2x + 2y + 1)$$

$$\therefore 2(x + y)^2 - 9(x + y) - 5 = (x + y - 5)(2x + 2y + 1)$$

Q34 . Give the possible expression for the length & breadth of the rectangle having $35y^2 - 13y - 12$ as its area.

SOLUTION :

Area is given as $35y^2 - 13y - 12$

Splitting the middle term,

$$\text{Area} = 35y^2 + 21y - 15y - 12$$

$$= 7y(5y + 4) - 3(5y + 4)$$

$$= (5y + 4)(7y - 3)$$

We also know that area of rectangle = length $\times$ breadth

$$\therefore \text{Possible length} = (5y + 4) \text{ and breadth} = (7y - 3)$$

$$\text{Or possible length} = (7y - 3) \text{ and breadth} = (5y + 4)$$

Q35 . What are the possible expression for the cuboid having volume $3x^2 - 12x$.

SOLUTION :

$$\text{Volume} = 3x^2 - 12x$$

$$= 3x(x - 4)$$

$$= 3 \times x(x - 4)$$

Also volume = Length $\times$ Breadth $\times$ Height

$$\therefore \text{Possible expression for dimensions of cuboid are} = 3, x, (x - 4)$$