

$$Q 1. a^3 + 8b^3 + 64c^3 - 24abc$$

SOLUTION :

$$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$$

$$= (a + 2b + 4c)(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$$

$$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$$

$$\therefore a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$$

$$Q 2. x^3 - 8y^3 + 27z^3 + 18xyz$$

SOLUTION :

$$= x^3 - (2y)^3 + (3z)^3 - 3 \times x \times (-2y)(3z)$$

$$= (x + (-2y) + 3z)(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x))$$

$$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$$

$$\therefore x^3 - 8y^3 + 27z^3 + 18xyz = (x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$$

$$Q 3. \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$$

SOLUTION :

$$= \left(\frac{x}{3}\right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3}(-y)(5z)$$

$$= \left(\frac{x}{3} + (-y) + 5z\right)\left(\left(\frac{x}{3}\right)^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z\left(\frac{x}{3}\right)\right)$$

$$= \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3}\right)$$

$$\therefore \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3}\right)$$

$$Q 4. 8x^3 + 27y^3 - 216z^3 + 108xyz$$

SOLUTION :

$$= (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$$

$$\begin{aligned}
&= (2x + 3y + (-6z))((2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)2x) \\
&= (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx) \\
\therefore 8x^3 + 27y^3 - 216z^3 + 108xyz &= (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)
\end{aligned}$$

Q 5. $125 + 8x^3 - 27y^3 + 90xy$

SOLUTION :

$$\begin{aligned}
&= (5)^3 + (2x)^3 + (-3y)^3 - 3 \times 5 \times 2x \times (-3y) \\
&= (5 + 2x + (-3y))(5^2 + (2x)^2 + (-3y)^2 - 5(2x) - 2x(-3y) - (-3y)5) \\
&= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y) \\
\therefore 125 + 8x^3 - 27y^3 + 90xy &= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)
\end{aligned}$$

Q 6. $(3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$

SOLUTION :

Let $(3x - 2y) = a$, $(2y - 4z) = b$, $(4z - 3x) = c$

$$\therefore a + b + c = 3x - 2y + 2y - 4z + 4z - 3x = 0$$

$$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$$

$$= 3(3x - 2y)(2y - 4z)(4z - 3x)$$

$$\therefore (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$$

Q 7. $(2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

SOLUTION :

Let $2x - 3y = a$, $4z - 2x = b$, $3y - 4z = c$

$$\therefore a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$$

$$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$$

$$= 3(2x - 3y)(4z - 2x)(3y - 4z)$$

$$\therefore (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z)$$

Q 8. $\left[\frac{x}{2} + y + \frac{z}{3}\right]^3 + \left[\frac{x}{3} - \frac{2y}{3} + z\right]^3 + \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right]^3$

SOLUTION :

$$\text{Let } \left[\frac{x}{2} + y + \frac{z}{3}\right] = a, \left[\frac{x}{3} - \frac{2y}{3} + z\right] = b, \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right] = c$$

$$a + b + c = \frac{x}{2} + y + \frac{z}{3} + \frac{x}{3} - \frac{2y}{3} + z - \frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}$$

$$a + b + c = \left(\frac{x}{2} + \frac{x}{3} - \frac{5x}{6}\right) + \left(y - \frac{2y}{3} - \frac{y}{3}\right) + \left(\frac{z}{3} + z - \frac{4z}{3}\right)$$

$$a + b + c = \frac{3x}{6} + \frac{2x}{6} - \frac{5x}{6} + \frac{3y}{3} - \frac{2y}{3} - \frac{y}{3} + \frac{z}{3} + \frac{3z}{3} - \frac{4z}{3}$$

$$a + b + c = \frac{5x-5x}{6} + \frac{3y-3y}{3} + \frac{4z-4z}{3}$$

$$a + b + c = 0$$

$$\therefore a + b + c = 0 \quad \therefore a^3 + b^3 + c^3 = 3abc$$

$$= 3\left(\frac{x}{2} + y + \frac{z}{3}\right)\left(\frac{x}{3} - \frac{2y}{3} + z\right)\left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$$

$$\therefore \left[\frac{x}{2} + y + \frac{z}{3}\right]^3 + \left[\frac{x}{3} - \frac{2y}{3} + z\right]^3 + \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right]^3 = 3\left(\frac{x}{2} + y + \frac{z}{3}\right)\left(\frac{x}{3} - \frac{2y}{3} + z\right)\left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$$

$$Q 9. (a - 3b)^3 + (3b - c)^3 + (c - a)^3$$

SOLUTION :

$$\text{Let } a - 3b = x, 3b - c = y, c - a = z$$

$$x + y + z = a - 3b + 3b - c + c - a = 0$$

$$(\because x + y + z = 0) \quad \therefore x^3 + y^3 + z^3 = 3xyz$$

$$= 3(a - 3b)(3b - c)(c - a)$$

$$\therefore (a - 3b)^3 + (3b - c)^3 + (c - a)^3 = 3(a - 3b)(3b - c)(c - a)$$

$$Q 10. 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6abc}$$

SOLUTION :

$$= (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + c^3 - 3 \times \sqrt{2}a \times \sqrt{3}b \times c$$

$$= (\sqrt{2}a + \sqrt{3}b + c)((\sqrt{2}a)^2 + (\sqrt{3}b)^2 + c^2 - (\sqrt{2}a)(\sqrt{3}b) - (\sqrt{3}b)c - (\sqrt{2}a)c)$$

$$= (\sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$$

$$\therefore 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6abc} = (\sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$$

$$Q 11. 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$$

SOLUTION :

$$= (\sqrt{3}a)^3 + (-b)^3 - (\sqrt{5}c)^3 - 3(\sqrt{3}a)(-b)(-\sqrt{5}c)$$

=

$$(\sqrt{3}a + (-b) + (-\sqrt{5}c))((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - \sqrt{3}a(-b) - (-b)(-\sqrt{5}c)$$

$$- (-\sqrt{5}c)\sqrt{3}a)$$

$$= (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$$

$$\therefore 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$$

Q 12. $8x^3 - 125y^3 + 216 + 180xy$

SOLUTION :

$$= (2x)^3 + (-5y)^3 + 6^3 - 3 \times (2x)(-5y)(6)$$

$$= (2x + (-5y) + 6)((2x)^2 + (-5y)^2 + 6^2 - 2x \times (-5y) - (-5y)6 - 6(2x))$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

$$\therefore 8x^3 - 125y^3 + 216 + 180xy = (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

Q 13. $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

SOLUTION :

$$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c$$

$$= (\sqrt{2}a + 2\sqrt{2}b + c)((\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)c - (\sqrt{2}a)c)$$

$$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$$

$$\therefore 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$$

Q 14. Find the value of $x^3 + y^3 - 12xy + 64$ when $x + y = -4$.

SOLUTION :

$$= x^3 + y^3 + 64 - 12xy$$

$$\begin{aligned}
&= x^3 + y^3 + 4^3 - 3(x)(y)(4) \\
&= (x + y + 4)(x^2 + y^2 + 4^2 - xy - y \times 4 - 4 \times x) \\
&= (-4 + 4)(x^2 + y^2 + 16 - xy - 4y - 4x) \quad [\because x + y = -4] \\
&= 0 \\
\therefore x^3 + y^3 - 12xy + 64 &= 0
\end{aligned}$$

Q 15. MULTIPLY :

(i). $x^2 + y^2 + z^2 - xy + xz + yz$ by $x + y - z$

SOLUTION :

$$\begin{aligned}
&= (x^2 + y^2 + z^2 - xy + xz + yz)(x + y - z) \\
&= x^3 + y^3 + z^3 - 3xyz
\end{aligned}$$

(ii). $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $x - 2y - z$

SOLUTION :

$$\begin{aligned}
&x^2 + (-2y)^2 + (-z)^2 - (-2y)(-z) - (-z)(x) = x^3 + (-2y)^3 + (-z)^3 - 3x(-2y)(-z) \\
\Rightarrow x^2 + 4y^2 + z^2 + 2xy - 2yz + zx &= x^3 - 8y^3 - z^3 - 6xyz
\end{aligned}$$

(iii). $x^2 + 4y^2 + 2xy - 3x + 6y + 9$ by $(x - 2y + 3)$

SOLUTION :

$$\begin{aligned}
&(x)^2 + (-2y)^2 + (3)^2 - (x)(-2y) - (-2y)(3) - 3(x) = (x)^3 + (-2y)^3 + 3^3 - 3(x)(-2y)(3) \\
\Rightarrow x^2 + 4y^2 + 9 + 2xy + 6y - 3x &= x^3 - 8y^3 + 27 + 18xy
\end{aligned}$$

(iv). $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y + 4$

SOLUTION :

$$\begin{aligned}
&(3x)^2 + (5y)^2 + 4^2 - (-3x)(5y) - (5y)(4) - (4)(-3x) = (-3x)^3 + (5y)^3 + 4^3 \\
&\quad - 3(-3x)(5y)(4) \\
\Rightarrow 9x^2 + 25y^2 + 16 + 15xy - 20y + 12x &= -27x^3 + 125y^3 + 64 + 180xy
\end{aligned}$$