

Ch-1Real NumbersExercise 1.1

Let a be $2q+3$ and b be $2q+1$

ATQ,

$$2q+3 > 2q+1$$

$$\frac{a+b}{2} = \frac{2q+3+2q+1}{2} = \frac{4q+4}{2} = \frac{4(q+1)}{2}$$

$$= [2q+1]$$

$\therefore$ It is odd.

$$\frac{a-b}{2} = \frac{2q+3-2q+1}{2} = \frac{4}{2}$$

$$= [2]$$

$\therefore$ It is even.

Let the two consecutive no. be $n, n+1$

$$\text{Product} = n(n+1)$$

$$= n^2 + n$$

Positive integers are of the form $2q, 2q+1$ or $2q+2$ (Euclid's lemma)

When $n = 2q$

$$n^2 + n = (2q)^2 + 2q = 4q^2 + 2q = 2q(2q+1)$$

When $n = 2q+1$

$$\begin{aligned} n^2 + n &= (2q+1)^2 + 2q+1 = 4q^2 + 1 + 4q + 2q + 1 \\ &= 4q^2 + 6q + 2 = 2(2q^2 + 3q + 1) \end{aligned}$$

$\therefore$ The product of two consecutive positive integers is divisible by 2.

Q.3

Let the three consecutive no. be $n, n+1, n+2$
 Positive integers are of the form $6q, 6q+1,$
 $6q+2, 6q+3$ (Euclid's division lemma)

$$\text{If } n = 6q$$

$$n(n+1)(n+2) = 6q(6q+1)(6q+2)$$

$$\text{If } n = 6q+1$$

$$\begin{aligned} n(n+1)(n+2) &= (6q+1)(6q+2)(6q+3) \\ &= 6(6q+1)(3q+1)(2q+1) \end{aligned}$$

$$\text{If } n = 6q+2$$

$$\begin{aligned} n(n+1)(n+2) &= (6q+2)(6q+3)(6q+4) \\ &= 12(3q+1)(2q+1)(2q+3) \end{aligned}$$

$\therefore$ The product of three consecutive positive integers is divisible by 6.

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$$n^3 - n = n(n^2 - 1) \\ = n(n-1)(n+1)$$

$$\text{If } n = 6q$$

$$n(n-1)(n+1) = (6q)(6q-1)(6q+1) \\ = 6 [(q)(6q-1)(6q+1)] \\ = 6m$$

$$\text{where } m = [(q)(6q-1)(6q+1)]$$

$$\text{If } n = 6q+1$$

$$n(n-1)(n+1) = (6q+1)(6q)(6q+2) \\ = 6 [(6q+1)(q)(6q+2)] \\ = 6m$$

$$\text{where } m = [(6q+1)(q)(6q+2)]$$

$$\text{If } n = 6q+2$$

$$n(n-1)(n+1) = (6q+2)(6q+1)(6q+3) \\ = 6 [(3q+1)(6q+1)(2q+1)] \\ = 6m$$

$$\text{where } m = [(3q+1)(6q+1)(2q+1)]$$

$$\text{If } n = 6q+3$$

$$n(n-1)(n+1) = (6q+3)(6q+2)(6q+4) \\ = 6 [(2q+1)(3q+1)(6q+4)] \\ = 6m$$

$$\text{where } m = [(2q+1)(3q+1)(6q+4)]$$

$$\text{If } n = 6q+4$$

$$\begin{aligned}
 n(n-1)(n+1) &= (6q+4)(6q+3)(6q+5) \\
 &= 6[(3q+2)(2q+1)(6q+5)] \\
 &= 6m
 \end{aligned}$$

$$\text{where } m = [(3q+2)(2q+1)(6q+5)]$$

$$\begin{aligned}
 \text{If } n &= 6q+5 \\
 n(n-1)(n+1) &= (6q+5)(6q+4)(6q+6) \\
 &= 6[(6q+5)(6q+4)(q+1)] \\
 &= 6m
 \end{aligned}$$

$$\text{where } m = [(6q+5)(6q+4)(q+1)]$$

$\therefore$ For any positive integer n , $n^3 - n$ is divisible by 6.

Q.5

Let $n = 6q + 5$ for some integer q .

Positive integers can be of the form $3q, 3q+1, 3q+2$ ($b=3$)

$\therefore q$ can be $3q, 3q+1, 3q+2$

$$\begin{aligned}
 \text{If } q &= 3q \\
 n &= 6q+5 \\
 &= 6(3q)+5 \\
 &= 18q+5 \\
 &= (18q+3)+2 \\
 &= 3(6q+1)+2 \\
 &= 3m+2
 \end{aligned}$$

$$\text{where } m = 6q+1$$

$$\begin{aligned}
 \text{If } q &= 3q+1 \\
 n &= 6q+5 \\
 &= 6(3q+1)+5 \\
 &= 18q+6+5 \\
 &= 18q+11 \\
 &= 18q+9+2 \\
 &= 3(6q+3)+2 \\
 &= 3m+2
 \end{aligned}$$

$$\text{where } m = 6q+3$$

$$\begin{aligned}
 \text{If } q &= 3q+2 \\
 n &= 6q+5 \\
 &= 6(3q+2)+5 \\
 &= 18q+12+5
 \end{aligned}$$

$$\begin{aligned}
 &= 18q + 15 + 2 \\
 &= 3(6q + 5) + 2 \\
 &= 3m + 2
 \end{aligned}$$

$$\text{Where } m = 6q + 5$$

$\therefore$ If a positive integer is of the form $6q + 5$, then it is of the form $3q + 2$ for some integer q .

Converse

$$\text{Let } n = 3q + 2$$

$$\text{If } q = 6q$$

$$n = 3q + 2$$

$$= 3(6q) + 2$$

$$= 18q + 2$$

$$= 6(3q) + 2$$

$$= 6m + 2$$

$$\text{Where } m = 3q$$

$\therefore$ The converse is not true.

$$n = 5q + 1$$

Squaring on both sides

$$n^2 = (5q + 1)^2$$

$$n^2 = 25q^2 + 10q + 1$$

$$n^2 = 5(5q^2 + 2q) + 1$$

$$n^2 = 5m + 1$$

$$\text{Where } m = 5q^2 + 2q$$

$\therefore$ The square of any positive integer of the form $5q + 1$ is of the same form.

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Q.7 Let any positive integer of the form $3q$, $3q+1$ or $3q+2$ ($b=3$)

$$\begin{aligned} \text{If } n &= 3q \\ \text{Squaring both sides} \\ n^2 &= (3q)^2 \\ n^2 &= 9q^2 \\ n^2 &= 3(3q^2) \\ n^2 &= 3m \\ \text{Where } m &= 3q^2 \end{aligned}$$

$$\begin{aligned} \text{If } n &= 3q+1 \\ \text{Squaring both sides} \\ n^2 &= (3q+1)^2 \\ n^2 &= 9q^2 + 6q + 1 \\ n^2 &= 3(3q^2 + 2q) + 1 \\ n^2 &= 3m + 1 \\ \text{Where } m &= 3q^2 + 2q \end{aligned}$$

$$\begin{aligned} \text{If } n &= 3q+2 \\ \text{Squaring both sides} \\ n^2 &= (3q+2)^2 \\ n^2 &= 9q^2 + 12q + 4 \\ n^2 &= 9q^2 + 12q + 3 + 1 \\ n^2 &= 3(3q^2 + 4q + 1) + 1 \\ n^2 &= 3m + 1 \\ \text{Where } m &= 3q^2 + 4q + 1 \end{aligned}$$

$\therefore$ The square of any positive integer is of the form $3q$ or $3q+1$ but not of the form $3q+2$

Q.8 Let 'a' be any a positive integer.

$$b=4$$

According to Euclid's division lemma,

$$a = 4q + r$$

$$0 \leq r < 4$$

$\therefore$ The possible remainders are 0, 1, 2, 3

When $r = 0$,

$$a = 4q$$

$$a^2 = 16q^2$$

$$a^2 = 4(4q^2)$$

$$a^2 = 4m$$

$$\text{Where } m = 4q^2$$

When $r = 1$

$$a = 4q + 1$$

$$a^2 = 16q^2 + 8q + 1$$

$$a^2 = 4(4q^2 + 2q) + 1$$

$$a^2 = 4m + 1$$

$$\text{Where } m = 4q^2 + 2q$$

When $r = 2$

$$a = 4q + 2$$

$$a^2 = 16q^2 + 16q + 4$$

$$a^2 = 4(4q^2 + 4q + 1)$$

$$a^2 = 4m$$

$$\text{Where } m = 4q^2 + 4q + 1$$

When $r = 3$

$$a = 4q + 3$$

$$a^2 = 16q^2 + 24q + 9$$

$$a^2 = 16q^2 + 24q + 8 + 1$$

$$a^2 = 4(4q^2 + 8q + 2) + 1$$

$$a^2 = 4m + 1$$

$$\text{Where } m = 4q^2 + 8q + 2$$

$\therefore$ The square of any positive integer is of the form $4q$ or $4q + 1$ for some integer q .

Ex 9 Let 'a' be any positive integer.

$$a = 5$$

According to Euclid's division lemma,

$$a = 5q + r$$

$$0 \leq r < 5$$

$\therefore$ The possible remainders are 0, 1, 2, 3, 4.

When $r = 0$

$$a = 5q$$

$$a^2 = 25q^2$$

$$a^2 = 5(5q)^2$$

$$a = 5m$$

$$\text{Where } m = 5q^2$$

When $x = 1$

$$a = 5q + 1$$

$$a^2 = 25q^2 + 10q + 1$$

$$a^2 = 5(5q^2 + 2q) + 1$$

$$a^2 = 5m + 1$$

$$\text{where } m = 5q^2 + 2q$$

When $x = 3$

$$a = 5q + 3$$

$$a^2 = 25q^2 + 30q + 9$$

$$a^2 = 25q^2 + 30q + 5 + 4$$

$$a^2 = 5(5q^2 + 6q + 1) + 4$$

$$a^2 = 5m + 4$$

$$\text{where } m = 5q^2 + 6q + 1$$

When $x = 2$

$$a = 5q + 2$$

$$a^2 = 25q^2 + 20q + 4$$

$$a^2 = 5(5q^2 + 4q) + 4$$

$$a^2 = 5m + 4$$

$$\text{where } m = 5q^2 + 4q$$

When $x = 4$

$$a = 5q + 4$$

$$a^2 = 25q^2 + 40q + 16$$

$$a^2 = 25q^2 + 40q + 15 + 1$$

$$a^2 = 5(5q^2 + 8q + 3) + 1$$

$$a^2 = 5m + 1$$

$$\text{where } m = 5q^2 + 8q + 3$$

∴ The square of any positive integer is of the form $5q$, $5q + 1$ or $5q + 4$ for some integer q .

Q.10

Let 'n' be a positive integer.

$$b = 4$$

According to Euclid's division lemma,

$$n = 4m + r$$

$$0 \leq r < 4$$

∴ The possible remainders are 0, 1, 2, 3

∴ $4m + 1$ and $4m + 3$ are odd positive integers (not divisible by 2)

$$(4m + 1)^2 = 16m^2 + 1 + 8m$$

$$= 16m^2 + 8m + 1$$

$$= 8(2m^2 + m) + 1$$

$$= 8q + 1$$

$$\text{where } q = 2m^2 + m$$

$$\begin{aligned}
 (4m+3)^2 &= 16m^2 + 24m + 9 \\
 &= 16m^2 + 24m + 8 + 1 \\
 &= 8(2m^2 + 4m + 1) + 1 \\
 &= 8q + 1
 \end{aligned}$$

where $q = 2m^2 + 4m + 1$

$\therefore$ The square of an odd positive integer is of the form $8q + 1$, for some integer q .

Level - 2

12 Let 'a' be any positive integer.

$$b = 6$$

According to Euclid's division lemma,

$$a = 6q + r$$

$$0 \leq r < 6$$

$\therefore$ The possible remainders are 0, 1, 2, 3, 4, 5.

When $r = 0$

$$a = 6q$$

$$a^2 = 36q^2$$

$$a^2 = 6(6q^2)$$

$$a^2 = 6m$$

$$\text{Where } m = 6q^2$$

When $r = 1$

$$a = 6q + 1$$

$$a^2 = 36q^2 + 12q + 1$$

$$a^2 = 6(6q^2 + 2q) + 1$$

$$a^2 = 6m + 1$$

$$\text{Where } m = 6q^2 + 12q$$

Q.12

When $n=2$

$$a = 6q + 2$$

$$a^2 = 36q^2 + 24q + 4$$

$$a^2 = 6(6q^2 + 4q) + 4$$

$$a^2 = 6m + 4$$

$$\text{Where } m = 6q^2 + 4q$$

Where $n=4$

$$a = 6q + 4$$

$$a^2 = 36q^2 + 48q + 16$$

$$a^2 = 36q^2 + 48q + 12 + 4$$

$$a^2 = 6(6q^2 + 8q + 2) + 4$$

$$a^2 = 6m + 4$$

$$\text{Where } m = 6q^2 + 8q + 2$$

When $n=3$

$$a = 6q + 3$$

$$a^2 = 36q^2 + 36q + 9$$

$$a^2 = 36q^2 + 36q + 6 + 3$$

$$a^2 = 6(6q^2 + 6q + 1) + 3$$

$$a^2 = 6m + 3$$

$$\text{Where } m = 6q^2 + 6q + 1$$

When $n=5$

$$a = 6q + 5$$

$$a^2 = 36q^2 + 60q + 25$$

$$a^2 = 36q^2 + 60q + 24 + 1$$

$$a^2 = 6(6q^2 + 10q + 4) + 1$$

$$a^2 = 6m + 1$$

$$\text{Where } m = 6q^2 + 10q + 4$$

$\therefore$ The square of any positive integer cannot be of the form $6m+2$ or $6m+5$ for any integer m .

Q.13

$6q+n$ is a positive integer of the form $6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

$$n = 0, 1, 2, 3, 4$$

$$\begin{aligned} & 6q \\ (6q)^3 &= 216q^3 \\ &= 6(36q^3) \\ &= 6m + 0 \end{aligned}$$

$$\text{Where } m = 36q^3$$

$$\begin{aligned} & 6q+1 \\ (6q+1)^3 &= 216q^3 + 1 + \\ & \quad 108q^2 + 18q \\ &= 6(36q^3 + 18q^2 + 3q) + 1 \\ &= 6m + 1 \end{aligned}$$

$$\text{Where } m = 36q^3 + 18q^2 + 3q$$

Q13

$$6q+2$$

$$\begin{aligned}(6q+2)^3 &= 216q^3 + 8 + 216q^2 + 72q \\ &= 6(36q^3 + 36q^2 + 12q + 1) + 2 \\ &= 6m + 2\end{aligned}$$

$$\text{Where } m = 36q^3 + 36q^2 + 12q + 1$$

$$6q+3$$

$$\begin{aligned}(6q+3)^3 &= 216q^3 + 27 + 324q^2 + 162q \\ &= 216q^3 + 324q^2 + 162q + 24 + 3 \\ &= 6(36q^3 + 54q^2 + 27q + 4) + 3 \\ &= 6m + 3\end{aligned}$$

$$\text{where } m = 36q^3 + 54q^2 + 27q + 4$$

$$6q+4$$

$$\begin{aligned}(6q+4)^3 &= 216q^3 + 64 + 432q^2 + 288q \\ &= 216q^3 + 432q^2 + 288q + 60 + 4 \\ &= 6(36q^3 + 72q^2 + 48q + 10) + 4 \\ &= 6m + 4\end{aligned}$$

$$\text{where } m = 36q^3 + 72q^2 + 48q + 10$$

$$6q+5$$

$$\begin{aligned}(6q+5)^3 &= 216q^3 + 125 + 540q^2 + 450q \\ &= 216q^3 + 540q^2 + 450q + 120 + 5 \\ &= 6(36q^3 + 90q^2 + 75q + 20) + 5 \\ &= 6m + 5\end{aligned}$$

$$\text{where } m = 36q^3 + 90q^2 + 75q + 20$$

∴ The cube of a positive integer is of the form $6q+r$ and $r = 0, 1, 2, 3, 4, 5$.

Q. 14

Let 'n' be a positive integer

$$b = 5$$

According to Euclid's division lemma,

$$n = 5q + r$$

$$0 \leq r < 5$$

 $\therefore$ The possible remainders are 0, 1, 2, 3, 4,

$$\text{When } n = 5q$$

$$n+4 = 5q+4$$

$$n+8 = 5q+8 \quad \therefore n \text{ is divisible by 5}$$

$$n+12 = 5q+12$$

$$n+16 = 5q+16$$

$$\text{When } n = 5q+1$$

$$n+4 = 5q+5 = 5(q+1)$$

$$n+8 = 5q+9$$

$$n+12 = 5q+13 \quad \therefore n+4 \text{ is divisible by 5}$$

$$n+16 = 5q+17$$

$$\text{When } n = 5q+2$$

$$n+4 = 5q+6$$

$$n+8 = 5q+10 = 5(q+2)$$

$$n+12 = 5q+14 \quad \therefore n+8 \text{ is divisible by 5}$$

$$n+16 = 5q+18$$

$$\text{When } n = 5q+3$$

$$n+4 = 5q+7$$

$$n+8 = 5q+11$$

$$n+12 = 5q+15 = 5(q+3)$$

$$n+16 = 5q+19$$

$$\therefore n+12 \text{ is divisible by 5}$$

Q14

$$\text{When } n = 5q + 4$$

$$n + 4 = 5q + 8$$

$$n + 8 = 5q + 12$$

$$n + 12 = 5q + 16$$

$$n + 16 = 5q + 20 = 5(q + 4)$$

$\therefore n + 16$ is divisible by 5

Therefore, in every case, one and only one out of n , $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5 where n is any positive integer.

Let 'a' be a positive integer.

$$b = 6$$

According to Euclid's division lemma,

$$a = 6m + r$$

$$0 \leq r < 6$$

$\therefore$ The possible remainders are, 0, 1, 2, 3, 4, 5

$\therefore 6m + 1$, $6m + 3$ and $6m + 5$ are odd integers (not divisible by 2)

$$\text{When } a = 6m + 1$$

$$(6m + 1)^2 = 36m^2 + 1 + 12m$$

$$= 36m^2 + 12m + 1$$

$$= 6(6m^2 + 2m) + 1$$

$$= 6q + 1$$

$$\text{where } q = 6m^2 + 2m$$

$$\text{When } a = 6m + 3$$

$$(6m + 3)^2 = 36m^2 + 9 + 36m$$

$$= 36m^2 + 36m + 9$$

$$= 6(6m^2 + 6m + 1) + 3$$

$$= 6q + 3$$

$$\text{where } q = 6m^2 + 6m + 1$$

$$\text{When } a = 6m + 5$$

$$(6m + 5)^2 = 36m^2 + 25 + 60m$$

$$= 36m^2 + 60m + 25$$

$$= 6(6m^2 + 10m + 4) + 1$$

$$= 6q + 1$$

$$\text{where } q = 6m^2 + 10m + 4$$

$\therefore$ The square of an odd positive integer can be of the form $6q+1$ or $6q+3$ for some integer q .

Q.16

Let 'a' be a positive integer.

$$b = 3$$

According to Euclid's division lemma,

$$a = 3q + r$$

$$0 \leq r < 3$$

$\therefore$ The possible remainders are 0, 1, 2

When $a = 3q$

$$\begin{aligned} (3q)^2 &= 9q^2 \\ &= 3(3q^2) \\ &= 3m \end{aligned}$$

$$\text{Where } m = 3q^2$$

When $a = 3q + 1$

$$\begin{aligned} (3q+1)^2 &= 9q^2 + 6q + 1 \\ &= 9q^2 + 6q + 3 + 1 \\ &= 3(3q^2 + 2q + 1) + 1 \\ &= 3m + 1 \end{aligned}$$

$$\text{Where } m = 3q^2 + 2q + 1$$

When $a = 3q + 2$

$$\begin{aligned} (3q+2)^2 &= 9q^2 + 12q + 4 \\ &= 9q^2 + 12q + 3 + 1 \\ &= 3(3q^2 + 4q + 1) + 1 \\ &= 3m + 1 \end{aligned}$$

$$\text{Where } m = 3q^2 + 4q + 1$$

$\therefore$ No, we cannot write the square of $3q+1$ in any form other than $3m+1$, $3m$ or $3m+2$ for some integer m .

Q.17

Let 'a' be a positive integer

$$b = 3$$

According to Euclid's division lemma,

$$a = 3q + r$$

$$0 \leq r < 3$$

$\therefore$ The possible remainders are 0, 1, 2

Q17

When $a = 3q$

$$(3q)^2 = 9q^2$$

$$= 3(3q^2)$$

$$= 3m$$

$$\text{where } m = 3q^2$$

When $a = 3q + 1$

$$(3q+1)^2 = 9q^2 + 6q + 1$$

$$= 3(3q^2 + 2q) + 1$$

$$= 3m + 1$$

$$\text{where } m = 3q^2 + 2q$$

When $a = 3q + 2$

$$(3q+2)^2 = 9q^2 + 12q + 4$$

$$= 9q^2 + 12q + 3 + 1$$

$$= 3(3q^2 + 4q + 1) + 1$$

$$= 3m + 1$$

$$\text{where } m = 3q^2 + 4q + 1$$

$\therefore$ No, ~~we cannot write~~ the square of any positive integer cannot of the form $3m + 2$, where m is a natural no.