

Q1. Find the following products

(a) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

(b) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

(c) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

(d) $(\frac{x}{2} + 2y)($

$\frac{x^2}{4} - xy + 4y^2)$

(e) $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

(f) $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$

(g) $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

(h) $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

(i) $(1 - x)(1 + x + x^2)$

(j) $(1 + x)(1 - x + x^2)$

(k) $(x^2 - 1)(x^4 + x^2 + 1)$

(l) $(x^2 + 1)(x^6 - x^3 + 1)$

Sol :

(a) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

Given, $(3x + 2y)(9x^2 - 6xy + 4y^2)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(3x + 2y)(9x^2 - 6xy + 4y^2)$ can be written as

$\Rightarrow (3x + 2y)[(3x)^2 - (3x)(2y) + (2y)^2]$

$\Rightarrow (3x)^3 + (2y)^3$

$\Rightarrow 27x^3 + 8y^3$

Hence, the value of $(3x + 2y)(9x^2 - 6xy + 4y^2) = 27x^3 + 8y^3$

(b) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

Given, $(4x - 5y)(16x^2 + 20xy + 25y^2)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(4x - 5y)(16x^2 + 20xy + 25y^2)$ can be written as

$\Rightarrow (4x - 5y)[(4x)^2 + (4x)(5y) + (5y)^2]$

$\Rightarrow (4x)^3 - (5y)^3$

$\Rightarrow 16x^3 - 125y^3$

Hence, the value of $(4x - 5y)(16x^2 + 20xy + 25y^2) = 16x^3 - 125y^3$

$$(c) (7p^4 + q)(49p^8 - 7p^4q + q^2)$$

$$\text{Given, } (7p^4 + q)(49p^8 - 7p^4q + q^2)$$

$$\text{We know that, } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$(7p^4 + q)(49p^8 - 7p^4q + q^2) \text{ can be written as}$$

$$\Rightarrow (7p^4 + q)[(7p^4)^2 - (7p^4)(q) + (q)^2]$$

$$\Rightarrow (7p^4)^3 + (q)^3$$

$$\Rightarrow 343p^{12} + q^3$$

$$\text{Hence, the value of } (7p^4 + q)(49p^8 - 7p^4q + q^2) = 343p^{12} + q^3$$

$$(d) \left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$$

Sol :

$$\text{Given, } \left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$$

$$\text{We know that, } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right) \text{ can be written as}$$

$$\Rightarrow \left(\frac{x}{2} + 2y\right)\left[\left(\frac{x}{2}\right)^2 - \frac{x}{2}(2y) + (2y)^2\right]$$

$$\Rightarrow \left(\frac{x}{2}\right)^3 + (2y)^3$$

$$\Rightarrow \frac{x^3}{8} + 8y^3$$

$$(e) \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

Sol:

$$\text{Given, } \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

$$\text{We know that, } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

$$\left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

Can be written as,

$$\Rightarrow \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{3}{x}\right)^2 + \left(\frac{5}{y}\right)^2 + \left(\frac{3}{x}\right)\left(\frac{5}{y}\right)$$

$$\Rightarrow \left(\frac{3}{x}\right)^3 - \left(\frac{5}{y}\right)^3$$

$$\Rightarrow \left(\frac{27}{x^3}\right) - \left(\frac{125}{y^3}\right)$$

$$\text{Hence, the value of } \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right) = \left(\frac{27}{x^3}\right) - \left(\frac{125}{y^3}\right)$$

$$(f) \left(3 + \frac{5}{x}\right)\left(9 - \frac{15}{x} + \frac{25}{x^2}\right)$$

Sol:

Given, $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$ can be written as,

$$\Rightarrow (3 + \frac{5}{x})[(3^2) - 3(\frac{5}{x}) + (\frac{5}{x})^2]$$

$$\Rightarrow (3)^3 + (\frac{5}{x})^3$$

$$\Rightarrow 27 + \frac{125}{x^3}$$

Hence, the value of $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$ is $27 + \frac{125}{x^3}$

(g) $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

Sol:

Given, $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$ can be written as,

$$\Rightarrow (\frac{2}{x} + 3x)[(\frac{2}{x})^2 + (3x)^2 - (\frac{2}{x})(3x)]$$

$$\Rightarrow (\frac{2}{x})^3 + (3x)^3$$

$$\Rightarrow \frac{8}{x^3} + 9x^3$$

Hence, the value of $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$ is $\frac{8}{x^3} + 9x^3$

(h) $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

Sol:

Given, $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$ can be written as,

$$\Rightarrow (\frac{3}{x} - 2x^2)[(\frac{3}{x})^2 + (2x^2)^2 - (\frac{3}{x})(2x^2)]$$

$$\Rightarrow (\frac{3}{x} - 2x^2)[(\frac{9}{x^2}) + 4x^4 - (\frac{3}{x})(2x^2)]$$

$$\Rightarrow (\frac{3}{x})^3 - (2x^2)^3$$

$$\Rightarrow \frac{27}{x^3} - 8x^6$$

Hence, $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$ is $\frac{27}{x^3} - 8x^6$

(i) $(1 - x)(1 + x + x^2)$

Sol:

Given, $(1 - x)(1 + x + x^2)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(1 - x)(1 + x + x^2)$ can be written as,

$$\Rightarrow (1 - x)[(1^2 + (1)(x) + x^2)]$$

$$\Rightarrow (1)^3 - (x)^3$$

$$\Rightarrow 1 - x^3$$

Hence, the value of $(1 - x)(1 + x + x^2)$ is $1 - x^3$

(j) $(1 + x)(1 - x + x^2)$

Sol:

Given, $(1 + x)(1 - x + x^2)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(1 + x)(1 - x + x^2)$ can be written as,

$$\Rightarrow (1 + x)[(1^2 - (1)(x) + x^2)]$$

$$\Rightarrow (1)^3 + (x)^3$$

$$\Rightarrow 1 + x^3$$

Hence, the value of $(1 + x)(1 - x + x^2)$ is $1 + x^3$

(k) $(x^2 - 1)(x^4 + x^2 + 1)$

Sol:

Given, $(x^2 - 1)(x^4 + x^2 + 1)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(x^2 - 1)(x^4 + x^2 + 1)$ can be written as,

$$\Rightarrow (x^2 - 1)[(x^2)^2 - 1^2 + (x^2)(1)]$$

$$\Rightarrow (x^2)^3 - 1^3$$

$$\Rightarrow x^6 - 1$$

Hence, $(x^2 - 1)(x^4 + x^2 + 1)$ is $x^6 - 1$

(l) $(x^2 + 1)(x^6 - x^3 + 1)$

Sol:

Given, $(x^2 + 1)(x^6 - x^3 + 1)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(x^2 + 1)(x^6 - x^3 + 1)$ can be written as,

$$\Rightarrow (x^3 + 1)[(x^3)^2 - (x^3)(1) + 1^2]$$

$$\Rightarrow (x^3)^3 + 1^3$$

$$\Rightarrow x^9 + 1$$

Hence, the value of $(x^2 + 1)(x^6 - x^3 + 1)$ is $x^9 + 1$

Q2. Find $x = 3$ and $y = -1$, Find the values of each of the following using identity:

$$(a) (9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$$

$$(b) \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

$$(c) \left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$$

$$(d) \left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21}\right)$$

$$(e) \left(\frac{5}{x} + 5x\right)\left(\frac{25}{x^2} - 25 + 25x^2\right)$$

Sol:

$$(a) (9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$ can be written as,

$$\Rightarrow (9x^2 - 4x^2)[(9y^2)^2 + (9)(4)x^2y^2 + (4x^2)^2]$$

$$\Rightarrow (9y^2)^3 - (4x^2)^3$$

$$\Rightarrow 729y^6 - 64x^6$$

Substitute the value $x = 3$, $y = -1$ in $729y^6 - 64x^6$ we get,

$$\Rightarrow 729y^6 - 64x^6$$

$$\Rightarrow 729(-1)^6 - 64(3)^6$$

$$\Rightarrow 729(1) - 64(729)$$

$$\Rightarrow 729 - 46656$$

$$\Rightarrow -45927$$

Hence, the product value of $(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4) = -45927$

$$(b) \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

Sol:

Given,

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$$

Can be written as,

$$\Rightarrow \left(\frac{3}{x} - \frac{x}{3}\right)\left[\left(\frac{3}{x}\right)^2 + \left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)\left(\frac{x}{3}\right)\right]$$

$$\Rightarrow \left(\frac{3}{x}\right)^3 - \left(\frac{x}{3}\right)^3$$

$$\Rightarrow \left(\frac{27}{x^3}\right) - \left(\frac{x^3}{27}\right) = 1$$

Substitute $x = 3$ in eq 1

$$\Rightarrow \left(\frac{27}{3^3}\right) - \left(\frac{3^3}{27}\right)$$

$$\Rightarrow \left(\frac{27}{27}\right) - \left(\frac{27}{27}\right)$$

$$\Rightarrow 0$$

Hence, the value of $\left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}\right)$ is 0

$$(c) \left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$$

Sol:

Given,

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$$

Can be written as,

$$\Rightarrow \left(\frac{x}{7} + \frac{y}{3}\right)\left[\left(\frac{x}{7}\right)^2 + \left(\frac{y}{3}\right)^2 - \left(\frac{x}{7}\right)\left(\frac{y}{3}\right)\right]$$

$$\Rightarrow \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3$$

$$\Rightarrow \left(\frac{x^3}{343}\right) + \left(\frac{y^3}{27}\right) = 1$$

Substitute $x = 3, y = -1$ in eq 1

$$\Rightarrow \left(\frac{3^3}{343}\right) + \left(\frac{(-1)^3}{27}\right)$$

$$\Rightarrow \left(\frac{27}{343}\right) - \left(\frac{1}{27}\right)$$

Taking least common multiple, we get

$$\Rightarrow \frac{27 \times 27}{343 \times 27} - \frac{1 \times 343}{27 \times 343}$$

$$\Rightarrow \frac{729}{9261} - \frac{343}{9261}$$

$$\Rightarrow \frac{729 - 343}{9261}$$

$$\Rightarrow \frac{386}{9261}$$

Hence, the value of $\left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right) = \frac{386}{9261}$

$$(d) \left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21}\right)$$

Sol:

Given,

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21}\right)$$

Can be written as,

$$\Rightarrow \left(\frac{x}{4} - \frac{y}{3}\right)\left[\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{x}{4}\right)\left(\frac{y}{3}\right)\right]$$

$$\Rightarrow \left(\frac{x}{4}\right)^3 - \left(\frac{y}{3}\right)^3$$

$$\Rightarrow \left(\frac{x^3}{64}\right) - \left(\frac{y^3}{27}\right) = 1$$

Substitute $x = 3$, $y = -1$ in eq 1

$$\Rightarrow \left(\frac{3^3}{64}\right) - \left(\frac{(-1)^3}{27}\right)$$

$$\Rightarrow \left(\frac{27}{64}\right) + \left(\frac{1}{27}\right)$$

Taking least common multiple, we get

$$\Rightarrow \frac{27 \times 27}{64 \times 27} + \frac{1 \times 64}{27 \times 64}$$

$$\Rightarrow \frac{729}{1728} + \frac{64}{1728}$$

$$\Rightarrow \frac{729 + 64}{1728}$$

$$\Rightarrow \frac{793}{1728}$$

Hence, the value of $\left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21}\right) = \frac{793}{1728}$

$$(e) \left(\frac{5}{x} + 5x\right)\left(\frac{25}{x^2} - 25 + 25x^2\right)$$

Sol:

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$\left(\frac{5}{x} + 5x\right)\left(\frac{25}{x^2} - 25 + 25x^2\right)$ can be written as,

$$\Rightarrow \left(\frac{5}{x} + 5x\right)\left[\left(\frac{5}{x}\right)^2 + (5x)^2 - \left(\frac{5}{x}\right)(5x)\right]$$

$$\Rightarrow \left(\frac{5}{x}\right)^3 + (5x)^3$$

$$\Rightarrow \frac{125}{x^3} + 125x^3 = 1$$

Substitute $x = 3$, in eq 1

$$\Rightarrow \frac{125}{3^3} + 125(3)^3$$

$$\Rightarrow \frac{125}{27} + 125 \times 27$$

$$\Rightarrow \frac{125}{27} + 3375$$

Taking least common multiple, we get

$$\Rightarrow \frac{125}{27} + \frac{3375 \cdot 27}{27 \cdot 1}$$

$$\Rightarrow \frac{125}{27} + \frac{91125}{27}$$

$$\Rightarrow \frac{125 + 91125}{27}$$

$$\Rightarrow \frac{91250}{27}$$

Hence, the value of $(\frac{5}{x} + 5x)(\frac{25}{x^2} - 25 + 25x^2)$ is $\frac{91250}{27}$

Q3. If $a + b = 10$ and $ab = 16$, find the value of $a^2 - ab + b^2$ and $a^2 + ab + b^2$

Sol :

Given, $a + b = 10$, $ab = 16$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\Rightarrow a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$\Rightarrow a^3 + b^3 = (10)^3 - 3(16)(10)$$

$$\Rightarrow a^3 + b^3 = 1000 - 480$$

$$\Rightarrow a^3 + b^3 = 520$$

Substitute, $a^3 + b^3 = 520$, $a + b = 10$ in $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$520 = 10(a^2 + b^2 - ab)$$

$$\frac{520}{10} = (a^2 + b^2 - ab)$$

$$\Rightarrow (a^2 + b^2 - ab) = 52$$

Now, we need to find $(a^2 + b^2 + ab)$

Add and subtract $2ab$ in $a^2 + b^2 + ab$

$$\Rightarrow a^2 + b^2 + ab - 2ab + 2ab$$

$$\Rightarrow (a + b)^2 - ab$$

Substitute $a + b = 10$, ab

$$\Rightarrow a^2 + b^2 + ab = 10^2 - 16$$

$$= 100 - 16$$

$$= 84$$

Hence, the values of $(a^2 + b^2 - ab) = 52$ and $(a^2 + b^2 + ab) = 84$

Q4. If $a + b = 8$ and $ab = 6$, find the value of $a^3 + b^3$

Sol:

Given, $a + b = 8$ and $ab = 6$

We know that, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$$\Rightarrow a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$\Rightarrow a^3 + b^3 = (8)^3 - 3(6)(8)$$

$$\Rightarrow a^3 + b^3 = 512 - 144$$

$$\Rightarrow a^3 + b^3 = 368$$

Hence, the value of $a^3 + b^3$ is 368

Q5. If $a - b = 6$ and $ab = 20$, find the value of $a^3 - b^3$

Sol:

Given, $a - b = 6$ and $ab = 20$

We know that, $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$$\Rightarrow a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

$$\Rightarrow a^3 - b^3 = (6)^3 + 3(20)(6)$$

$$\Rightarrow a^3 - b^3 = 216 + 360$$

$$\Rightarrow a^3 - b^3 = 576$$

Hence, the value of $a^3 - b^3$ is 576

Q6. If $x = -2$ and $y = 1$, by using an identity find the value of the following:

$$(a) (4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$$

$$(b) \left(\frac{2}{x} - \frac{x}{2}\right)\left(\frac{4}{x^2} + \frac{x^2}{4} + 1\right)$$

$$(c) \left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$$

Sol:

Given,

$$(a) (4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$ can be written as,

$$\Rightarrow (4y^2 - 9x^2)[(4x)^2 + 4y^2 \cdot 9x^2 + (9x^2)^2]$$

$$\Rightarrow (4y^2)^3 - (9x^2)^3$$

$$\Rightarrow 64y^6 - 729x^6 \text{ ---- 1}$$

Substitute $x = -2$ and $y = 1$ in eq 1

$$\Rightarrow 64y^6 - 729x^6$$

$$\Rightarrow 64(1)^6 - 729(-2)^6$$

$$\Rightarrow 64 - 729(64)$$

$$\Rightarrow 64(1 - 729)$$

$$\Rightarrow 64(-728)$$

$$\Rightarrow -46592$$

Hence, the value of $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$ is -46592

$$(b) \left(\frac{2}{x} - \frac{x}{2}\right) \left(\frac{4}{x^2} + \frac{x^2}{4} + 1\right)$$

here $x = -2$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$\left(\frac{2}{x} - \frac{x}{2}\right) \left(\frac{4}{x^2} + \frac{x^2}{4} + 1\right)$ can be written as,

$$\Rightarrow \left(\frac{2}{x} - \frac{x}{2}\right) \left[\left(\frac{2}{x}\right)^2 + \left(\frac{x}{2}\right)^2 + \left(\frac{2}{x}\right)\left(\frac{x}{2}\right)\right]$$

$$\Rightarrow \left(\frac{2}{x}\right)^3 - \left(\frac{x}{2}\right)^3$$

$$\Rightarrow \left(\frac{8}{x^3}\right) - \left(\frac{x^3}{8}\right) \text{ --- 1}$$

Substitute $x = -2$ in eq 1

$$\Rightarrow \left(\frac{8}{(-2)^3}\right) - \left(\frac{(-2)^3}{8}\right)$$

$$\Rightarrow \left(\frac{8}{-8}\right) - \left(\frac{-8}{8}\right)$$

$$\Rightarrow -1 + 1$$

$$\Rightarrow 0$$

Hence, the value of $\left(\frac{2}{x} - \frac{x}{2}\right) \left(\frac{4}{x^2} + \frac{x^2}{4} + 1\right)$ is 0

$$(c) \left(5y + \frac{15}{y}\right) \left(25y^2 - 75 + \frac{225}{y^2}\right)$$

Sol:

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$\left(5y + \frac{15}{y}\right) \left(25y^2 - 75 + \frac{225}{y^2}\right)$ can be written as,

$$\Rightarrow \left(5y + \frac{15}{y}\right) \left[(5y)^2 + \left(\frac{15}{y}\right)^2 - (5y)\left(\frac{15}{y}\right)\right]$$

$$\Rightarrow (5y)^3 + \left(\frac{15}{y}\right)^3$$

$$\Rightarrow 125y^3 + \left(\frac{3375}{y^3}\right) \text{ --- 1}$$

Substitute $y = 1$ in eq 1

$$\Rightarrow 125(1)^3 + \left(\frac{3375}{(1)^3}\right)$$

$$\Rightarrow 125 + 3375$$

$$\Rightarrow 3500$$

Hence, the value of $\left(5y + \frac{15}{y}\right) \left(25y^2 - 75 + \frac{225}{y^2}\right)$ is 3500.

EXERCISE 4.5

Q1. Find the following products:

$$(a) (3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$$

$$(b) (4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$$

$$(c) (2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$$

$$(d) (3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$$

Sol:

Given,

$$(a) (3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$$

we know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

so,

$$\begin{aligned}(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx) &= (3x)^3 + (2y)^3 + (2z)^3 - 3(3x)(2y)(2z) \\ &= 27x^3 + 8y^3 + 8z^3 - 36xyz\end{aligned}$$

Hence, the value of $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$ is $27x^3 + 8y^3 + 8z^3 - 36xyz$

$$(b) (4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$$

we know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

so,

$$\begin{aligned}(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx) &= (4x)^3 + (-3y)^3 + (2z)^3 - 3(4x)(-3y)(2z) \\ &= 64x^3 - 27y^3 + 8z^3 + 72xyz\end{aligned}$$

Hence, the value of $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$ is $64x^3 - 27y^3 + 8z^3 + 72xyz$

$$(c) (2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$$

we know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

so,

$$\begin{aligned}(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca) &= (2a)^3 + (-3b)^3 + (-2c)^3 - 3(2a)(-3b)(-2c) \\ &= 8a^3 - 27b^3 - 8c^3 - 36abc\end{aligned}$$

Hence, the value of (c) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$ is $8a^3 - 27b^3 - 8c^3 - 36abc$

$$(d) (3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$$

we know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

so,

$$\begin{aligned}(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz) &= (3x)^3 + (-4y)^3 + (5z)^3 - 3(3x)(-4y)(5z) \\ &= 27x^3 - 64y^3 + 125z^3 + 180xyz\end{aligned}$$

Hence, the value of $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$ is $27x^3 - 64y^3 + 125z^3 + 180xyz$

Q2. If $x + y + z = 8$ and $xy + yz + zx = 20$, Find the value of $x^3 + y^3 + z^3 - 3xyz$

Sol:

given, $x + y + z = 8$ and $xy + yz + zx = 20$

We know that,

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(20)$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 40$$

$$8^2 = x^2 + y^2 + z^2 + 40$$

$$64 - 40 = x^2 + y^2 + z^2$$

$$x^2 + y^2 + z^2 = 24$$

we know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[(x^2 + y^2 + z^2) - (xy + yz + zx)]$$

here, $x + y + z = 8$, $xy + yz + zx = 20$, $x^2 + y^2 + z^2 = 24$

$$x^3 + y^3 + z^3 - 3xyz = 8[(24 - 20)]$$

$$= 8 * 4$$

$$= 32$$

Hence, the value of $x^3 + y^3 + z^3 - 3xyz$ is 32

Q3. If $a + b + c = 9$ and $ab + bc + ca = 26$, Find the value of $a^3 + b^3 + c^3 - 3abc$

Sol :

Given, $a + b + c = 9$ and $ab + bc + ca = 26$

We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(26)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 52$$

$$9^2 = a^2 + b^2 + c^2 + 52$$

$$81 - 52 = a^2 + b^2 + c^2$$

$$a^2 + b^2 + c^2 = 29$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$$

here, $a + b + c = 9$, $ab + bc + ca = 26$, $a^2 + b^2 + c^2 = 29$

$$a^3 + b^3 + c^3 - 3abc = 9[(29 - 26)]$$

$$= 9 * 3$$

$$= 27$$

Hence, the value of $a^3 + b^3 + c^3 - 3abc$ is 27

Q4. If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$, Find the value of $a^3 + b^3 + c^3 - 3abc$

Sol:

$$\text{Given, } a + b + c = 9 \text{ and } a^2 + b^2 + c^2 = 35$$

We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$9^2 = 35 + 2(ab + bc + ca)$$

$$81 = 35 + 2(ab + bc + ca)$$

$$81 - 35 = 2(ab + bc + ca)$$

$$\frac{46}{2} = ab + bc + ca$$

$$ab + bc + ca = 23$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$$

$$\text{here, } a + b + c = 9, ab + bc + ca = 23, a^2 + b^2 + c^2 = 35$$

$$a^3 + b^3 + c^3 - 3abc = 9[(35 - 23)]$$

$$= 9 * 12$$

$$= 108$$

Hence, the value of $a^3 + b^3 + c^3 - 3abc$ is 108

Q5. Evaluate:

$$(a) 25^3 - 75^3 + 50^3$$

$$(b) 48^3 - 30^3 - 18^3$$

$$(c) \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$$

$$(d) (0.2)^3 - (0.3)^3 + (0.1)^3$$

Sol:

Given,

$$(a) 25^3 - 75^3 + 50^3$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{here, } a = 25, b = -75, c = 50$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (25 - 75 + 50)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 3abc$$

$$25^3 + (-75)^3 + 50^3 = 3abc$$

$$= 3(25)(-75)(50)$$

$$= -281250$$

$$\text{Hence, the value } 25^3 + (-75)^3 + 50^3 = -281250$$

$$(b) 48^3 - 30^3 - 18^3$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{here, } a = 48, b = -30, c = -18$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (48 - 30 - 18)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 3abc$$

$$48^3 + (-30)^3 + (-18)^3 = 3abc$$

$$= 3(48)(-30)(-18)$$

$$= 77760$$

$$\text{Hence, the value } 48^3 + (-30)^3 + (-18)^3 = 77760$$

$$(c) \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{here, } a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{-5}{6}$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = \left(\frac{1}{2} + \frac{1}{3} - \frac{5}{6}\right)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

by using least common multiple

$$a^3 + b^3 + c^3 = \left(\frac{1*6}{2*6} + \frac{1*4}{3*4} - \frac{5*2}{6*2}\right)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = \left(\frac{6}{12} + \frac{4}{12} - \frac{10}{12}\right)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 0(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{-5}{6}\right)^3 = 3 * \frac{1}{2} * \frac{1}{3} * \frac{-5}{6}$$

$$= \frac{1}{2} * \frac{-5}{6}$$

$$= \frac{-5}{12}$$

Hence, the value of $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ is $\frac{-5}{12}$

$$(d) (0.2)^3 - (0.3)^3 + (0.1)^3$$

we know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

here, $a = 0.2$, $b = 0.3$, $c = 0.1$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (0.2 - 0.3 + 0.1)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 3abc$$

$$(0.2)^3 - (0.3)^3 + (0.1)^3 = 3abc$$

$$= 3(0.2)(-0.3)(0.1)$$

$$= -0.018$$

Hence, the value $(0.2)^3 - (0.3)^3 + (0.1)^3$ is 0.018