

1) Evaluate each of the following using identities:

(i) $(2x - \frac{1}{x})^2$

Solution:

Given,

$$(2x - \frac{1}{x})^2 = (2x)^2 + (\frac{1}{x})^2 - 2 * 2x * \frac{1}{x}$$

$$(2x - \frac{1}{x})^2 = 4x^2 + \frac{1}{x^2} - 4 \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

Where, $a = 2x$, $b = \frac{1}{x}$

$$\therefore (2x - \frac{1}{x})^2 = 4x^2 + \frac{1}{x^2} - 4$$

(ii) $(2x+y)(2x-y)$

Solution:

Given, $(2x+y)(2x-y)$

$$= (2x)^2 - (y)^2 \quad [\because (a + b)(a - b) = a^2 - b^2]$$

$$= 4x^2 - y^2$$

$$\therefore (2x + y)(2x - y) = 4x^2 - y^2$$

(iii) $(a^2b - ab^2)^2$

Solution:

Given, $(a^2b - ab^2)^2$

$$= (a^2b)^2 + (ab^2)^2 - 2 * a^2b * ab^2 \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

Where, $a = a^2b$, $b = ab^2$

$$= a^4b^2 + b^4a^2 - 2a^3b^3$$

$$\therefore (a^2b - ab^2)^2 = a^4b^2 + b^4a^2 - 2a^3b^3$$

(iv) $(a-0.1)(a+0.1)$

Solution:

Given, $(a-0.1)(a+0.1)$

$$= a^2 - (0.1)^2 \quad [\because (a + b)(a - b) = a^2 - b^2]$$

Where, $a = a$ and $b = 0.1$

$$= a^2 - 0.01$$

$$\therefore (a - 0.1)(a + 0.1) = a^2 - 0.01$$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

Solution:

$$\begin{aligned}
 &\text{Given, } (1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2) \\
 &= (1.5x^2)^2 - (0.3y^2)^2 \quad [\because (a+b)(a-b) = a^2 - b^2] \\
 &\text{Where, } a = 1.5x^2, \quad b = 0.3y^2 \\
 &= 2.25x^4 - 0.09y^4 \\
 &\therefore (1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2) = 2.25x^4 - 0.09y^4
 \end{aligned}$$

2) Evaluate each of the following using identities:

(i) $(399)^2$

Solution:

We have,

$$\begin{aligned}
 399^2 &= (400-1)^2 \\
 &= (400)^2 + (1)^2 - 2 \times 400 \times 1 \quad [(a-b)^2 = a^2 + b^2 - 2ab]
 \end{aligned}$$

Where, $a = 400$ and $b = 1$

$$= 160000 + 1 - 8000$$

$$= 159201$$

Therefore, $(399)^2 = 159201$.

(ii) $(0.98)^2$

Solution:

We have,

$$\begin{aligned}
 (0.98)^2 &= (1-0.02)^2 \\
 &= (1)^2 + (0.02)^2 - 2 \times 1 \times 0.02 \\
 &= 1 + 0.0004 - 0.04 \quad [\text{Where, } a=1 \text{ and } b=0.02] \\
 &= 1.0004 - 0.04 \\
 &= 0.9604
 \end{aligned}$$

Therefore, $(0.98)^2 = 0.9604$

(iii) 991×1009

Solution:

We have,

$$\begin{aligned}
 &991 \times 1009 \\
 &= (1000-9)(1000+9) \\
 &= (1000)^2 - (9)^2 \quad [(a+b)(a-b) = a^2 - b^2] \\
 &= 1000000 - 81 \quad [\text{Where } a=1000 \text{ and } b=9] \\
 &= 999919
 \end{aligned}$$

Therefore, $991 \times 1009 = 999919$

(iv) 117×83

Solution:

We have,

$$117 \times 83$$

$$= (100+17) (100-17)$$

$$= (100)^2 - (17)^2 \quad [(a+b) (a-b) = a^2 - b^2]$$

$$= 10000 - 289 \quad [\text{Where } a=100 \text{ and } b=17]$$

$$= 9711$$

Therefore, $117 \times 83 = 9711$

3) Simplify each of the following:

(i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

Solution:

We have,

$$175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = (175)^2 + 2 (175) (25) + (25)^2$$

$$= (175+25)^2 \quad [a^2 + b^2 + 2ab = (a+b)^2]$$

$$= (200)^2 \quad [\text{Where } a=175 \text{ and } b=25]$$

$$= 40000$$

Therefore, $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000$.

(ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

Solution:

We have,

$$322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$$

$$= (322-22)^2 \quad [a^2 + b^2 - 2ab = (a-b)^2]$$

$$= (300)^2 \quad [\text{Where } a=322 \text{ and } b=22]$$

$$= 90000$$

Therefore, $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22 = 90000$.

(iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

Solution:

We have,

$$0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$$

$$= (0.76+0.24)^2 \quad [a^2 + b^2 + 2ab = (a+b)^2]$$

$$= (1.00)^2 \quad [\text{Where } a=0.76 \text{ and } b=0.24]$$

$$= 1$$

Therefore, $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24 = 1$.

(iv) $\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$

Solution:

We have,

$$\begin{aligned} & \frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} \\ &= \frac{(7.83 + 1.17)(7.83 - 1.17)}{6.66} \quad [\because (a - b)^2 = (a + b)(a - b)] \\ &= \frac{(9.00)(6.66)}{(6.66)} \\ &= 9 \\ \therefore \frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} &= 9 \end{aligned}$$

4) If $x + \frac{1}{x} = 11$, find the value of $x^2 + \frac{1}{x^2}$

Solution:

We have, $x + \frac{1}{x} = 11$

$$\begin{aligned} \text{Now, } (x + \frac{1}{x})^2 &= x^2 + (\frac{1}{x})^2 + 2 * x * \frac{1}{x} \\ \Rightarrow (x + \frac{1}{x})^2 &= x^2 + \frac{1}{x^2} + 2 \\ \Rightarrow (11)^2 &= x^2 + \frac{1}{x^2} + 2 \quad [\because x + \frac{1}{x} = 11] \\ \Rightarrow 121 &= x^2 + \frac{1}{x^2} + 2 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 119 \end{aligned}$$

5) If $x - \frac{1}{x} = -1$, find the value of $x^2 + \frac{1}{x^2}$

Solution:

We have, $x - \frac{1}{x} = -1$

$$\begin{aligned} \text{Now, } (x - \frac{1}{x})^2 &= x^2 + (\frac{1}{x})^2 - 2 * x * \frac{1}{x} \\ \Rightarrow (x - \frac{1}{x})^2 &= x^2 + \frac{1}{x^2} - 2 \\ \Rightarrow (-1)^2 &= x^2 + \frac{1}{x^2} - 2 \quad [\because x - \frac{1}{x} = -1] \\ \Rightarrow 2 + 1 &= x^2 + \frac{1}{x^2} \\ \Rightarrow x^2 + \frac{1}{x^2} &= 3 \end{aligned}$$

6) If $x + \frac{1}{x} = \sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$

Solution:

We have,

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 + 2 * x * \frac{1}{x}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow (\sqrt{5})^2 = x^2 + \frac{1}{x^2} + 2 \quad [\because x + \frac{1}{x} = \sqrt{5}]$$

$$\Rightarrow 5 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 3 \dots\dots(1)$$

$$\text{Now, } \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 * x^2 * \frac{1}{x^2}$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$$

$$\Rightarrow 9 = x^4 + \frac{1}{x^4} + 2 \quad [\because x^2 + \frac{1}{x^2} = 3]$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 7$$

$$\text{Hence, } x^2 + \frac{1}{x^2} = 3; \quad x^4 + \frac{1}{x^4} = 7.$$

7) If $x^2 + \frac{1}{x^2} = 66$, find the value of $x - \frac{1}{x}$

Solution:

We have,

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 - 2 * x * \frac{1}{x}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 66 - 2 \quad [\because x^2 + \frac{1}{x^2} = 66]$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 64$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (\pm 8)^2$$

$$\Rightarrow x - \frac{1}{x} = \pm 8$$

8) If $x^2 + \frac{1}{x^2} = 79$, find the value of $x + \frac{1}{x}$

Solution:

We have,

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 + 2 * x * \frac{1}{x}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 79 + 2 \quad [\because x^2 + \frac{1}{x^2} = 79]$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 81$$

$$\Rightarrow (x + \frac{1}{x})^2 = (\pm 9)^2$$

$$\Rightarrow x + \frac{1}{x} = \pm 9$$

9) If $9x^2 + 25y^2 = 181$ and $xy = -6$, find the value of $3x + 5y$.

Solution:

We have,

$$(3x + 5y)^2 = (3x)^2 + (5y)^2 + 2*3x*5y$$

$$\Rightarrow (3x + 5y)^2 = 9x^2 + 25y^2 + 30xy$$

$$= 181 + 30(-6) \quad [\text{Since, } 9x^2 + 25y^2 = 181 \text{ and } xy = -6]$$

$$\Rightarrow (3x+5y)^2 = 1$$

$$\Rightarrow (3x + 5y)^2 = (\pm 1)^2$$

$$\Rightarrow 3x + 5y = \pm 1$$

10) If $2x + 3y = 8$ and $xy = 2$, find the value of $4x^2 + 9y^2$.

Solution:

We have,

$$(2x + 3y)^2 = (2x)^2 + (3y)^2 + 2*2x*3y$$

$$\Rightarrow (2x + 3y)^2 = 4x^2 + 9y^2 + 12xy \quad [\text{Since, } 2x + 3y = 8 \text{ and } xy = 2]$$

$$\Rightarrow (8)^2 = 4x^2 + 9y^2 + 24$$

$$\Rightarrow 64 - 24 = 4x^2 + 9y^2$$

$$\Rightarrow 4x^2 + 9y^2 = 40$$

11) If $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$.

Solution:

We have,

$$(3x - 7y)^2 = (3x)^2 + (-7y)^2 - 2*3x*7y$$

$$\Rightarrow (3x - 7y)^2 = 9x^2 + 49y^2 - 42xy \quad [\text{Since, } 3x - 7y = 10 \text{ and } xy = -1]$$

$$\Rightarrow (10)^2 = 9x^2 + 49y^2 + 42$$

$$\Rightarrow 100 - 42 = 9x^2 + 49y^2$$

$$\Rightarrow 9x^2 + 49y^2 = 58$$

12) Simplify each of the following products:

$$(i) (\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2)$$

Solution:

$$(\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2)$$

$$\Rightarrow [(\frac{1}{2}a)^2 - (3b)^2] [(\frac{1}{4}a^2 + 9b^2)] \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$\Rightarrow [\frac{1}{4}a^2 - 9b^2] [\frac{1}{4}a^2 + 9b^2] \quad [\because (ab)^2 = a^2b^2]$$

$$= [(\frac{1}{4}a^2)^2 - (9b^2)^2] \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{1}{16}a^4 - 81b^4$$

$$\therefore (\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2) = \frac{1}{16}a^4 - 81b^4$$

$$(ii) (m + \frac{n}{7})^3 (m - \frac{n}{7})$$

Solution:

We have,

$$(m + \frac{n}{7})^3 (m - \frac{n}{7})$$

$$= (m + \frac{n}{7}) (m + \frac{n}{7}) (m + \frac{n}{7}) (m - \frac{n}{7})$$

$$= (m + \frac{n}{7})^2 (m^2 - (\frac{n}{7})^2) \quad [\because (a+b)(a+b) = (a+b)^2 \text{ and } (a+b)(a-b) = a^2 - b^2]$$

$$= (m + \frac{n}{7})^2 (m^2 - \frac{n^2}{49})$$

$$\therefore (m + \frac{n}{7})^3 (m - \frac{n}{7}) = (m + \frac{n}{7})^2 (m^2 - \frac{n^2}{49})$$

$$(iii) (\frac{x}{2} - \frac{2}{5}) (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$$

Solution:

We have,

$$(\frac{x}{2} - \frac{2}{5}) (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$$

$$\Rightarrow -(\frac{2}{5} - \frac{x}{2}) (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$$

$$\Rightarrow -(\frac{2}{5} - \frac{x}{2})^2 - x^2 + 2x \quad [\because (a-b)(a-b) = (a-b)^2]$$

$$\Rightarrow -[(\frac{2}{5})^2 + (\frac{x}{2})^2 - 2(\frac{2}{5})(\frac{x}{2})] - x^2 + 2x$$

$$\Rightarrow -(\frac{4}{25} + \frac{x^2}{4} - \frac{2x}{5}) - x^2 + 2x$$

$$\Rightarrow -\frac{x^2}{4} - x^2 + \frac{2x}{5} + 2x - \frac{4}{25}$$

$$\Rightarrow -\frac{5x^2}{4} + \frac{12x}{5} - \frac{4}{25}$$

$$\therefore (\frac{x}{2} - \frac{2}{5}) (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x = -\frac{5x^2}{4} + \frac{12x}{5} - \frac{4}{25}$$

$$(iv) (x^2 + x - 2) (x^2 - x + 2)$$

Solution:

$$(x^2 + x - 2) (x^2 - x + 2)$$

$$[(x)^2 + (x - 2)] [(x^2 - (x + 2))]$$

$$\Rightarrow (x^2)^2 - (x - 2)^2 \quad [(a - b)(a + b) = a^2 - b^2]$$

$$\Rightarrow x^4 - (x^2 + 4 - 4x) \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow x^4 - x^2 + 4x - 4$$

$$\therefore (x^2 + x - 2)(x^2 - x + 2) = x^4 - x^2 + 4x - 4$$

$$(v) (x^3 - 3x - x)(x^2 - 3x + 1)$$

Solution:

We have,

$$(x^3 - 3x - x)(x^2 - 3x + 1)$$

$$\Rightarrow x(x^2 - 3x - 1)(x^2 - 3x + 1)$$

$$\Rightarrow x[(x^2 - 3x)^2 - (1)^2] \quad [\because (a + b)(a - b) = a^2 - b^2]$$

$$\Rightarrow x[(x^2)^2 + (-3x)^2 - 2(3x)(x^2) - 1]$$

$$\Rightarrow x[x^4 + 9x^2 - 6x^3 - 1]$$

$$\Rightarrow x^5 - 6x^4 + 9x^3 - x$$

$$\therefore (x^3 - 3x - x)(x^2 - 3x + 1) = x^5 - 6x^4 + 9x^3 - x$$

$$(vi) (2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$$

Solution:

We have,

$$(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$$

$$\Rightarrow [(2x^4 - 4x^2)^2 - (1)^2] \quad [\because (a + b)(a - b) = a^2 - b^2]$$

$$\Rightarrow [(2x^4)^2 + (4x^2)^2 - 2(2x^4)(4x^2) - 1]$$

$$\Rightarrow 4x^8 - 16x^6 + 16x^4 - 1 \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\therefore (2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1) = 4x^8 - 16x^6 + 16x^4 - 1$$

13) Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ is always non-negative for all values of a, b and c .

Solution:

We have,

$$a^2 + b^2 + c^2 - ab - bc - ca$$

Multiply and divide by '2'

$$= \frac{2}{2} [a^2 + b^2 + c^2 - ab - bc - ca]$$

$$= \frac{1}{2} [2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$$

$$= \frac{1}{2} [a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca]$$

$$= \frac{1}{2} [(a^2 + b^2 - 2ab) + (a^2 + c^2 - 2ca) + (b^2 + c^2 - 2bc)]$$

$$= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] \quad [\because (a-b)^2 = a^2 + b^2 - 2ab]$$

$$= \frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2} \geq 0$$

$$\therefore a^2 + b^2 + c^2 - ab - bc - ca \geq 0$$

Hence, $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$ is always non-negative for all values of a, b and c.