

MATHEMATICS

Standard



Question Pool



Government of Kerala
Department of Education

Prepared by

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*Preface*_____

This book contains a collection of model questions of the public examination. For each question, corresponding learning outcomes, score, time and scoring indicators are given.

Students themselves can assess their achievements by practicing these questions. We hope that this will be a great help for the students to face the public examination with confidence.

Regards,

Director

Contents

1.	Arithmetic Sequences	5
2.	Circles	23
3.	Mathematics of Chance	36
4.	Second Degree Equations	43
5.	Trigonometry	63
6.	Coordinates	83
7.	Tangents	93
8.	Solids	105
9.	Geometry and Algebra	124
10.	Polynomials	142
11.	Statistics	152
	Model Question Papers	156

ARITHMETIC SEQUENCES

1. Learning Outcomes

- Formation of sequences from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.

? Write the sequence obtained by adding two adjacent consecutive terms in counting numbers starting from 1. Write the algebraic expression of this sequence.

(Score: 3, Time: 4 minutes)

■ Scoring indicators

- Counting numbers : 1, 2, 3, 4, 5, ... (1)
- Sequence obtained by adding two adjacent consecutive terms : $1 + 2, 2 + 3, 3 + 4, 4 + 5, \dots$ (1)
: 3, 5, 7, 9, ...
- Algebraic expression of above sequence : $n + (n + 1) = 2n + 1$ (1)

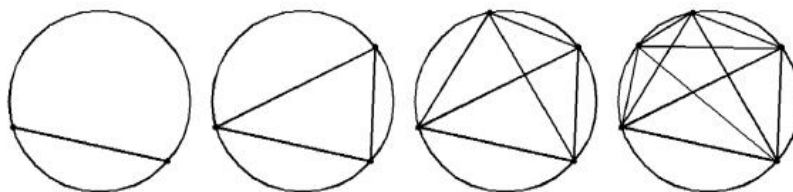
2. Learning Outcomes

- Formation of sequence from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.

? Consider circles, points on its circumference and chords as shown in the figure. Mark two points on the circle and draw a chord. Mark one more point and draw three chords. Continue this process by adding one more point each time.

- Write the number of chords in each figure as a sequence.
- Write the algebraic expression of this sequence
- Find the number of chords in the 10th figure.

(Score: 4, Time: 7 minutes)



■ Scoring indicators

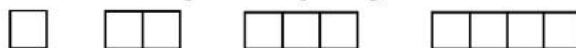
- (a) No. of chords in figure.1 = 1
 No. of chords in figure.2 = $1 + 2 = 3$
 No. of chords in figure.3 = $1 + 2 + 3 = 6$ (1)
 Sequence of number of chords = 1, 3, 6, 10, ... (1)
- (b) No. of chords in figure. n = $1 + 2 + 3 + \dots + n$
 $= \frac{n(n+1)}{2}$ (1)
- (c) No. of chords in the 10th figure = $\frac{10 \times 11}{2} = 55$ (1)

3. Learning Outcomes

- Formulating sequence from practical situations.
- Formulating the relation between terms, position of terms and expressing it in algebraic form.
- Finding a particular term from the algebraic expression.



A pattern is formed using sticks of equal length as shown below:



- a) Write the sequence of number of sticks used in each figure.
 b) Write the sequence of number of squares and rectangles in each figure.
 c) Write the algebraic expression in the above two sequences
 d) Find the number of sticks and squares in the 10th figure

(Score: 4, Time: 9 minutes)

■ Scoring indicators

- (a) No of sticks in the figure.1 = $1 + 3 = 4$
 No of sticks in the figure.2 = $1 + 3 + 3 = 1 + 2 \times 3 = 7$
 No of sticks in the figure.3 = $1 + 3 \times 3 = 10$
 No of sticks in the figure.4 = $1 + 4 \times 3 = 13$
 Sequence of number of sticks = 4, 7, 10, 13, ... (1)
- (b) No of squares and rectangles in the figure.1 = 1
 No of squares and rectangles in the figure.2 = $2 + 1 = 3$
 No of squares and rectangles in the figure.3 = $3 + 2 + 1 = 6$
 No of squares and rectangles in the figure.4 = $4 + 3 + 2 + 1 = 10$
 Sequence of squares and rectangles = 1, 3, 6, 10, ... (1)
- (c) No of sticks in the n^{th} figure = $1 + n \times 3 = 3n + 1$
 No of squares and rectangles in the n^{th} figure = $1 + 2 + 3 \dots + n$
 $= \frac{n(n+1)}{2}$ (1)

$$(d) \text{ No of sticks in the } 10^{\text{th}} \text{ figure} = 3 \times 10 + 1 = 31$$

$$\text{No of squares and rectangles in the } 10^{\text{th}} \text{ figure} = \frac{10 \times 11}{2} = 55 \quad (1)$$

4. Learning Outcomes

- Formulating an arithmetic sequence when a term, position of the term, common difference are known.
- The difference between any two terms of an arithmetic sequence will be a multiple of common difference.

? Consider an arithmetic sequence with common difference 6 and 7th term 52. Find the 15th term of the arithmetic sequence. Is it possible, to get a difference of 100 between any two terms of this sequence?

(Score: 3, Time: 5 minutes)

■ Scoring indicators

- 15th term can be obtained by adding 8 times the common difference to the 7th term (1)

$$\begin{aligned} \text{ie.,} \quad x_{15} &= x_7 + 8d \\ &= 52 + 8 \times 6 = 100 \end{aligned} \quad (1)$$

The difference between any two terms of an Arithmetic sequence will be a multiple of common difference

100 can't be the difference between any two terms of this sequence, since it is not a multiple of 6. (1)

5. Learning Outcomes

- In an arithmetic sequence the difference between any two terms will be the product of common difference and difference of term positions of each term.
- Finding any term of an arithmetic sequence where a term, its position and, common difference are given

? Consider an arithmetic sequence whose 7th term is 34 and 15th term is 66.

- Find the common difference.
- Find the 20th term.

(Score: 3, Time: 5 minutes)

■ Scoring indicators

- 15th term can be obtained by adding 7th term and 8 times the common difference.

$$x_{15} = x_7 + 8d \quad (1)$$

$$66 = 34 + 8d$$

$$8d = 66 - 34 = 32$$

$$d = \frac{32}{8} = 4 \quad (1)$$

- (b) 20th term can be obtained by adding 15th term and 5 times the common difference.

$$\begin{aligned}x_{20} &= x_{15} + 5d \\ &= 66 + 5 \times 4 = 86\end{aligned}\quad (1)$$

6. Learning Outcomes

- Formulating an algebraic expression connecting terms and its position in arithmetic sequence.
- Finding a particular term using algebraic expression in an arithmetic sequence.

? Consider an arithmetic sequence $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \dots$

- (a) Write the algebraic expression of the sequence.
 (b) Write the sequence of counting numbers in the above given sequence. Is the newly obtained sequence an arithmetic sequence.

(Score: 4, Time: 6 minutes)

■ Scoring indicators

(a) Common Difference = $\frac{20}{7} - \frac{17}{7} = \frac{3}{7}$ (1)

Algebraic expression of the sequence = $\frac{17}{7} + (n-1)\frac{3}{7}$

$$\begin{aligned}x_n &= \frac{3}{7}n + \frac{17}{7} - \frac{3}{7} \\ &= \frac{3}{7}n + 2\end{aligned}\quad (1)$$

(b)
$$\begin{aligned}x_n &= \frac{3}{7}n + 2 \\ x_7 &= \frac{3}{7} \times 7 + 2 = 5 \\ x_{14} &= \frac{3}{7} \times 14 + 2 = 8 \\ x_{21} &= \frac{3}{7} \times 21 + 2 = 11 \\ x_n &= \frac{3}{7} \times n + 2\end{aligned}\quad (1)$$

If we give multiples of 7 to n in the expression $\frac{3}{7}n + 2$, we get counting numbers.

$$\begin{aligned}\text{ie., } x_n &= \frac{3}{7} \times 7m + 2 \\ &= 3m + 2\end{aligned}$$

Thus 5, 8, 11..... is an arithmetic sequence with common difference 3 (1)

7. Learning Outcomes

- Formulating an algebraic expression connecting terms and its position in arithmetic sequence.
- Finding a particular term using algebraic expression in an arithmetic sequence.

? Considering an arithmetic sequence $\frac{17}{7}, \frac{31}{7}, \frac{45}{7}, \dots$

- Write the algebraic expression of the sequence.
- Is there any counting number in this sequence? Justify your answer?

(Score: 3, Time: 6 minutes)

■ **Scoring indicators**

a. Common difference $= \frac{31}{7} - \frac{17}{7} = \frac{14}{7} = 2$ (1)

$$\begin{aligned} \text{Algebraic expression of the sequence} &= \frac{17}{7} + (n-1)2 \\ &= 2n + \frac{17}{7} - 2 \\ &= 2n + \frac{3}{7} \end{aligned} \quad (1)$$

For any counting number 'n', 2n will be a counting number. But when $\frac{3}{7}$ is added to 2n it becomes a fraction. So this sequence will not have a counting number. (1)

8. Learning Outcomes

- In an arithmetic sequence the difference between any two terms will be the product of common difference and difference of term position of each term.

? x_n is the n^{th} term of an arithmetic sequence. If $x_a, x_b, x_c, \dots$ are in arithmetic sequence, then prove that a,b,c are in arithmetic sequence.

(Score: 4, Time: 8 minutes)

■ **Scoring indicators**

- If 'd' is the common difference of an arithmetic sequence, then

$$\begin{aligned} x_n - x_m &= (n - m)d \\ \Rightarrow x_b - x_a &= (b - a)d \quad (1) \\ x_c - x_b &= (c - b)d \quad (1) \end{aligned}$$

Given ; $x_a, x_b, x_c, \dots$ are in arithmetic sequence.

$$\begin{aligned} \Rightarrow x_b - x_a &= x_c - x_b \\ \Rightarrow (b - a)d &= (c - b)d \quad (1) \\ b - a &= c - b \end{aligned}$$

$$\therefore a, b, c, \dots \text{ are in arithmetic sequence.} \quad (1)$$

9. Learning Outcomes

- In an arithmetic sequence the difference between any two terms will be equal to the product of common difference and difference between the position of the terms.
- Finding other terms in an arithmetic sequence when common difference, a term and its position are given.

? Find the 20th term of an arithmetic sequence if its 6th term is 14 and 14th term is 6.

(Score: 2, Time: 3 minutes)

■ Scoring indicators

$$\begin{aligned}x_6 &= 14; x_{14} = 6 \\x_{14} - x_6 &= (14 - 6)d & (1) \\6 - 14 &= 8d \\d &= -1 & (1) \\x_{20} &= x_{14} + 6d \\&= 6 + 6 \times -1 = 0 & (1)\end{aligned}$$

10. Learning Outcomes

- In an arithmetic sequence the difference between any two terms will be equal to the product of common difference and difference between the position of the terms.
- Finding other terms in an arithmetic sequence when common difference, a term and its position are given.

? Consider an arithmetic sequence whose m^{th} term is ' n ' and n^{th} term is ' m '.

- Find the common difference of the sequence
- Prove that $(m + n + p)^{\text{th}}$ term of the sequence is ' $-p$ '

(Score: 3, Time: 7 minutes)

■ Scoring indicators

$$\begin{aligned}\text{(a) } x_m &= n; x_n = m \\x_m - x_n &= (m - n)d & (1) \\n - m &= (m - n)d \\d &= \frac{n - m}{m - n} = \frac{-(m - n)}{m - n} = -1 & (1) \\\text{(b) } x_{m+n+p} &= x_m + (m + n + p - m)d \\&= n + (n + p) \times -1 \\&= n - n - p = -p & (1)\end{aligned}$$

11. Learning Outcomes

- Finding other terms of an arithmetic sequence whose common difference, a term and its position are given

? Find the 13th term of an arithmetic sequence if 5 times the 5th term is equal to 8 times the 8th term

(Score: 4, Time: 6 minutes)

■ Scoring indicators

- 5th term = x_5
 - 8th term = $x_5 + 3d$ (1)
- $$5x_5 = 8(x_5 + 3d) \quad (1)$$
- $$5x_5 = 8x_5 + 24d$$
- $$-3x_5 = 24d \quad (1)$$
- $$x_5 = -8d$$
- $$x_{13} = x_5 + 8d$$
- $$= -8d + 8d = 0 \quad (1)$$

12. Learning Outcomes

Understanding properties of an arithmetic sequence from its algebraic expression

? Prove that the arithmetic sequence 7, 11, 15, ... does not contain perfect square.

(Score: 4, Time: 6 minutes)

■ Scoring indicators

- The algebraic expression of the arithmetic sequence is $x_n = 4n + 7 - 4 = 4n + 3$ (1)
- Each term in this sequence will give a remainder 3 when divided by 4. All counting numbers greater than 7, when divided by 4 will give a remainder 3 will be a term in this sequence. (1)

Any counting number can be written in the form $4n, 4n + 1, 4n + 2$

$$\Rightarrow (4n)^2 = 16n^2$$

$$(4n + 1)^2 = 16n^2 + 8n + 1$$

$$(4n + 2)^2 = 16n^2 + 16n + 4 \quad (1)$$

- This shows that $(4n)^2, (4n + 2)^2$ are exactly divisible by 4, i.e., remainder will be zero. But when $(4n + 1)^2$ is divided by 4 the remainder is 1. This shows that when a perfect square is divided by 4 there remainder is 0 or 1. But in the sequence 7, 11, 15, ... when terms are divided by 4 remainder is 3. So a perfect square will not be a term in this sequence. (1)

13. Learning Outcomes

Understanding the properties of an arithmetic sequence from its algebraic expression.

- ?** Let the algebraic expression of an arithmetic sequence be $5n + b$. If there is no perfect square in this sequence, find the counting number less than 5 that can be the value of 'b'.

(Score: 4, Time: 5 minutes)

■ **Scoring indicators**

- Any counting number can be written in the form $5n, 5n + 1, 5n + 2$
 $(5n)^2 = 25n^2$; when divided by 5, remainder = 0 (1)
 $(5n + 1)^2 = 25n^2 + 10n + 1$; when divided by 5, remainder = 1 (1)
 $(5n + 2)^2 = 25n^2 + 20n + 4$; when divided by 5, remainder = 4 (1)
 $\Rightarrow$ Any perfect square when divided by 5 leaves remainder 0, 1, 4. So if remainder is 2 and 3 then it will not be a perfect square.
 $\Rightarrow$ If there is not perfect square in the sequence, the only possibility for value of 'b' less than 5 is 2 and 3 (1)

14. Learning Outcomes

- Understanding the properties of an arithmetic sequence from its algebraic expression.

- ?** Consider the arithmetic sequence 10, 16, 22, ... Can you find out any terms as the sum or difference of any two terms of this sequence.

(Score: 5, Time: 5 minutes)

■ **Scoring indicators**

- Algebraic expressions of the sequence $x_n = 6n + 4$
Each term when divided by 6, remainder is 4. (1)
 n^{th} term $= 6n + 4$
 m^{th} term $= x_m = 6m + 4$
 $x_n + x_m = 6n + 4 + 6m + 4$ (1)
 $= 6(n + m) + 8 = 6(n + m) + 6 + 2$
 $\Rightarrow x_n + x_m$ when divided by 6 will give remainder '2'. Therefore $x_n + x_m$ can't be a term in the given sequence (1)
Similarly; $x_n - x_m = (6n + 4) - (6m + 4)$ (1)
 $= 6n - 6m = 6(n - m)$ (1)
 $x_n - x_m$ when divided by 6 will give remainder '0'. Therefore $x_n - x_m$ can't be a term in the given sequence. (1)

15. Learning Outcome

- Understanding the properties of an arithmetic sequence from its algebraic expression.

- ❓ Prove that the square of any term of the arithmetic sequence 7, 11, 15, ... will not be a term of the sequence.

(Score: 3, Time: 6 minutes)

■ Scoring indicators

- Algebraic expression of the sequence $x_n = 4n + 3$. (1)

⇒ Each term when divided by 4 will leave the remainder 3.

$$\begin{aligned} x_n^2 &= (4n + 3)^2 = 16n^2 + 24n + 9 \\ &= 16n^2 + 24n + 8 + 1 \end{aligned} \quad (1)$$

x_n^2 when divided by '4' will leave remainder '1'.

⇒ x_n^2 will not be a term of the given sequence. (1)

16. Learning Outcomes

- Formulating the algebraic expression of an arithmetic sequence when a term and common difference is given
- Finding the position of term using the algebraic expression.

- ❓ Find the 110th term in the arithmetic sequence 5, 12, 19, ...

(Score: 2, Time: 4 minutes)

■ Scoring indicators

- Common difference = 7

110 will be equal to sum of 'n' times the common difference and first term.

$$5 + 7n = 110 \quad (1)$$

$$7n = 110 - 5 = 105$$

$$n = \frac{105}{7} = 15$$

$$\begin{aligned} \Rightarrow 110 &= x_1 + 15d \\ &= x_{16} \end{aligned} \quad (1)$$

∴ 110 be the 16th term in the given sequence.

17. Learning Outcomes

- Comparing sequences using their algebraic expressions.

- ❓ Consider two arithmetic sequences given below: 11, 19, 27, ... and 50, 55, 60, ...

Is there a common number to these sequences at same term position? If yes, find the term positions. Find the term?

(Score: 4, Time: 8 minutes)

■ Scoring indicators

For the sequence 11, 19, 27, ... ; $x_n = 8n + 11 - 8 = 8n + 3$ (1)

For the sequence 50, 55, 60, ... ; $x_n = 5n + 50 - 5 = 5n + 45$ (1)

If n^{th} terms are equal for both sequences

$$8n + 3 = 5n + 45 \quad (1)$$

$$8n - 5n = 45 - 3 = 42$$

$$3n = 42$$

$$n = \frac{42}{3} = 14 \quad (1)$$

The 14^{th} term of both sequences are equal.

$$14^{\text{th}} \text{ term} = 8 \times 14 + 3 = 112 + 3 = 115$$

18. Learning Outcomes

- Understanding the properties of an arithmetic sequence using their algebraic expression.

? Consider the arithmetic sequence $-74, -68, -62, \dots$. How many negative numbers are there in this sequence? Find the first positive number in this sequence?

(Score: 5, Time: 9 minutes)

■ Scoring indicators

- Common difference = 6
- Algebraic expression of the sequence $= 6n + -74 - 6$

$$\Rightarrow x_n = 6n - 80 \quad (1)$$

If zero is a term in the sequence, then

$$6n - 80 = 0$$

$$6n = 80 \quad (1)$$

$$n = \frac{80}{6} = 13 + \frac{2}{6} \quad (1)$$

Here n is not a counting number. This shows that zero is not a term in the sequence. Hence upto zero there are 13 terms.

Therefore there are 13 negative numbers in this sequence. (1)

$$\begin{aligned} \text{The first positive number will be } 14^{\text{th}} \text{ term} &= x_{14} \\ &= 6 \times 14 - 80 \\ &= 84 - 80 = 4 \end{aligned} \quad (1)$$

19. Learning Outcome

- Understanding the properties of an arithmetic sequence using their algebraic expression.

? Is it possible that the product of any two terms in the arithmetic sequence $9, 16, 23, \dots$ a term in it? Justify your answer.

(Score: 4, Time: 8 minutes)

■ Scoring indicators

- $9, 16, 23, \dots$; the algebraic expression $= 7n + 2$ (1)
Assume that $7n + 2$ and $7m + 2$ are two terms in this sequence (1)

$$(7n + 2)(7m + 2) = 49mn + 14n + 14m + 4 \quad (1)$$

This shows that when $(7n + 2)(7m + 2)$ divided by '7' will leave the remainder '4'.
But the terms of the sequence when divided by '7' should leave a remainder '2'.
Hence the product of any two terms is not a term in the sequence. (1)

20. Learning Outcome

- The sum of first and third term is equal to twice the second term.



$2x + 1, 4x - 1, 5x + 1, \dots$ are in an arithmetic sequence.

- Find x ?
- Write the algebraic expression of the sequence.
- Find the position of 195 in this sequence

(Score: 4, Time: 7 minutes)

■ Scoring indicators

- If a, b, c are three consecutive terms of an arithmetic sequence, then
 $2b = a + c$

$$\begin{aligned} 8x - 2 &= 7x + 2 \\ 2(4x - 1) &= 2x + 1 + 5x + 1 \end{aligned} \quad (1)$$

$$\begin{aligned} 8x - 2 &= 7x + 2 \\ 8x - 7x &= 2 + 2 \\ x &= 4 \end{aligned} \quad (1)$$

$\therefore$ the sequence is 9, 15, 21, ...

The algebraic expression of this sequence ; $x_n = 6n + 3$

$$6n + 3 = 195 \quad (1)$$

$$\begin{aligned} 6n &= 195 - 3 \\ &= 192 \end{aligned}$$

$$\begin{aligned} n &= \frac{192}{6} \\ &= 32 \end{aligned} \quad (1)$$

$\therefore$ 32nd term of the sequence is 195

21. Learning Outcome

- In an arithmetic sequence when the sum of odd number of terms divided by the number of terms we get the middle term.



The angles in a 9 sided polygon are in arithmetic sequence. Is 100° the smallest angle of the polygon? Justify your answer.

(Score: 5, Time: 10 minutes)

■ Scoring indicators

$$\text{The sum of angles in a 9 sided polygon} = 7 \times 180 \quad (1)$$

$$\text{The 5th term in the arithmetic sequence} = \frac{7 \times 180}{9} = 140 \quad (1)$$

If 100° is the first angle, then

$$\begin{aligned} x_5 &= x_1 + 4d \\ 140 &= 100 + 4d \\ d &= 10 \end{aligned} \quad (1)$$

$$\begin{aligned} 9^{\text{th}} \text{ angle ; } \quad x_9 &= x_5 + 4d \\ 140 + 4 \times 10 &= 180 \end{aligned} \quad (1)$$

Therefore 100° is not the smallest angle in the polygon since 180° is not an angle of the polygon (1)

22. Learning Outcome

- The sum of first n counting numbers is $\frac{n(n+1)}{2}$

- ?** (a) Find the sum of first 25 counting numbers
 (b) Find the sum of first 25 even numbers
 (c) Find the sum of first 25 odd numbers

(Score: 4, Time: 9 minutes)

■ Scoring indicators

$$(a) \quad 1 + 2 + 3 + \dots + 25 = \frac{25 \times 26}{2} = 25 \times 13 = 325 \quad (1)$$

$$(b) \quad 2 + 4 + 6 + \dots + 50 = 2(1 + 2 + 3 + \dots + 25) = 2 \times \frac{25 \times 26}{2} = 650 \quad (1)$$

$$\begin{aligned} (c) \quad 1 + 3 + 5 + \dots + 49 &= (2 - 1) + (4 - 1) + (6 - 1) + \dots + (50 - 1) \\ &= 2 + 4 + 6 + \dots + 50 - (1 + 1 + \dots + 1) \\ &= 2(1 + 2 + 3 + \dots + 25) - (1 \times 25) \\ &= 650 - 25 = 625 \end{aligned} \quad (1)$$

23. Learning Outcome

- The sum of first n counting numbers is $\frac{n(n+1)}{2}$

- ?** Prove that the sum of first n odd numbers is n^2 .

(Score: 4, Time: 7 minutes)

■ Scoring indicators

$$\bullet \quad 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad (1)$$

$$\bullet \quad 1 + 3 + 5 + \dots + 2n - 1 = (2 - 1) + (4 - 1) + (6 - 1) + \dots + (2n - 1) \quad (1)$$

$$\begin{aligned} &= (2 + 4 + 6 + \dots + 2n) - \underbrace{(1 + 1 + \dots + 1)}_{n \text{ times}} \\ &= 2(1 + 2 + 3 + \dots + n) - (1 \times n) \end{aligned} \quad (1)$$

$$= 2 \frac{n(n+1)}{2} - n = n^2 + n - n = n^2 \quad (1)$$

24. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term ' $an + b$ ' is

$$a \frac{n(n+1)}{2} + bn$$



Find the sum of first 25 terms of the arithmetic sequence 5, 8, 11, ...

(Score: 4, Time: 6 minutes)

■ Scoring indicators

The n^{th} term of the sequence 5, 8, 11, ... is

$$x_n = 3n + 2 \quad (1)$$

$$x_1 = 3 \times 1 + 2$$

$$x_2 = 3 \times 2 + 2$$

$$x_3 = 3 \times 3 + 2$$

$$x_{25} = 3 \times 25 + 2 \quad (1)$$

$$x_1 + x_2 + \dots + x_{25} = 3(1 + 3 + \dots + 25) + 2 \times 25 \quad (1)$$

$$= 3 \times \frac{25 \times 26}{2} + 50 \quad (1)$$

$$= 1025$$

25. Learning Outcomes

- In an arithmetic sequence the difference between two terms is equal to the product of common difference and difference between the term position.
- The sum of first n terms of an arithmetic sequence whose first term is x_1 and n^{th} term x_n is $\frac{n}{2}(x_1 + x_n)$



Consider an arithmetic sequence whose 6th term is 40 and 9th term is 58

- Find the 25th term of the sequence
- Find the sum of first 25 terms of the sequence
- Find the sum of first n terms of the sequence

(Score: 5, Time: 7 minutes)

■ Scoring indicators

$$(a) \quad x_9 = x_6 + 3d \quad (1)$$

$$58 = 40 + 3d$$

$$3d = 58 - 40 = 18$$

$$d = 6$$

$$x_1 = x_6 - 5d = 40 - 5 \times 6 = 10 \quad (1)$$

$$x_{25} = x_6 + 19d = 40 + 19 \times 6 = 154 \quad (1)$$

$$\begin{aligned} \text{(b) Sum of first 25 terms} &= \frac{25}{2} (x_1 + x_{25}) \\ &= \frac{25}{2} (10 + 154) \\ &= \frac{25}{2} \times 164 = 2050 \end{aligned} \quad (1)$$

$$\text{(c) } n^{\text{th}} \text{ term; } x_n = 6n + 4$$

$$\begin{aligned} \text{Sum of first 'n' terms} &= \frac{n}{2} (x_1 + x_n) \\ &= \frac{n}{2} (10 + 6n + 4) \\ &= n(3n + 7) \\ &= 3n^2 + 7n \end{aligned} \quad (1)$$

26. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term $an + b$ is $\frac{an(n+1)}{2} + bn$

? Let the algebraic expression of an arithmetic sequence be $6n + 3$.

- Find the sum of first 20 terms of the sequence
- Write the algebraic expression of the sum.

(Score: 5, Time: 9 minutes)

■ Scoring indicators

$$\text{Given ; } x_n = 6n + 3 \quad (1)$$

$$\begin{aligned} \text{(a) Sum of 20 terms} &= \frac{6 \times 20 \times 21}{2} + 3 \times 20 \\ &= 1260 + 60 = 1320 \end{aligned} \quad \begin{matrix} (1) \\ (1) \end{matrix}$$

$$\begin{aligned} \text{(b) Sum of } n \text{ terms} &= \frac{6n(n+1)}{2} + 3 + n \\ &= 3n^2 + 3n + 3n \\ &= 3n^2 + 6n \end{aligned} \quad \begin{matrix} (1) \\ (1) \end{matrix}$$

27. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term is ' $an + b$ ' is $\frac{an(n+1)}{2} + bn$.

? (a) Find the sum of first 20 terms of counting numbers.

- Consider an arithmetic sequence whose common difference is '7' and sum of first 20 terms is '1530'. Write the algebraic expression of the sequence.

(Score: 4, Time: 6 minutes)

■ Scoring indicators

$$(a) \quad 1 + 2 + 3 + \dots + 20 = \frac{20 \times 21}{2} = 210 \quad (1)$$

(b) The n^{th} term of an arithmetic sequence with common difference 7 is $x_n = 7n + b$

$$\text{Sum of first '20' terms} = 7 \times 210 + b \times 20 = 1530 \quad (1)$$

$$\Rightarrow 1470 + b \times 20 = 1530$$

$$20b = 60 \quad (1)$$

$$b = 3$$

Therefore the algebraic expression of the sequence is $7n + 3$ (1)

28. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term $an + b$ is $\frac{an(n+1)}{2} + bn$ ×



Consider an arithmetic sequence whose sum of first 10 terms is 250 and sum of first 16 terms is 592

- Write the algebraic expression of the sequence
- Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 10 minutes)

■ Scoring indicators

- Assume that the algebraic expression of the sequence $x_n = an + b$

$$\text{Sum of first 10 terms} = a \times \frac{10 \times 11}{2} + b \times 10 = 250$$

$$\Rightarrow \frac{11}{2}a + b = 25 \quad (1)$$

$$\text{Sum of first 16 terms} = a \times \frac{16 \times 17}{2} + b \times 16 = 592$$

$$\Rightarrow \frac{17}{2}a + b = 37 \quad (2)$$

$$(2) - (1) \Rightarrow 3a = 12$$

$$a = 4$$

$$(1) \quad \Rightarrow \frac{11}{2} \times 4 + b = 25$$

$$b = 3 \quad (1)$$

Therefore the algebraic expression,

$$x_n = an + b$$

$$= 4n + 3 \quad (1)$$

- Assume that the algebraic expression for the sum of first n terms,

$$S_n = \frac{an(n+1)}{2} + bn$$

$$\begin{aligned}
 &= \frac{4n(n+1)}{2} + 3n \\
 &= 2n(n+1) + 3n \\
 &= 2n^2 + 2n + 3n \\
 &= 2n^2 + 5n \quad (1)
 \end{aligned}$$

29. Learning Outcome

- The sum of first n terms of an arithmetic sequence whose n^{th} term is $an + b$ is $\frac{an(n+1)}{2} + bn$

? If the sum of first $(n+1)$ terms of an arithmetic sequence is $pn^2 + qn + r$, then show that $p+r=q$. Which of the following is the sum of first $(n+1)$ term of an arithmetic sequence

- a) $2n^2 + 3n + 4$ b) $2n^2 + 3n + 1$

(Score: 5, Time: 10 minutes)

■ Scoring indicators

- n^{th} term of an arithmetic sequence $x_n = an + b$ (1)
- The sum of first ' n ' terms of the sequence $= \frac{a(n+1)(n+2)}{2} + b(n+1)$ (1)

$$\begin{aligned}
 &= \frac{a}{2} (n^2 + 3n + 2) + bn + b \\
 &= \frac{a}{2} n^2 + \left(\frac{3}{2}a + b\right)n + a + b \\
 &= pn^2 + qn + r \\
 p = \frac{a}{2}, q &= \frac{3}{2}a + b, r = a + b \\
 p + r &= \frac{a}{2} + a + b = \frac{3}{2}a + b = q \quad (1) \\
 p + r &= q
 \end{aligned}$$

- (a) Since in $2n^2 + 3n + 4$; $2+4 \neq 3$, so $2n^2 + 3n + 4$ can't be the sum of first $(n+1)$ terms of an arithmetic sequence (1)
- (b) Since in $2n^2 + 3n + 1$; $2+1 = 3$. So $2n^2 + 3n + 1$ is a sum of first n terms of an arithmetic sequence (1)

30. Learning Outcome

- Explaining the method of finding the sum of a particular number of terms in arithmetic sequence whose two terms and their term positions are given.

- ❓ Consider an arithmetic sequence whose sum of first 9 terms is 261 and sum of next 6 terms is 444.

- Find the first term and common difference
- Write the algebraic expression of the sequence
- Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 6 minutes)

■ Scoring indicators

- $$\text{Sum of first 9 terms} = 261$$

$$\text{Middle term} = 5^{\text{th}} \text{ term} = x_5 = \frac{261}{9} = 29 \quad (1)$$

$$\text{Sum of first 15 terms} = 261 + 444 = 705 \quad (1)$$

$$\text{Middle term} = 8^{\text{th}} \text{ term} = x_8 = \frac{705}{15} = 47$$

$$x_8 = x_5 + 3d$$

$$3d = x_8 - x_5 = 47 - 29 = 18$$

$$d = \frac{18}{3} = 6 \quad (1)$$

$$x_1 = x_5 - 4d = 29 - 4 \times 6 = 5 \quad (1)$$
- $$x_n = 6n - 1$$
- $$s_n = 3n^2 + 2n \quad (1)$$

31. Learning Outcome

- The method of finding an arithmetic sequence when the sum of a particular number of terms is given.

- ❓ Consider an arithmetic sequence whose sum of first 10 terms is 230 and sum of first 16 terms is 560

- Find the first term and common difference
- Write the algebraic expression of the sequence
- Write the algebraic expression of the sum of the sequence

(Score: 5, Time: 7 minutes)

■ Scoring indicators

- $$\text{The sum to first 10 terms} = 230$$

$$S_{10} = 230$$

$$\frac{10}{2} (x_1 + x_{10}) = 230$$

$$x_1 + x_{10} = 46 \quad \dots\dots\dots (1)$$

$$\text{The sum to first 16 terms} = 560$$

$$S_{16} = 560$$

$$\frac{16}{2} (x_1 + x_{16}) = 560$$

$$x_1 + x_{16} = 70 \quad \dots\dots\dots (2) \quad (1)$$

$$\begin{aligned}
 (2) - (1) &\Rightarrow & x_{16} - x_{10} &= 24 \\
 && 6d &= 24 \\
 && d &= 4 \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 (1) &\Rightarrow & x_1 + x_{10} &= 46 \\
 && x_1 + x_1 + 9d &= 46 \\
 && 2x_1 &= 46 - 9d = 10 \quad (1) \\
 && x_1 &= 5 \quad (1)
 \end{aligned}$$

$$(b) \quad x_n = 4n + 1$$

$$(c) \quad s_n = 3n^2 + 3n \quad (1)$$

32. Learning Outcome

- Comparing the sum of terms of two arithmetic sequences.



Consider two arithmetic sequences with common difference 4 and difference between their 5th terms is 8

- Find the difference between the first terms
- Find the difference between the sum of first 20 terms of the arithmetic sequences.

(Score: 5, Time: 7 minutes)

■ Scoring indicators

a) Sequence 1: $a, a + 4, a + 8, a + 12, a + 16, \dots$

Sequence 2: $b, b + 4, b + 8, b + 12, b + 16, \dots$

Difference between the 5th terms $= (a + 16) - (b + 16) = 8$

$$a - b = 8 \quad (1)$$

(b) $a + (a + 4) + (a + 8) + (a + 12) + \dots + a + 76$

$b + (b + 4) + (b + 8) + ((b + 12) + \dots + b + 76$

$$\begin{array}{ccccccc}
 8 & + & 8 & + & 8 & + & 8 & + & \dots & + & 8 \\
 \hline
 \end{array} \quad (1)$$

Difference between the sum of first 20 terms $= 8 \times 20$

$$= 160 \quad (1)$$

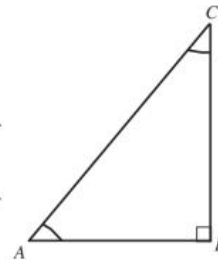
CIRCLES

1. Learning Outcome

- The lines drawn from the end points of a diameter of a circle to any point on the circle make a right angle.

? In the figure $\triangle ABC$ is a right triangle

- If a circle is drawn with AC as diameter find the position of B ?
- If a circle is drawn with BC as diameter, find the position of A ?



(Score: 2, Time: 3 minute)

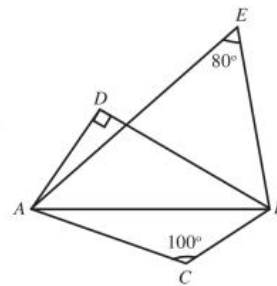
■ Scoring indicators

- | | |
|--------------------|-----|
| On the circle | (1) |
| Outside the circle | (1) |

2. Learning Outcome

- Angle in a semi circle is right.

? A circle is drawn with AB as diameter. Find the positions of the points C, D, E related to the circle.



(Score: 3, Time: 5 minutes)

■ Scoring indicators

- | | |
|-------------------------|-----|
| C inside the circle. | (1) |
| D on the circle. | (1) |
| E outside the circle. | (1) |

3. Learning Outcome

- The half of the central angle of an arc is equal to the angle in the alternate arc.

- ❓ Construct an angle $22\frac{1}{2}^\circ$ without using a protractor.

(Score: 3, Time: 5 minutes)

■ Scoring indicators

Constructing 90°

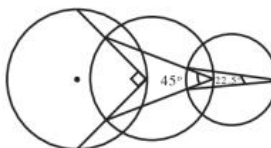
(1)

Constructing 45°

(1)

Constructing $22\frac{1}{2}^\circ$

(1)



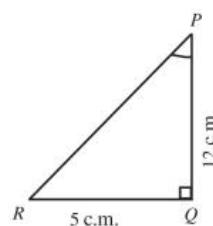
4. Learning Outcome

- Angles in the same arc are equal.
- Angles in a semi circle is 90°

- ❓ In $\triangle ABC$ and $\triangle PQR$, $BC = QR$, $\angle A = \angle P$, $\angle Q = 90^\circ$, $QR = 5$ cm, $PQ = 12$ cm.

Find the diameter of the circumcircle of $\triangle ABC$

(Score : 4, Time 6 : minutes)



■ Scoring indicators

$QR = BC$

(1)

$\angle A = \angle P$

(1)

PR = Diameter of the circumcircle of $\triangle ABC$

(1)

$PR = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm

(1)

5. Learning Outcome

- Angle in a segment are equal.

- ❓ PQ & RS are two mutually perpendicular chords of a circle. $\angle QPR = 50^\circ$ find $\angle PQS$?

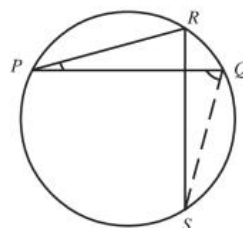
(Score : 3, Time : 4 minutes)

■ Scoring indicators

Draw the picture and mark the angles.

$\angle PRS = 90 - 50 = 40^\circ$

$\angle PQS = 40^\circ$



(1)

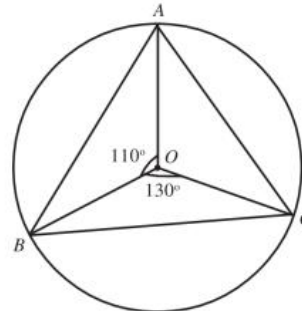
(1)

(1)

6. Learning Outcome

- Relation between the central angle of an arc and an angle in the point on a circle.

- ?** O is the centre of the circle. If $\angle BOC = 130^\circ$ and $\angle AOB = 110^\circ$. What is $\angle AOC$? Find all angles of $\triangle ABC$.



(Score 3, Time 3 minutes)

■ Scoring indicators

$$\angle A = \frac{130}{2} = 65^\circ \quad (1)$$

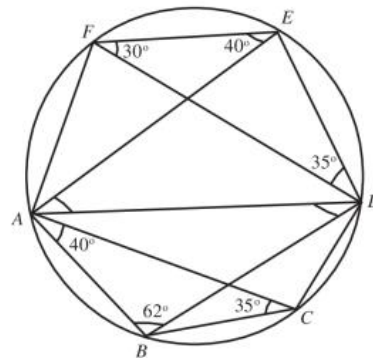
$$\angle C = \frac{110}{2} = 55^\circ \quad (1)$$

$$\angle B = 180 - (65 + 55) = 60^\circ \quad (1)$$

7. Learning Outcome

- Relation between the central angle of an arc and the angles made by that arc points on a circle.

- ?** Find all angles of the hexagon $ABCDEF$.



(Score: 4, Time: 3 minutes)
(1)

■ Scoring indicators

$$\angle EFD = \angle EAD = 30^\circ$$

$$\angle FEA = \angle FDA = 40^\circ$$

$$\angle FDE = \angle FAE = 35^\circ$$

$$\angle BAC = \angle BDC = 45^\circ$$

$$\angle ABD = \angle ACD = 62^\circ$$

$$\angle ACB = \angle ADB = 35^\circ$$

$$\angle A = 148^\circ$$

$$\angle B = 100^\circ$$

$$\angle C = 97^\circ$$

$$\angle D = 155^\circ$$

$$\angle E = 115^\circ$$

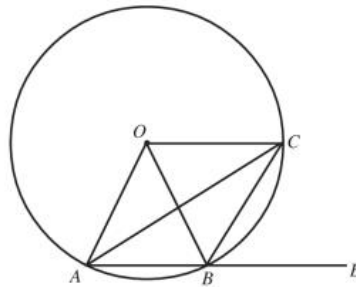
$$\angle F = 105^\circ$$

8. Learning Outcome

- If the vertices of a quadrilateral are on a circle, the angles at the opposite vertices are supplementary.

? O is the centre of the circle and AB is a chord. AC is the bisector of $\angle OAB$, $\angle OAB = 56^\circ$.

- Prove that OC and AB are parallel
- Find $\angle ABC$ and $\angle OBE$.



(Score : 5, Time: 8 minutes)

■ Scoring indicators

$$(a) \angle OAC = \angle BAC \text{ (bisector)} \quad (1)$$

$$\angle OAC = \angle OCA \text{ (Isosceles triangle)}$$

$$\therefore \angle BAC = \angle OCA$$

$$\therefore AB \text{ and } OC \text{ are parallel} \quad (1)$$

$$(b) \angle AOC = 180 - 56 = 124^\circ$$

$$\angle ABC = \frac{360 - 124}{2}$$

$$= \frac{236}{2} = 118^\circ \quad (1)$$

$$\angle CBE = 180 - 118 = 62^\circ$$

$$\angle COB = 2 \times 28 = 56^\circ$$

$$\angle AOB = 124 - 56 = 68^\circ \quad (1)$$

$$\angle OBA = 180 - (56 + 68)$$

$$= 180 - 124 = 56^\circ$$

$$\therefore \angle OBC = 180 - (56 + 62) \quad (1)$$

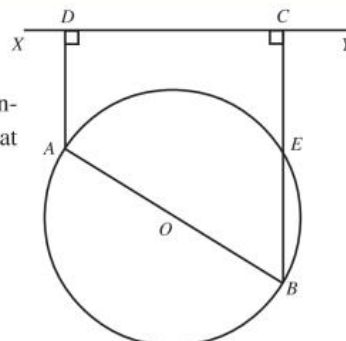
$$= 180 - 118 = 62^\circ$$

$$\therefore \angle OBE = 62 + 62 = 124^\circ$$

9. Learning Outcome

- Angle in a semi circle is right angle.

? O is the centre of the circle. AD and BC are perpendicular to XY . CB cuts the circle at E . Prove that $CE = AD$.



(Score : 3, Time: 4 minutes)

■ Scoring indicators

$$\angle AEB = 90^\circ \text{ (Angle in a semi circle)} \quad (1)$$

$$\angle AEC = 90^\circ$$

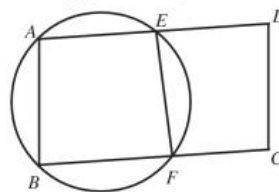
$$\therefore AECD \text{ is a rectangle} \quad (1)$$

$$\therefore AD = CE \quad (1)$$

10. Learning Outcome

- The opposite angles of a cyclic quadrilateral are supplementary.

- **?** $ABCD$ is a parallelogram. A, B, E, F are the points on a circle. $\angle DEF = 80^\circ$. Find out the angles of the quadrilateral $AEFB$.



(Score : 4, Time : 4 minutes)

■ Scoring indicators

$$\angle AEF = 180 - 80 = 100^\circ \quad (1)$$

$$\angle ABF = 180 - 100 = 80^\circ \quad (1)$$

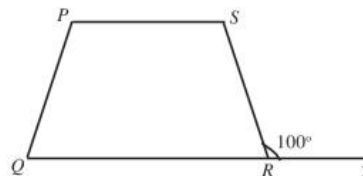
$$\angle A = 180 - 80 = 100^\circ \quad (1)$$

$$\angle EFB = 180 - 100 = 80^\circ \quad (1)$$

11. Learning Outcome

- Opposite angles of a cyclic quadrilateral are supplementary.

- **?** $PQRS$ is a cyclic quadrilateral. QR is extended upto X . $\angle SRX = 100^\circ$, $\angle RPS = 50^\circ$. Find $\angle RPQ$.



(Score: 3, Time : 4 minutes)

■ Scoring indicators

$$\angle SRQ = 180 - 100 = 80^\circ \quad (1)$$

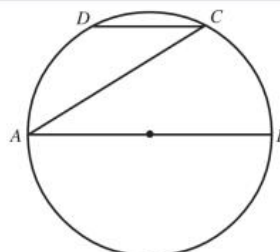
$$\angle SPQ = 180 - 80 = 100^\circ \quad (1)$$

$$\angle RPQ = 100 - 50 = 50^\circ \quad (1)$$

12. Learning Outcome

- Angle in a semi circle is right.

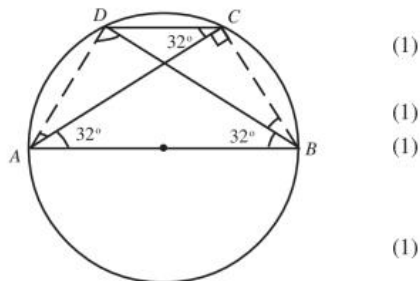
- **?** AB is the diameter of the circle. CD is parallel to AB , $\angle CAB = 32^\circ$. Find $\angle ADC$ and $\angle DAC$.



(Score 4, Time: 8 minutes)

■ Scoring indicators

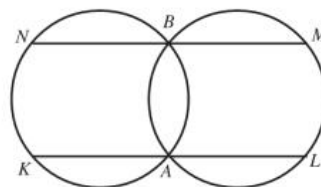
$$\begin{aligned}\angle ACB &= \angle ADB = 90^\circ \\ \angle CAB &= \angle ACD = 32^\circ \\ \angle DCA &= \angle DBA = 32^\circ \\ \angle DAB &= 90 - 32 = 58^\circ \\ \angle CDB &= 32^\circ \\ \angle ADC &= 90 + 32 = 122^\circ \\ \angle DAC &= 58^\circ - 32^\circ \\ &= 26^\circ\end{aligned}$$



13. Learning Outcome

- Recognising if the vertices of a quadrilateral are on a circle, opposite angles are supplementary.

? Two circles intersect at A & B. The lines KAL, NBM are parallel. Prove that KLMN is a parallelogram.



(Score: 4, Time: 4 minutes)

■ Scoring indicators

$$\begin{aligned}\angle N &= x & (1) \\ \angle BAL &= x & (1) \\ \angle M &= 180 - x & (1)\end{aligned}$$

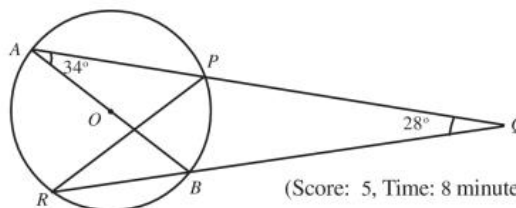
$\therefore$ KLMN is a parallelogram (1)

14. Learning Outcome

- Understanding the relationship between the central angle of an arc and angles formed at various points on a circle.

? AB is a diameter of the circle. A, P, B, R are four points on the circle.

The lines AP, RB intersect at Q. Find $\angle PRB$, $\angle PBR$, $\angle BPR$.



(Score: 5, Time: 8 minutes)

■ Scoring indicators

$$\begin{aligned}\angle PAB &= \angle PRB = 34^\circ & (1) \\ \angle APB &= 90^\circ & (1) \\ \angle ARB &= 90^\circ & (1) \\ \therefore \angle ARP &= 90 - 34 = 56^\circ & (1)\end{aligned}$$

$$\begin{aligned}
 \angle ABP &= 56^\circ \\
 \angle RPQ &= 180 - (28 + 34) = 118^\circ \\
 \angle APR &= 62 = \angle ABR \\
 \angle PBR &= 62 + 56 = 118^\circ \\
 \angle RAP &= 180 - 118 = 62^\circ \\
 \angle RAB &= 62 - 34 = 28^\circ \\
 \angle BPR &= 28^\circ
 \end{aligned}$$

(1)

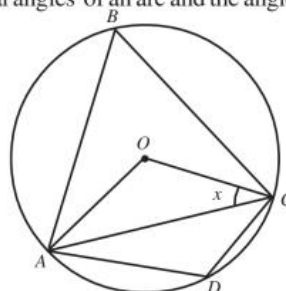
(1)

15. Learning Outcome

- Understanding the relationship between the central angles of an arc and the angles by that arc at various points on a circle.

? O is the centre of the circle. $\angle OCA = x^\circ$. Find

- Find $\angle OAC$
- Prove that $\angle OCA + \angle ABC = 90^\circ$.
- Prove that $\angle ADC - \angle OCA = 90^\circ$.



(Score: 5, Time: 8 minutes)

■ Scoring indicators

- $\angle OAC = x$ (1)
- $\angle AOC = 180 - 2x$ (1)
- $\angle B = \frac{180 - 2x}{2} = 90 - x$ (1)
- $\angle OCA + \angle ABC = x + 90 - x = 90^\circ$ (1)
- $\angle ADC = 180 - (90 - x)$ (1)
- $= 90 + x$ (1)
- $\angle ADC - \angle OCA = 90 + x - x = 90^\circ$ (1)

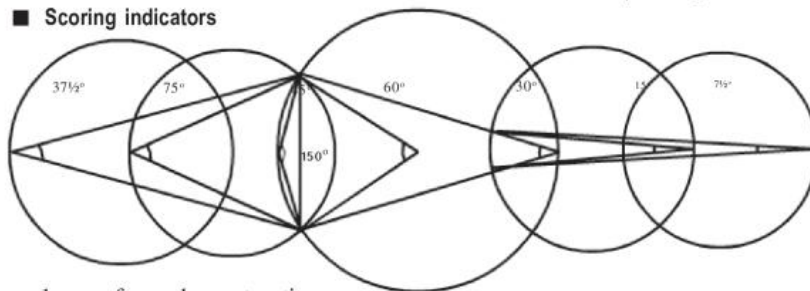
16. Learning Outcome

- Understanding half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

? Draw an equilateral triangle. Draw a circle with centre as the vertex of the equilateral triangle and passes through other two vertices of it. Construct angles with measurements 30° , 15° , $7\frac{1}{2}^\circ$, 150° , 75° using scale and compasses with the help of the drawn figure.

(Score 5, Time: 5 minutes)

■ Scoring indicators



1 score for each construction.

17. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

? Draw a circle with radius 5cm. Draw a triangle with its vertices on the circle and having angles 35° , 72° , 73° .

(Score: 4, Time: 5 minutes)

■ Scoring indicators

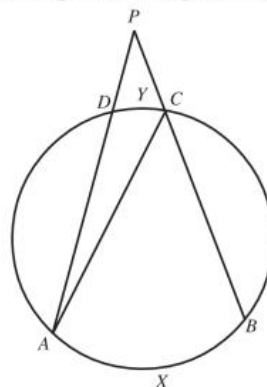
- Draw the circle. (1)
- Draw 70° , 144° angles at the centre. (1)
- Completing the triangle. (1)
- Measuring the sides. (1)

18. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

? In the figure central angle of arc AXB is 100° central angle of arc CYD is 30° .

Find $\angle CAD$ and $\angle ACB$. Find the angles of $\triangle APC$.



(Score: 4, Time: 5 minutes)

■ Scoring indicators

$$\angle CAD = \frac{30}{2} = 15^\circ \quad (1)$$

$$\angle ACB = \frac{100}{2} = 50^\circ \quad (1)$$

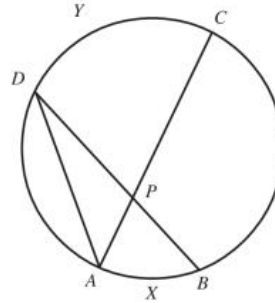
$$\angle PCA = 180 - 50 = 130^\circ \quad (1)$$

$$\angle P = 35^\circ \quad (1)$$

19. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

- ?** In the figure, the central angle of arc AXB is 40° , central angle of arc CYD is 70° . Find the angles of $\triangle APD$



(Score : 3, Time: 4 minutes)

■ Scoring indicators

$$\angle D = \frac{40}{2} = 20^\circ \quad (1)$$

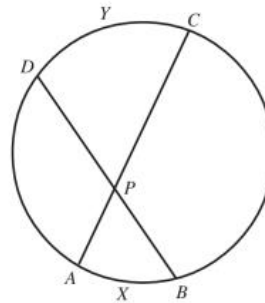
$$\angle A = \frac{70}{2} = 35^\circ \quad (1)$$

$$\begin{aligned} \angle APD &= 180 - (35 + 20) \\ &= 125^\circ \end{aligned} \quad (1)$$

20. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

- ?** In the figure, prove that $\angle APB$ is equal to half the sum of the central angles of arc AXB and arc DYC.



(Score : 3, Time: 4 minutes)

■ Scoring indicators

$$\angle ADP = \frac{\text{Central angle of arc AXB}}{2} \quad (1)$$

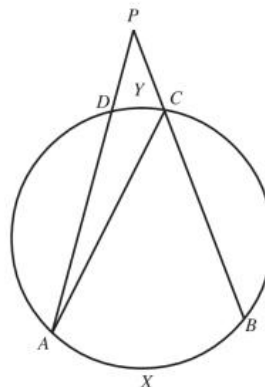
$$\angle DAC = \frac{\text{Central angle of arc DYC}}{2} \quad (1)$$

$$\angle APB = \angle ADP + \angle DAC \quad (1)$$

21. Learning Outcome

- Understanding the half the central angle of an arc is equal to the angle made by that arc on the alternate arc.

- ❓ Prove that $\angle APB$ is equal to half the difference of the central angles of arc AXB and arc DYC .



(Score: 3, Time: 4 minutes)

■ Scoring indicators

$$\angle ACB = \frac{\text{Central angle of arc } AXB}{2} \quad (1)$$

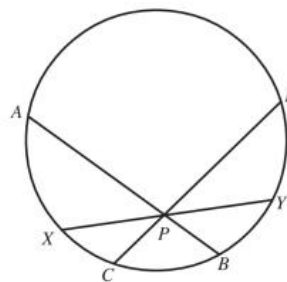
$$\angle DAC = \frac{\text{Central angle of arc } CYD}{2} \quad (1)$$

$$\angle P = \angle ACB - \angle DAC \quad (1)$$

22. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

- ❓ In the figure, the chords AB , CD , XY intersect at P . $AP = 9$ cm, $AB = 13$ cm, $PD = 12$ cm. Find CD . If $PX = PY$, find XY .



(Score :3, Time: 4 minutes)

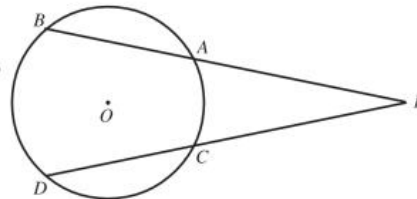
■ Scoring indicators

$$\begin{aligned} PA \times PB &= PC \times PD \\ 9 \times 4 &= PC \times 12 \\ PC &= 3 \\ CD &= 12 + 3 = 15 & (1) \\ PX \times PY &= 36 & (1) \\ PX &= PY = \sqrt{36} = 6 \text{ cm} & (1) \end{aligned}$$

23. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

- ?** In the figure $PA = 3$ cm, $AB = 9$ cm, $PC = 4$ cm Find CD .



(Score : 3, Time: 4 minutes)

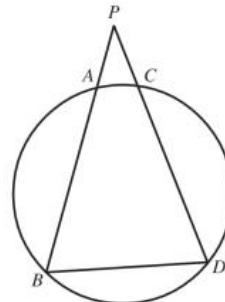
■ Scoring indicators

$$\begin{aligned}
 PA \times PB &= PC \times PD \\
 3 \times 12 &= 4(4 + CD) & (1) \\
 36 &= 16 + 4CD & (1) \\
 CD &= 5 \text{ cm} & (1)
 \end{aligned}$$

24. Learning Outcome

- Understanding the relation between the parts into which chords of a circle cut each other.

- ?** In the figure $PA = PC$. Prove that the sides PB and PD of the triangle are equidistant from the centre of the circle.



(Score: 5, Time :5 minutes)

■ Scoring indicators

$$\begin{aligned}
 PA \times PB &= PC \times PD & (1) \\
 PA &= PC & (1) \\
 \therefore PB &= PD & (1) \\
 \therefore AB &= CD & (1)
 \end{aligned}$$

Equal chords of a circle are equidistant from the centre. (1)

25. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

- ?** The sides of a rectangle are 5 cm and 4 cm. Construct a rectangle of the same area with width 7 cm.

(Score: 4, Time: 6 minutes)

■ Scoring indicators

- Drawing the rectangle (1)
- Drawing a circle with a chord of length equal to the sum of the sides of the rectangle (1)
- Drawing a chord $PC = 7$ cm in the circle. (1)
- Construct the rectangle with sides PC and PD (1)

26. Learning Outcome

- Realising that the product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.

- ?** Draw a line of length $\sqrt{7}$ cm.

(Score 3, Time: 4 minutes)

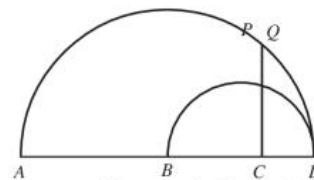
■ Scoring indicators

- Draw a semicircle having diameter $7 + 1 = 8$ cm. (2)
- Draw a perpendicular line. (1)

27. Learning Outcome

- Realising that the product of the parts into which a diameter of a circle is cut by a perpendicular chord, is equal to the square of half the chord.

- ?** In the figure $AD = 10$ cm, $BD = 6$ cm, $CD = 2$ cm. Find PQ .



(Score : 4, Time: 4 minutes)

■ Scoring indicators

$$\begin{aligned}
 AC &= 8 \text{ cm} \\
 AC \times CD &= CQ^2 & (1) \\
 8 \times 2 &= 16 \\
 CQ &= \sqrt{16} = 4 & (1) \\
 BC \times CD &= CP^2 \\
 4 \times 2 &= CP^2 \\
 CP &= \sqrt{8} & (1) \\
 PQ &= 4 - \sqrt{8} & (1)
 \end{aligned}$$

28. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

? Draw a rectangle of sides 5 cm and 3 cm. Construct a square of the same area.

(Score: 4, Time: 4 minutes)

■ **Scoring indicators**

Drawing a rectangle of sides 5 cm and 3 cm.	(1)
Drawing a semicircle having diameter as $5 + 3 = 8$ cm	(1)
Drawing a perpendicular	(1)
Constructing the square	(1)

29. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

? The sides of a triangle are 4 cm, 7 cm and 8 cm. Draw it and construct a square of the same area.

(Score 5, Time: 8 minutes)

■ **Scoring indicators**

Drawing the triangle	(1)
Drawing rectangle with equal area	(1)
Drawing semi circle with diameter equal to the sum of the sides of the rectangle	(1)
Drawing the perpendicular	(1)
Construct the square	(1)

30. Learning Outcome

- Converting a rectangle into another rectangle or square without changing the area.

? Draw an isosceles triangle of hypotenuse 7 cm

(Score: 3, Time: 4 minutes)

■ **Scoring indicators**

Drawing a semicircle of diameter 7 cm.	(1)
Drawing the perpendicular to the diameter from the centre.	(1)
Construct the isosceles right triangle.	(1)

MATHEMATICS OF CHANCE

1. Learning Outcomes

- Explaining probability as a number.
- Explaining the method to calculate probability in different contexts.

? One is asked to say a two digit number. What is the probability of being the number not a perfect square?

(Score : 3, Time : 3 Minutes)

■ Scoring Indicators

Total number of two digit number = 90 (1)

Total number of two digit perfect squares = 6 (1)

Number of two digit numbers which are not perfect squares = $90 - 6 = 84$

Probability = $\frac{84}{90} = \frac{42}{45} = \frac{14}{15}$ (1)

2. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts.

? A bag contains 10 blue balls and 12 yellow balls. Another contains 15 blue balls and 7 yellow balls.

- What is the probability of getting a yellow ball from the first bag?
- What is the probability of getting a yellow ball from the second bag?
- If all the balls are put in a single bag, what is the probability of getting a yellow ball from it?

(Score : 4, Time: 4 Minutes)

■ Scoring Indicators

a) Total number of balls in the first bag = $10 + 12 = 22$

Number of yellow balls = 12

Probability of getting a yellow ball = $\frac{12}{22} = \frac{6}{11}$ (1)

b) Total number of balls in the second bag = $15 + 7 = 22$

Number of a yellow ball = 7

$$\text{Probability of getting a yellow ball} = \frac{7}{22} \quad (1)$$

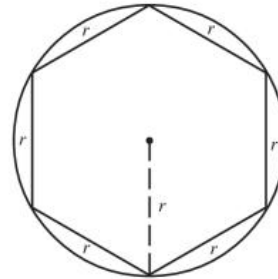
$$\begin{aligned} \text{c) Total number of balls} &= 22 + 22 = 44 \\ \text{Number of yellow balls} &= 12 + 7 = 19 \end{aligned} \quad (1)$$

$$\text{Probability of getting a yellow ball} = \frac{19}{44} \quad (1)$$

3. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts.

? A regular hexagon, is drawn with its vertices on a circle. Without looking into the picture, if one put dot in that picture, what is the probability of being the dot not in the regular hexagon?



(Score : 4 Time: 3 Minutes)

■ Scoring Indicators

$$\text{Area of the circle} = \pi r^2 \quad (1)$$

$$\text{Area of regular hexagon} = \frac{\sqrt{3}r^2}{4} \times 6 = \frac{3\sqrt{3}r^2}{2} \quad (1)$$

$$\text{Required probability} = \frac{\frac{3\sqrt{3}r^2}{2}}{\pi r^2} = \frac{3\sqrt{3}r^2}{2} \times \frac{1}{\pi r^2} \quad (1)$$

$$= \frac{3\sqrt{3}}{2\pi} \quad (1)$$

4. Learning Outcomes

- Explaining probability as a number
- Explaining the method to calculate probability in different contexts

? A box contains slips numbered 1, 2, 3, 4. Another box contains slips numbered 1, 2, 3. If one slip is taken from each, what is the probability of getting a sum which is a multiple of three? Also find the probability of getting a sum, which is multiple of 2

(Score: 5, Time: 6 Minutes)

■ Scoring Indicators

Pair of numbers obtained by taking one slip from each box at a time:

(1, 1) (2, 1) (3, 1) (4, 1)

(1, 2) (2, 2) (3, 2) (4, 2) 12 Numbers (1)

(1, 3) (2, 3) (3, 3) (4, 3)

pairs having a sum, which is a multiple of 3 (1)

(2, 1) (1, 2) (4, 2) (3, 3) - 4 Numbers (1)

$$\text{Probability} = \frac{4}{12} = \frac{1}{3}$$

pairs having a sum, which is a multiple of 2

(1, 1) (3, 1) (1, 3) (2, 2) (3, 3) (4, 2) - 6 Numbers (1)

$$\text{Probability} = \frac{6}{12} = \frac{1}{2} \quad (1)$$

5. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.



A bag contains 6 red beads and 4 green beads. Another contains 2 red and 2 green seeds more. The probability of getting red bead from which bag is more?

(Score: 4, Time: 3 Minutes)

■ Scoring Indicators

Number of red beads in first bag = 6

$$\text{Probability of getting red beads from the first bag} = \frac{6}{10} \quad (1)$$

$$\text{Number of red beads in the second bag} = 6 + 2 = 8 \quad (1)$$

$$\text{Probability of getting red beads from the second bag} = \frac{8}{14} \quad (1)$$

$$\frac{6}{10} > \frac{8}{14}$$

∴ Probability of getting red bead from the first bag is more (1)

6. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.



One is asked to say a two digit number: what is the probability of

- Both digits being different?
- The first digit being larger?
- The first digit being smaller?

(Score: 6, Time: 8 Minutes)

■ Scoring Indicators

(1) Total two digit numbers : 90

Numbers having equal digits

11, 22, 33, 44, 55, 66, 77, 88, 99 - (9 Numbers) (1)

Therefore, numbers having different digits - 81 Numbers (1)

$$\text{Probability} = \frac{81}{90} = \frac{9}{10} \quad (1)$$

(2) The probability of the first digit being larger and second being smaller

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = \frac{9 \times 10}{2} = 45$$

$$\text{Probability} = \frac{45}{90} = \frac{1}{2} \quad (1)$$

10	- 1
20 21	- 2
30 31 32	- 3
40 41 42 43	- 4
.....	
90 91 92 93 99	- 9
	$\frac{9 \times 10}{2} = 45$

(3) The probability of the first digit being smaller and second digit being larger

$$= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{8 \times 9}{2} \quad (1)$$

$$= 36$$

$$\text{Probability} = \frac{36}{90} = \frac{4}{10} = \frac{2}{5} \quad (1)$$

7. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.



There are 60 students in a class among which 30 are boys. In another class there are 50 students among which 25 of them are boys. If one from each class is selected,

- What is the probability of both being girls?
- What is the probability of having atleast one girl?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

Total number of students in the first class = 60

No. of boys = 30

No. of girls = 30

Total number of students in the second class = 50

No. of boys = 25

No. of girls = 25

$$1) \quad \text{Probability of both being girls} = \frac{30 \times 25}{60 \times 50} \quad (1)$$

$$= \frac{750}{3000}$$

$$= \frac{1}{4} \quad (1)$$

$$\begin{aligned}
 2) \quad \text{Probability of at least one girl} &= \frac{30 \times 25 + 30 \times 25 + 30 \times 25}{3000} & (1) \\
 &= \frac{2250}{3000} \\
 &= \frac{3}{4} \quad \text{or} \quad \frac{3000 - 750}{3000} \\
 &= \frac{3}{4} & (1)
 \end{aligned}$$

8. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.



A vessel contains 4 black beads, 6 white beads and 10 red beads. In another vessel contains 7 black beads, 5 white beads and 8 red beads. If we take one bead from each vessel, without looking into it.

- What is the probability of both being same colour?
- What is the probability of both being different colours?
- What is the probability of getting atleast one black bead?

(Score: 5, Time: 8 Minutes)

■ Scoring Indicators

4 B	7 B
6 W	5 W
10 R	8 R

Number of beads in each vessel = 20

Total number of ways of selecting two beads = $20 \times 20 = 400$ (1)

$$(a) \quad \text{No. of beads both being same colour} = 4 \times 7 + 6 \times 5 + 10 \times 8 = 138$$

$$= \frac{138}{400} \quad (1)$$

$$(b) \quad \text{Probability of both being different colour} = \frac{400 - 138}{400}$$

$$= \frac{262}{400} \quad (1)$$

(c) Number of beads having atleast one black bead

$$= 4 \times 7 + 4 \times 5 + 4 \times 8 + 6 \times 7 + 10 \times 7 = 28 + 20 + 32 + 42 + 70 \quad (1)$$

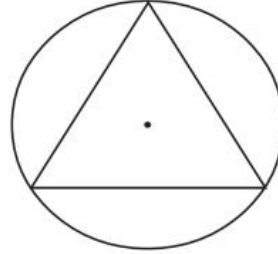
$$= 192$$

$$\text{Its probability} = \frac{192}{400} = \frac{12}{25} \quad (1)$$

9. Learning Outcomes

- Justifying the need to analyse probability using numbers in practical situations.

- ❓ An equilateral triangle is drawn with its vertices if we put a dot init without looking into the picture, what is the probability of being the dot inside the triangle? Also find the probability of being the dot outside the triangle?



(Score: 4, Time: 7 Minutes)

■ Scoring Indicators

If the radius of the circle is r , then the side of the triangle will be $\sqrt{3}r$

$$\text{Area of the circle} = \pi r^2 \quad (1)$$

$$\text{Area of the equilateral triangle} = \frac{\sqrt{3}}{4}(\sqrt{3}r)^2 = \frac{\sqrt{3}}{4} \times 3r^2 \quad (1)$$

$$\text{Proabability of being the dot inside the triangle} = \frac{\frac{\sqrt{3}}{4} \times 3r^2}{\pi r^2} = \frac{3\sqrt{3}}{4\pi} \quad (1)$$

$$\text{Probability of being the dot out side the triangle} = \frac{4\pi - 3\sqrt{3}}{4\pi} \quad (1)$$

10. Learning Outcome

- Justifying the need to analyze probability using numbers on practical situations

- ❓ In the figure all the four shaded semicircles have same area. If we put a dot in the figure without looking into it, what is the probability of being the dot in the shaded semicircles?



(Score : 3 , Time: 5 Minutes)

■ Scoring Indicators

If the radius of the shaded semicircle is r ,

$$\text{Area of all 4 semicircles} = 4 \times \frac{\pi r^2}{2} = 2\pi r^2 \quad (1)$$

$$\text{Area of the bigger circle} = \pi(4r)^2 = 16\pi r^2 \quad (1)$$

$$\text{Probability} = \frac{2\pi r^2}{16\pi r^2} = \frac{1}{8} \quad (1)$$

11. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

? What is the probability of getting 5 sundays in December in a calendar year?

(Score: 3, Time: 5 Minutes)

■ Scoring Indicators

There are 31 days in December.

That means 4 full weeks and 3 days.

The probable three days are as shown below.

Sunday	Monday	Tuesday
Monday	Tuesday	Wednesday
Tuesday	Wednesday	Thursday
Wednesday	Thursday	Friday
Thursday	Friday	Saturday
Friday	Saturday	Sunday
Saturday	Sunday	Monday

$$\text{Total possibilities} = 7 \quad (1)$$

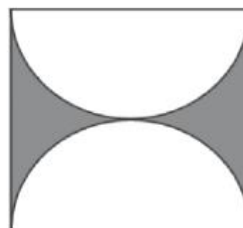
$$\begin{aligned} \text{Number of possible ways having sunday} \\ = 3 \end{aligned} \quad (1)$$

$$\text{Probability} = \frac{3}{7} \quad (1)$$

12. Learning Outcome

- Justifying the need to analyse probability using numbers in practical situations.

? Two semicircles are drawn in a square as shown. If we put a dot in the figure, without looking into it, what is the probability of being the dot in the shaded region?



(Score: 3, Time: 5 Minutes)

■ Scoring Indicators

If a is the side of a square,

$$\text{Area of the two circular part} = \pi \frac{a^2}{2} = \frac{\pi a^2}{2} \quad (1)$$

$$\text{Area of the shaded portion} = a^2 - \frac{\pi a^2}{2} = \frac{4 - \pi}{2} a^2 \quad (1)$$

$$\text{Probability} = \frac{\frac{4 - \pi}{2} a^2}{a^2} = \frac{4 - \pi}{2} \quad (1)$$

SECOND DEGREE EQUATIONS

1. Learning outcome

- Framing and solving second degree equations

? When the sides of a square are increased by 8 cm each, its area becomes 1225 sq. cm. Frame an equation using the above data by taking the side of the smaller square as x cm. Find the sides of both the squares.

(Score: 3, Time: 4 Minutes)

■ Scoring Indicators

- One side of smaller square = x
- One side of bigger square = $x + 8$
- Area = $(x + 8)^2 = 1225$ (1)
- $x + 8 = 35$
- $x = 35 - 8 = 27$ (1)
- Side of smaller square = 27 centimetre
- Side of bigger square = 35 centimetre (1)

2. Learning outcome

- Framing and solving second degree equations

? The difference of two positive numbers is 6. Their product is 216. Find the numbers.

(Score: 3, Time: 4 Minutes)

■ Scoring Indicators

Let the numbers be $x, x + 6$

$$\text{Products: } x(x + 6) = 216 \quad (1)$$

$$x^2 + 6x = 216$$

$$x^2 + 6x + 9 = 216 + 9 = 225$$

$$(x + 3)^2 = 225 \quad (1)$$

$$x + 3 = 15$$

$$x = 15 - 3 = 12$$

$$\text{Numbers: } 12, 18 \quad (1)$$

3. Learning outcome

- Framing and solving second degree equations



In a right triangle one of the perpendicular sides is one less than 2 times the smaller side. Hypotenuse is one more than 2 times the same smaller side. By taking the smaller side as x cm, write the algebraic expression for the other two sides. Compute all the 3 sides of the right triangle.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{Smaller side} = x$$

$$\text{Perpendicular side} = 2x - 1$$

$$\text{Hypotenuse} = 2x + 1 \quad (1)$$

$$x^2 + (2x - 1)^2 = (2x + 1)^2 \quad (1)$$

$$x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$$

$$x^2 - 8x = 0 \quad (1)$$

$$x(x - 8) = 0$$

$$x = 0 \text{ or } x = 8$$

$$\text{Sides : } 8 \text{ cm, } 15 \text{ cm, } 17 \text{ cm} \quad (1)$$

4. Learning outcome

- Framing and solving second degree equations



Find the sides of a rectangle whose perimeter is 100 metres and area 600 sq. metres.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{Perimeter} = 100 \text{ metre}$$

$$\text{Length} + \text{breadth} = 50 \text{ metre}$$

$$\text{Length} = 25 + x$$

$$\text{Breadth} = 25 - x \quad (1)$$

$$\text{Area, } (25 + x)(25 - x) = 600$$

$$25^2 - x^2 = 600 \quad (1)$$

$$x^2 = 625 - 600 = 25$$

$$x = 5 \quad (1)$$

$$\text{Sides, } 25 + 5 = 30 \text{ metre}$$

$$25 - 5 = 20 \text{ metre} \quad (1)$$

5. Learning outcome

- Solving practical problems by forming second degree equations

? The one's place of a two digit number is 4. The product of the number and digit sum is 238.

- If ten's place digit is taken as x , write the number.
- Frame a second degree equation and find the number.

(Score: 4, Time: 6 Minutes)

■ Scoring Indicators

Digit in the ten's place = x

$$\text{Number} = 10x + 4 \quad (1)$$

The product of the number

$$\text{and digit sum} = (x + 4)(10x + 4) = 238 \quad (1)$$

$$10x^2 + 44x + 16 = 238$$

$$10x^2 + 44x - 222 = 0$$

$$5x^2 + 22x - 111 = 0$$

$$x = \frac{-22 \pm \sqrt{22^2 - 4 \times 5 \times -111}}{2 \times 5} \quad (1)$$

$$= \frac{-22 \pm \sqrt{2704}}{10}$$

$$= \frac{-22 \pm 52}{10}$$

$$\text{Number} = 34 \quad (1)$$

6. Learning outcome

- Framing and solving second degree equations

? How many consecutive natural numbers from 1 should be added to get 465?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{Sum of first } n \text{ natural numbers: } \frac{n(n+1)}{2} = 465 \quad (1)$$

$$n^2 + n = 930$$

$$n^2 + n + \frac{1}{4} = 930 + \frac{1}{4} \quad (1)$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{3721}{4}$$

$$n + \frac{1}{2} = \frac{61}{2} \quad (1)$$

$$n = 30$$

The sum of first 30 natural numbers = 465 (1)

7. Learning outcome

- Solving practical problems by framing second degree equations.

? The product of the digits of a two digit number is 12. When 36 is added to this number, got a new number with digits reversed. Find the number.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

Digits : $x, \frac{12}{x}$ (1)

When 36 is added to the number, the digits became reversed

$$\text{Then, } 10x + \frac{12}{x} + 36 = 10 \times \frac{12}{x} + x \quad (1)$$

$$10x^2 + 12 + 36x = 120 + x^2$$

$$9x^2 + 36x = 108$$

$$x = 2 \quad (1)$$

$$x^2 + 4x = 12$$

$$(x + 2)^2 = 16$$

$$x + 2 = 4 \quad (1)$$

$$\text{Number} = 26 \quad (1)$$

8. Learning outcome

- Solving practical problems by framing second degree equations.

? For a two digit number, one's place is 3 more than its ten's place. The product of this number and its digitsum is square of double the digitsum. What is the number.

(Score: 5, Time: 6 Minutes)

■ Scoring Indicators

digits: $x, x+3$

$$\text{Number: } 10x + x + 3 = 11x + 3 \quad (1)$$

$$\text{Product of the number and digit sum: } (2x+3)(11x+3) = (4x+6)^2 \quad (1)$$

$$22x^2 + 39x + 9 = 16x^2 + 48x + 36$$

$$6x^2 - 9x - 27 = 0$$

$$2x^2 - 3x - 9 = 0 \quad (1)$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 2 \times -9}}{2 \times 2}$$

$$= \frac{3 \pm \sqrt{9+72}}{4}$$

$$= \frac{3 \pm \sqrt{81}}{4}$$

$$= \frac{3+9}{4} \text{ or } \frac{3-9}{4} \text{ (being a fraction not acceptable)}$$

$$x = \frac{3+9}{4} = \frac{12}{4} = 3 \quad (1)$$

$$\text{Number} = 36 \quad (1)$$

9. Learning outcome

- Solving geometrical problems by framing second degree equations.



A pavement of width 2 metres is built around a square shaped garden. The area of the pavement alone is 116 square metres. Find one side of the garden.

(Score: 3, Time: 4 Minutes)

■ Scoring Indicators

Side of garden = x .

$$\text{Side of garden including pavement} = x + 4. \quad (1)$$

$$\text{Area of the pavement} = (x+4)^2 - x^2 = 116$$

$$x^2 + 8x + 16 - x^2 = 116 \quad (1)$$

$$8x + 16 = 116$$

$$x = \frac{116-16}{8} = 12.5 \text{ metre}$$

$$\text{Side of garden} = 12.5 \text{ metre} \quad (1)$$

10. Learning outcomes

- Solving geometrical problems by framing second degree equations.

? A rectangle of width 8 centimetres is cut off from a square sheet along its side. The remaining rectangular portion has an area 84 sq. metres. Calculate the side of the square.

(Score: 4, Time: 6 Minutes)

■ Scoring Indicators

Let the side of square = x

Sides of rectangle = x , $x - 8$ (1)

Area = $x(x - 8) = 84$

$x^2 - 8x = 84$ (1)

$x^2 - 8x + 16 = 84 + 16$

$(x - 4)^2 = 100$ (1)

$x - 4 = 10$

$x = 14$

Side of square = 14 centimetre (1)

11. Learning outcome

- Solving geometrical problems by framing second degree equations.

? The length of a rectangle is 3 metre more than 3 times its breadth. Its diagonal is 1 metre more than the length. Find the length and breadth of the rectangle.

(Score: 4, Time: 7 Minutes)

■ Scoring Indicators

Side of rectangle = x .

Length = $3x + 3$

Diagonal = $3x + 4$ (1)

$(3x + 4)^2 = x^2 + (3x + 3)^2$ (1)

$9x^2 + 24x + 16 = x^2 + 9x^2 + 18x + 9$

$x^2 - 6x = 7$

$x^2 - 6x + 9 = 16$ (1)

$(x - 3)^2 = 16$

$x - 3 = 4$

$x = 4 + 3 = 7$ (1)

length of rectangle = $3 \times 7 + 3 = 24$ metre

breadth of rectangle = 7 metre (1)

12. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.



In the figure AB is the diameter of the circle. CD = 10 cm. BC is 15 cm less than AC. Find AB?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$AC \times CB = CD^2 \quad (1)$$

$$AC = x$$

$$x(x - 15) = 10^2 \quad (1)$$

$$x^2 - 15x = 100$$

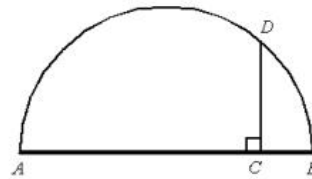
$$x^2 - 15x + \frac{225}{4} = 100 + \frac{225}{4}$$

$$\left(x - \frac{15}{2}\right)^2 = \frac{625}{4} \quad (1)$$

$$x - \frac{15}{2} = \frac{25}{2}$$

$$x = \frac{25}{2} + \frac{15}{2} = 20$$

$$\therefore AB = 20 + 5 = 25 \text{ centimetre} \quad (1)$$



13. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.



The chords AB and CD of a circle intersect at M. If MA = 6 cm, MB = 8 cm and CD = 16 cm, Find MC and MD.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$CD = 16 \text{ centimetre}$$

$$MC = 8 - x, MD = 8 + x \quad (1)$$

$$\text{then, } MA \times MB = MC \times MD$$

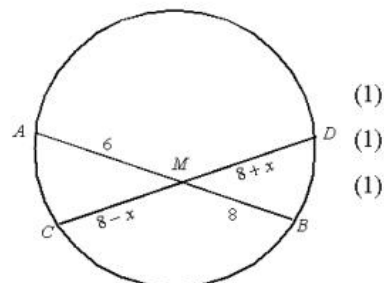
$$6 \times 8 = (8 - x)(8 + x)$$

$$48 = 64 - x^2$$

$$x^2 = 16, x = 4$$

$$MC = 8 - 4 = 4 \text{ centimetre}$$

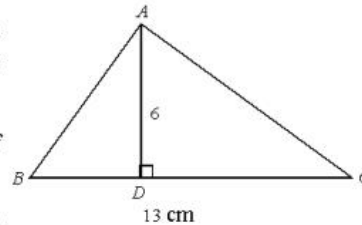
$$MD = 8 + 4 = 12 \text{ centimetre}$$



14. Learning outcome

- Applying the method to frame and solve a second degree equation in practical contexts.

? In the figure AD is drawn perpendicular to the side opposite to the right angled vertex A. $BC = 13$ cm and $AD = 6$ cm.



- Take $BD = x$ and express DC in terms of x .
- Frame a second degree equation and find the lengths BD and DC.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{let } BD = x$$

$$DC = 13 - x$$

$$BC \times DC = AD^2 \quad (1)$$

$$x(13 - x) = 6^2$$

$$13x - x^2 = 36$$

$$x^2 - 13x + 36 = 0$$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4 \times 1 \times 36}}{2 \times 1} \quad (1)$$

$$\therefore \frac{13 \pm \sqrt{169 - 144}}{2} \quad (1)$$

$$\frac{13 \pm \sqrt{25}}{2} \quad \frac{13 \pm 5}{2}$$

$$BD = 9 \text{ centimetre, } DC = 4 \text{ centimetre}$$

15. Learning outcome

- Identifying the nature of solution in problems involving second degree equations.

? Can the sum of a number and its reciprocal be $\frac{2}{3}$? Why?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{let number} = x, \text{ then its reciprocal} = \frac{1}{x}$$

$$\text{then, } x + \frac{1}{x} = \frac{2}{3} \quad (1)$$

$$(x^2 + 1) \times 3 = 2x$$

$$3x^2 - 2x + 3 = 0 \quad (1)$$

$$\begin{aligned}
 x &= \frac{+2 \pm \sqrt{(-2)^2 - 4 \times 3 \times 3}}{4 \times 3} \\
 &= \frac{2 \pm \sqrt{4 - 36}}{12} \\
 &= \frac{2 \pm \sqrt{-32}}{12} \quad (1)
 \end{aligned}$$

Since the discriminant is negative, we do not get a solution

$\therefore$ The sum of a number and its reciprocal never gives $\frac{2}{3}$. (1)

16. Learning outcome

- Framing and solving second degree equations.

? The sum of a number and its reciprocal is $\frac{13}{6}$. What is the number?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

Let number = x

then its reciprocal = $\frac{1}{x}$

$$x + \frac{1}{x} = \frac{13}{6} \quad (1)$$

$$(x^2 + 1) \times 6 = 13x$$

$$6x^2 - 13x + 6 = 0 \quad (1)$$

$$\begin{aligned}
 x &= \frac{13 \pm \sqrt{(-13)^2 - 4 \times 6 \times 6}}{2 \times 6} \\
 &= \frac{13 \pm \sqrt{169 - 144}}{12} \\
 &= \frac{13 \pm \sqrt{25}}{12} \\
 &= \frac{13 \pm 5}{12} \quad (1)
 \end{aligned}$$

$$\text{Number} = \frac{3}{2} \text{ or } \frac{2}{3} \quad (1)$$

17. Learning outcome

- Framing and solving second degree equations.

? The sum of two numbers is 12 and sum of its reciprocal is $\frac{3}{8}$. Find the numbers

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

If we take numbers as $6 + x$, $6 - x$ (1)

$$\frac{1}{6+x} + \frac{1}{6-x} = \frac{3}{8} \quad (1)$$

$$12 \times 8 = 3(36 - x^2)$$

$$96 = 108 - 3x^2 \quad (1)$$

$$3x^2 = 12 \quad x^2 = 4$$

$$x = 2 \text{ or } -2$$

$$\text{Number} = 6 + 2 = 8, 6 - 2 = 4 \quad (1)$$

18. Learning outcome

- Solving mathematical problems involving second degree equations.



Consider an arithmetic sequence with common difference 20. If the sum of reciprocals of two consecutive terms of this sequence is $\frac{1}{24}$, find the first term of the arithmetic sequence

(Score: 4, Time: 6 Minutes)

■ Scoring Indicators

Terms $x - 10$, $x + 10$ (1)

$$\frac{1}{x-10} + \frac{1}{x+10} = \frac{1}{24}$$

$$\frac{2x}{x^2-100} = \frac{1}{24}$$

$$2x \times 24 = x^2 - 100 \quad (1)$$

$$x^2 - 48x = 100$$

$$x^2 - 48x + 24^2 = 100 + 576$$

$$(x - 24)^2 = 676 \quad (1)$$

$$x - 24 = \pm 26$$

$$x = 26 + 24 = 50 \text{ or } x = -26 + 24 \quad (1)$$

First term = 40 or -12

19. Learning outcomes

- Solving practical problems with the help of second degree equations.



Prove that the difference of a number and its reciprocal will be always positive.

(Score: 3, Time: 5 Minutes)

■ Scoring Indicators

$$\begin{aligned} \text{Number} &= x \\ \text{Its reciprocal} &= \frac{1}{x} \end{aligned} \quad (1)$$

$$\text{then; } x - \frac{1}{x} = k \quad (1)$$

$$x^2 - kx - 1 = 0$$

$$x = \frac{+k \pm \sqrt{k^2 + 4}}{2}$$

$$\text{Since } k \text{ being a positive number, } k^2 + 4 \text{ also negative} \quad (1)$$

20. Learning outcome

- Solving geometrical problems with the help of second degree equations.

- ?** Two circles touch each other as shown below. Their centres are at a distance of 11 centimetres. The sum of the areas of the circles is 65π sq. centimetres. Find their radii.

(Score: 4, Time: 6 Minutes)

■ Scoring Indicators

Radius of first circle = r

Radius of second circle = $11 - r$

Sum of their areas:

$$\pi r^2 + \pi (11 - r)^2 = 65\pi$$

$$r^2 + 121 - 22r + r^2 = 65$$

$$2r^2 - 22r = -56$$

$$r^2 - 11r = -28 \quad (1)$$

$$r^2 - 11r + \left(\frac{11}{2}\right)^2 = -28 + \frac{121}{4}$$

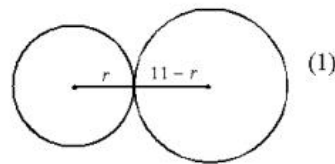
$$\left(r - \frac{11}{2}\right)^2 = \frac{9}{4} \quad (1)$$

$$r - \frac{11}{2} = \frac{3}{2}$$

$$r = \frac{3+11}{2} = 7 \text{ cm}$$

Radius of first circle = 7 cm

Radius of second circle = $11 - 7 = 4$ cm (1)



21. Learning outcome

- Applying the concept of second degree equations in different mathematical contexts.

- ?** The algebraic expression for the sum of 'n' consecutive terms of an arithmetic sequence is $2n^2 + 5n$. How many consecutive terms are needed to get a sum 1375?

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$2n^2 + 5n = 1375$$

$$2n^2 + 5n - 1375 = 0 \quad (1)$$

$$n = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -1375}}{2 \times 2} \quad (1)$$

$$= \frac{-5 \pm \sqrt{25 + 11000}}{4}$$

$$= \frac{-5 \pm 105}{4} \quad (1)$$

$$n = \frac{100}{4} = 25$$

Sum of first 25 terms is 1375 (1)

22. Learning outcome

- Solving problems using the nature of solution of second degree equations.

? If $x^2 + ax + b = 0$ has only one solution, show that $a^2 = 4b$.

(Score: 3, Time: 4 Minutes)

■ Scoring Indicators

$x^2 + ax + b = 0$, has only one solution

It means the left side of the equation is a perfect square (1)

$$\therefore \left(\frac{a}{2}\right)^2 = b \quad (1)$$

$$\frac{a^2}{4} = b$$

$$a^2 = 4b \quad (1)$$

23. Learning outcome

- Applying second degree equations in different mathematical contexts.

? Observe the given number pattern formed using natural numbers. Which line has the last number 210?

(Score: 4, Time: 5 Minutes)

1

2 3

4 5 6

7 8 9 10

.....

.....

■ Scoring Indicators

$$\frac{n(n+1)}{2} = 210 \quad (1)$$

$$n^2 + n = 420$$

$$n^2 + n + \frac{1}{4} = 420 + \frac{1}{4} \quad (1)$$

$$\left(n + \frac{1}{2}\right)^2 = \frac{1681}{4} \quad (1)$$

$$n + \frac{1}{2} = \frac{41}{2}$$

$$n = \frac{41-1}{2} = 20$$

20th line has the last number 210 (1)

24. Learning outcome

- Solving practical problems using the idea of second degree equations.

? Can 240 be the sum of consecutive terms of an arithmetic sequence 7, 11, 15,... Justify.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$\text{let } S_n = 2n^2 + 5n = 240$$

$$2n^2 + 5n - 24 = 0 \quad (1)$$

$$n = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -24}}{2 \times 2}$$

$$= \frac{-5 \pm \sqrt{25 + 1920}}{4} \quad (1)$$

$$= \frac{-5 \pm \sqrt{1945}}{4} \quad (1)$$

sin a $\sqrt{1945}$ is not a natural number, 'n' cannot be a natural number

'n' should be a natural number. Hence sum cannot be 240 (1)

25. Learning outcome

- Relation between second degree equations and polynomials.

? Can the polynomial $2x - x^2$ get 2 for any x ? What about getting $\frac{1}{2}$?

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$2x - x^2 = 2 \quad (1)$$

$$x^2 - 2x = -2$$

$$x^2 - 2x + 1 = -2 + 1$$

$$(x - 1)^2 = -1 \quad (1)$$

The square of any number cannot be negatives

So, for any x , we never get $2x - x^2 = 2$ (1)

$$\text{Now, } 2x - x^2 = \frac{1}{2}$$

$$x^2 - 2x = \frac{1}{2}$$

$$x^2 - 2x + 1 = -\frac{1}{2} + 1$$

$$(x - 1)^2 = \frac{1}{2}, \text{ a positive number} \quad (1)$$

$$\text{then } \therefore 2x - x = \frac{1}{2} \quad (1)$$

26. Learning outcome

- Applying the method of finding the solution of a second degree equation in different mathematical contexts.

? In triangle ABC , $AB = AC$ and $\angle A = 36^\circ$. The bisector of $\angle B$ intersect AC at D . if $\frac{BC}{CD} = x$, prove that $x = 1 + \frac{1}{x}$. Find the value of x also.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

The angles of triangle ABC and triangle BCD are equal

let $BC = 1$ and $AC = AB = x$ (1)

$$\text{then, we have } \frac{x-1}{1} = \frac{1}{x} \quad (1)$$

$$x = 1 + \frac{1}{x} \quad (1)$$

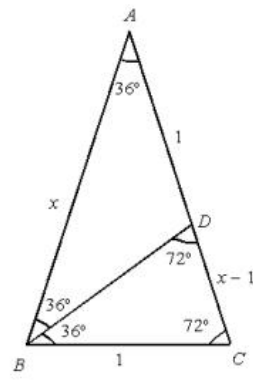
$$x = \frac{x+1}{x}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0 \quad (1)$$

$$x = \frac{1 \pm \sqrt{1 - (-4)}}{2} = \frac{1 \pm \sqrt{5}}{2}, \text{ not a negative number}$$

$$\text{Hence, } x = \frac{1 + \sqrt{5}}{2}$$



27. Learning outcome

- Applying the method of solving a second degree equation in practical problems.

? The distance from town A to town B is 180 kilometres. A cyclist rides half the distance from A to B in a specific speed, and the next half distance, at a speed of 15 kilometre/hr more than the first half. He travelled for 5 hours. Find the speed of the journey in each half?

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$\frac{90}{x} + \frac{90}{x+15} = 5 \quad (1)$$

$$90x + 1350 + 90x = 5x^2 + 75x$$

$$5x^2 - 105x - 1350 = 0 \quad (1)$$

$$x^2 - 21x - 270 = 0$$

$$= \frac{-21 \pm \sqrt{(-21)^2 - 4 \times 1 \times -270}}{2 \times 1} \quad (1)$$

$$= \frac{21 \pm \sqrt{1521}}{2}$$

$$= \frac{21 \pm 39}{2}, \text{ not a negative number} \quad (1)$$

$$\text{Hence, } x = \frac{60}{2} = 30$$

Speed in the first half: 30 km/hr

Speed in the second half: 45 km/hr (1)

28. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.



A traveller travels half of his journey in a specific speed and the next half, a speed 15 kilometre/hour more than the first half. The average speed of the total journey is 36 kilometre/hour. Find his speed in each half?

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$\frac{1}{x} + \frac{1}{x+15} = \frac{2}{36} = \frac{1}{18} \quad (1)$$

$$(2x + 15) \times 18 = x^2 + 15x$$

$$x^2 + 15x - 36x - 270 = 0 \quad (1)$$

$$x = \frac{-21 \pm \sqrt{441 - 4 \times -270}}{2 \times 1} \quad (1)$$

$$= \frac{21 \pm \sqrt{1521}}{2}$$

$$= \frac{21 \pm 39}{2}$$

$$x = \frac{60}{2} = 30 \quad (1)$$

Speed in the first half = 30 km/hr

Speed in the second half = 45 km/hr (1)

29. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

- ?** If the price of a toy is lessened by 2 rupees, one could have bought 2 more toys for Rs.360. Find the actual price of the toy.

(Score: 5, Time: 7 Minutes)

■ **Scoring Indicators**

let the price of a toy = x

$$\text{Number of toys for 360 rupees} = \frac{360}{x} \quad (1)$$

If the price is lessened by 2 rupees per toy

$$\text{Number of toys for 360 rupees} = \frac{360}{x-2} \quad (1)$$

$$\text{Difference: } \frac{360}{x-2} - \frac{360}{x} = 2$$

$$360x - 360x + 720 = 2x^2 - 4x$$

$$2x^2 - 4x = 720$$

$$x^2 - 2x = 360 \quad (1)$$

$$x^2 - 2x + 1 = 361$$

$$(x - 1)^2 = 361$$

$$x - 1 = 19 \quad (1)$$

$$x = 20$$

$$\text{Price of a toy} = 20 \text{ rupees} \quad (1)$$

30. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

- ?** Ravi needs 16 more days to complete a work than Raju to complete the same work. If both of them work together, only 15 days are sufficient. How many days Raju alone needed to complete that work.

(Score: 5, Time: 7 Minutes)

■ **Scoring Indicators**

$$\text{Number of days Raju needed for the job} = x \quad (1)$$

$$\text{Number of days Ravi needed for the same job} = x + 16$$

$$\therefore \frac{1}{x} + \frac{1}{x+16} = \frac{1}{15} \quad (1)$$

$$(2x + 16) \times 15 = x^2 + 16x$$

$$x^2 + 16x - 30x = 240 \quad (1)$$

$$x^2 - 14x = 240$$

$$x^2 - 14x + 49 = 269 \quad (1)$$

$$(x - 7)^2 = 269$$

$$x - 7 = 17$$

$$x = 17 + 7 = 24$$

$$\text{Number days Raju needed to complete the job} = 24$$

31. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

? Using 1 litre of petrol, car *B* travels 5 kilometres more than another car *A* for travelling 400 kilometres, car *B* uses 4 litres petrol less than that of car *A*. Find the distance travelled by car *A* using 1 litre of petrol.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

Distance travelled by car *A* using 1 litre of petrol = x

Distance travelled by car *B* using 1 litre of petrol = $x + 5$

$$\frac{400}{x} - \frac{400}{x+5} = 4 \quad (1)$$

$$400x + 2000 - 400x = 4x^2 + 20x$$

$$4x^2 + 20x = 2000$$

$$x^2 + 5x = 500 \quad (1)$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 500 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{2025}{4} \quad (1)$$

$$x + \frac{5}{2} = \frac{45}{2}$$

$$x = \frac{45-5}{2} = 20 \quad (1)$$

Car *A* travels 20 kilometres using 1 litre of petrol (1)

32. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

? While computing the sides of a rectangle having specific perimeter and area, perimeter was taken as 52 centimetres instead of 50 centimetres. Then the side was calculated as 18 centimetres. Find the actual length of the rectangle.

(Score: 5, Time: 7 Minutes)

■ Scoring Indicators

$$\text{Perimeter} = 52$$

$$\text{length} + \text{breadth} = 26$$

$$\text{Sides: } 18, 26 - 18 = 8 \quad (1)$$

$$\text{Area: } 18 \times 8 = 144 \text{ Sq. centimetres}$$

$$\text{Perimeter} = 50$$

$$\text{length} + \text{breadth} = 25$$

$$\text{length} = x, \text{ breadth} = 25 - x \quad (1)$$

$$\text{Perimeter: } x(25 - x) = 144$$

$$x^2 - 25x + 144 = 0$$

$$x = \frac{25 \pm \sqrt{25^2 - 4 \times 144}}{2 \times 1} \quad (1)$$

$$= \frac{25 \pm \sqrt{625 - 576}}{2}$$

$$= \frac{25 \pm \sqrt{49}}{2} \quad (1)$$

$$= \frac{25 \pm 7}{2}$$

$$x = \frac{25 + 7}{2} = 16 \text{ or } \frac{25 - 7}{2} = \frac{18}{2} = 9$$

$$\text{sides : } 16 \text{ cm, } 9 \text{ cm} \quad (1)$$

33. Learning outcome

- Finding the relation between polynomials and second degree equations.



Prove that the second degree polynomial $x^2 - 2x + 6$ will never be less than 5 for any number x . Also find for which number, gets the value 5.

(Score: 4, Time: 5 Minutes)

■ Scoring Indicators

$$p(x) = x^2 - 2x + 6$$

$$= x^2 - 2x + 1 + 5$$

$$= (x - 1)^2 + 5 \quad (1)$$

$$\text{Since } (x - 1)^2 \text{ a perfect square, the smallest value for } (x - 1)^2 \text{ is zero} \quad (1)$$

$$\text{Thus } x^2 - 2x + 6 \text{ never gets the value smaller than 5 for any } x \quad (1)$$

$$\text{When } x = 1, (x - 1)^2 = 0, (x - 1)^2 + 5 = 5 \quad (1)$$

34. Learning outcome

- Applying the method of solving a second degree equation in other practical problems.

- ❓ A water tank is connected to two pipes, the bigger one is to fill and a smaller one to drain. Time taken to fill the tank is 1 minute less than that of the time taken to drain out. When one uses both pipes simultaneously, the tank will fill in 56 minutes. Find the time taken to fill up the tank, if one uses the bigger one only.

(Score: 5, Time: 8 Minutes)

■ **Scoring Indicators**

Time taken to drain out the tank using smaller pipe = x only

Time taken to fill the tank using the bigger pipe = $x - 1$ only (1)

When both pipes are used simultaneously, we have

$$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{56} \quad (1)$$

$$(x - x + 1) \times 56 = x^2 - x$$

$$x^2 - x - 56 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \times -56}}{2} \quad (1)$$

$$= \frac{1 \pm \sqrt{1 + 224}}{2}$$

$$= \frac{1 \pm 15}{2}, \text{ which never gets a negative number}$$

$$\text{Hence, } x = \frac{1+15}{2} = 8 \text{ minute} \quad (1)$$

Time taken to fill the tank using the bigger pipe = 7 minutes only (1)

35. Learning outcome

- Applying the method of finding the solution of a second degree equation in solving practical problems.

- ❓ In copying a second degree equation, the number without x was written as -30 instead of 30 . The answers found were 15 and -2 . What are the answers of the correct problem.

(Score: 5, Time: 8 Minutes)

■ **Scoring Indicators**

$$ax^2 + bx - 30 = 0$$

$$x = 15, \quad 225a + 15b - 30 = 0 \quad 15a + b = 2 \quad (1) \quad (1)$$

$$x = -2 \quad 4a - 2b - 30 = 0 \quad 2a - b = 15 \quad (2) \quad (1)$$

$$(1) + (2) \rightarrow 17a = 17, \quad a = 1$$

$$2 \times 1 - b = 15 \quad b = -13 \quad (1)$$

Then the equation is $x^2 - 13x - 30 = 0$

$$\text{for that equation, } x = \frac{13 \pm \sqrt{169 - 120}}{2} \quad (1)$$

$$x = \frac{13 \pm 7}{2}$$

$$x = \frac{13+7}{2} = 10 \quad \text{or} \quad \frac{13-7}{2} = 3 \quad (1)$$

Solutions of the correct problem 10 or 3

TRIGONOMETRY

1. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ The diagonal of a square is 4cm long. Find its perimeter and area.

(Score : 2, Time : 3 minute)

■ Scoring indicators

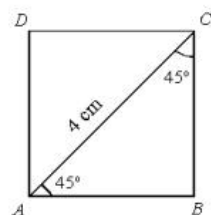
Angles of $\triangle ABC$ are 45° , 45° , 90° .

45°		45°		90°
1	:	1	:	$\sqrt{2}$
↓		↓		↓
$2\sqrt{2}$		$2\sqrt{2}$		4

$$\text{One side} = 2\sqrt{2} \text{ cm}$$

$$\text{Perimeter} = 4 \times 2\sqrt{2} = 8\sqrt{2} \text{ cm.}$$

$$\text{Area} = 2\sqrt{2} \times 2\sqrt{2} \text{ sq.cm.}$$



(1)

(1)

(1)

2. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ AC and BC are two equal chords of a circle with diameter AB. If the equal chords have lengths 10cm find the area of the circle.

(Score : 3, Time : 3 minute)

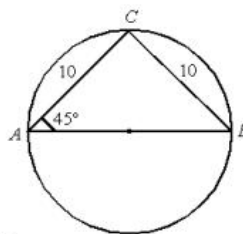
■ Scoring indicators

45°		45°		90°
1	:	1	:	$\sqrt{2}$
↓		↓		↓
10		10		$10\sqrt{2}$

$$\text{Diameter AB} = 10\sqrt{2} \text{ cm}$$

$$\text{Radius} = 5\sqrt{2} \text{ cm}$$

$$\text{Area} = \pi r^2 = \pi \times (5\sqrt{2})^2 = 50\pi \text{ sq.cm}$$



(1)

(1)

(1)

3. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $AB = 10\text{ cm}$, $AC = 8\text{ cm}$, $\angle A = 45^\circ$

- Find the perpendicular distance from C to AB .
- Find the area of the triangle.

(Score : 2, Time : 3 minute)

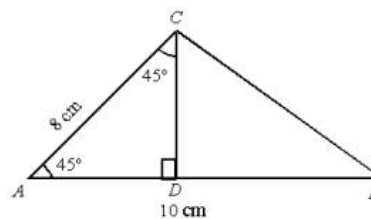
■ Scoring indicators

- Draw CD perpendicular to AB .

Angles of $\triangle ADC$ are $45^\circ, 45^\circ, 90^\circ$

$$\begin{array}{ccc} 45^\circ & 45^\circ & 90^\circ \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ 4\sqrt{2} & & 4\sqrt{2} & & 8 \end{array}$$

$$CD = AD = 4\sqrt{2} \text{ cm}$$



(1)

- $$\begin{aligned} \text{Area} &= \frac{1}{2} \times 10 \times 4\sqrt{2} \\ &= 20\sqrt{2} \text{ sq.cm} \end{aligned}$$

(1)

4. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? One side of a rhombus is 12 cm and one angle is 135°

- Find the distance between the parallel sides?
- Find the area of the rhombus?

(Score : 3, Time : 3 minute)

■ Scoring indicators

- $ABCD$ is a rhombus

$AB = AD = 12\text{ cm}$, $\angle B = 135^\circ$

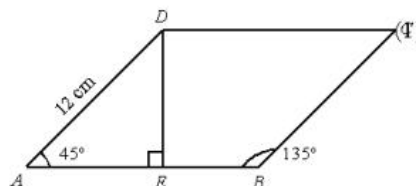
$\angle A = 180 - 135 = 45^\circ$

Angles of $\triangle ADE$ are $45^\circ, 45^\circ, 90^\circ$

$$\begin{array}{ccc} 45^\circ & 45^\circ & 90^\circ \\ 1 & : & 1 & : & \sqrt{2} \\ \downarrow & & \downarrow & & \downarrow \\ 6\sqrt{2} & & 6\sqrt{2} & & 12 \end{array}$$

$$DE = 6\sqrt{2} \text{ cm}$$

- $$\begin{aligned} \text{Area of the rhombus} &= 12 \times 6\sqrt{2} \\ &= 72\sqrt{2} \text{ sq.cm} \end{aligned}$$



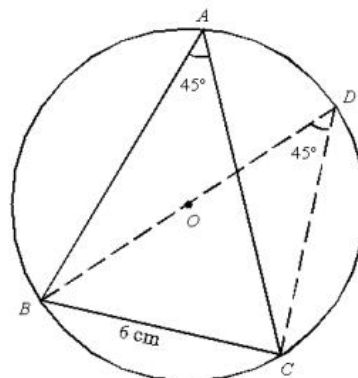
(1)

(1)

5. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

- ❓ In $\triangle ABC$, $\angle A = 45^\circ$, $BC = 6$ cm. Find the diameter of the circumcircle.



(Score : 4, Time : 4 minute)

■ Scoring indicators

- (a) Draw diameter BD and Join CD Angles of $\triangle BCD$ is $45^\circ, 45^\circ, 90^\circ$ (1)

45°	:	45°	:	90°	
1	:	1	:	$\sqrt{2}$	
↓		↓		↓	
6		6		$6\sqrt{2}$	(1)

$$BD = 6\sqrt{2} \text{ cm} \quad (1)$$

6. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

- ❓ 12 centimetre long diagonal of a rectangle makes an angle 30° with its one side. Find its perimeter and area.

(Score : 3, Time : 4 minute)

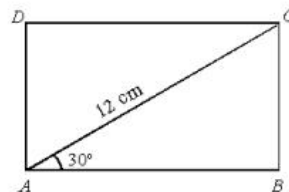
■ Scoring indicators

ABCD is a square

$AC = 12$ cm; $\angle BAC = 30^\circ$

Angles of $\triangle ABC$ are $30^\circ, 60^\circ, 90^\circ$

30°	:	60°	:	90°	
1	:	$\sqrt{3}$	:	2	
↓		↓		↓	
6		$6\sqrt{3}$		12	



$$AB = 6\sqrt{3} \text{ cm} \quad (1)$$

$$BC = 6 \text{ cm}$$

$$\text{perimeter} = 2(6 + 6\sqrt{3}) = (12 + 12\sqrt{3}) \text{ cm} \quad (1)$$

$$\text{Area} = 6 \times 6\sqrt{3} = 36\sqrt{3} \text{ sq.cm} \quad (1)$$

7. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In triangle ABC, $AB = 20\text{cm}$, $\angle A = 30^\circ$, $AC = 12\text{cm}$

- Find the length of the perpendicular from C to AB.
- Find the area of the triangle.

(Score : 3, Time : 4 minute)

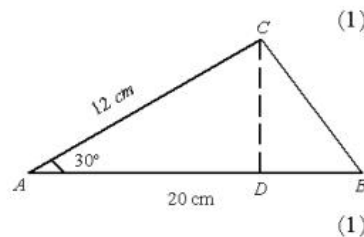
■ Scoring indicators

- Draw CD perpendicular to AB.

Angles of $\triangle ADC$ are 30° , 60° , 90°

30°	60°	90°
1	$\sqrt{3}$	2
$\downarrow$	$\downarrow$	$\downarrow$
6	$6\sqrt{3}$	12

$$CD = 6 \text{ cm}$$



- Area of the triangle $= \frac{1}{2} \times 20 \times 6 = 60 \text{ sq.cm}$ (1)

8. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $AB = 8 \text{ cm}$, $BC = 10\text{cm}$ and $\angle B = 60^\circ$

- Find the area of the triangle $\triangle ABC$?
- Find AC?

(Score : 4, Time : 6 minute)

■ Scoring indicators

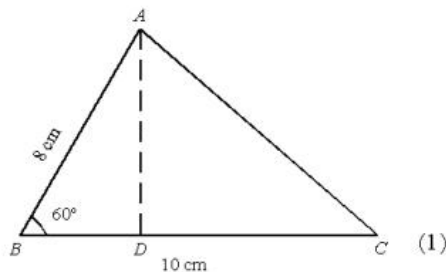
- Draw AD perpendicular to BC

Angles of $\triangle ADB$ are 30° , 60° , 90°

30°	60°	90°
1	$\sqrt{3}$	2
$\downarrow$	$\downarrow$	$\downarrow$
4	$4\sqrt{3}$	8

$$BD = 4 \text{ cm}; AD = 4\sqrt{3} \text{ cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 10 \times 4\sqrt{3} = 20\sqrt{3} \text{ sq.cm} \quad (1)$$



$$\begin{aligned} \text{(b)} \quad BC &= 10 \text{ cm} \\ CD &= 10 - 4 = 6 \text{ cm} \end{aligned} \quad (1)$$

$$\begin{aligned} \text{In } \triangle ADC \quad AC^2 &= AD^2 + CD^2 \\ &= (4\sqrt{3})^2 + 6^2 \\ &= 48 + 36 = 84 \\ AC &= \sqrt{84} = 2\sqrt{21} \text{ cm} \end{aligned} \quad (1)$$

9. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

- ❓ One angle of a triangle is 150° and its opposite side 3 centimetre. Find the diameter of its circum circle.

(Score: 3, Time: 5 minute)

■ Scoring indicators

In $\triangle ABC$

$$\angle B = 150^\circ, AC = 3 \text{ cm}$$

Draw diameter AD and join CD

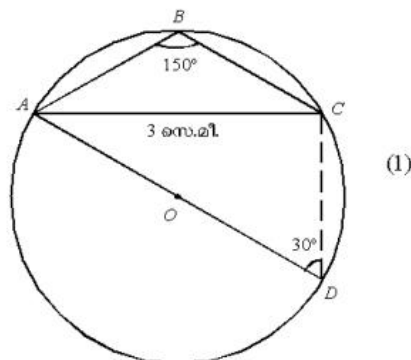
$$\angle ADC = 180 - 150 = 30^\circ$$

$$\angle ACD = 90^\circ$$

Angles of $\triangle ADC$ are $30^\circ, 60^\circ, 90^\circ$

30°	60°	90°
1	:	$\sqrt{3}$
:		2
↓		↓
3		$3\sqrt{3}$
		6

$$\text{Diameter, AD} = 6 \text{ cm}$$



10. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

- ❓ In $\triangle ABC$, $AB = 12 \text{ cm}$, $\angle A = 45^\circ$ and $\angle B = 30^\circ$

- Find the area of the triangle ABC.
- Find the ratio of the sides of the triangle having angles $30^\circ, 45^\circ, 105^\circ$.

(Score: 5, Time: 8 minute)

■ Scoring indicators

Angles of $\triangle BDC$ are 30° , 60° , 90°

$$\begin{array}{ccc} 30^\circ & 60^\circ & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ x & & x\sqrt{3} & & 2x \end{array}$$

$$x\sqrt{3} = 12 - x$$

$$x(\sqrt{3} + 1) = 12$$

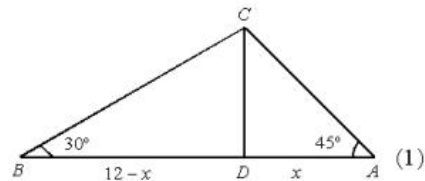
$$x = \frac{12}{\sqrt{3} + 1} = \frac{12(\sqrt{3} - 1)}{3 - 1} = 6(\sqrt{3} - 1) \quad (1)$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 12 \times 6(\sqrt{3} - 1) \\ &= 36(\sqrt{3} - 1) \text{ sq.cm} \end{aligned} \quad (1)$$

$$(b) AD = x, BD = \sqrt{3}x, AB = (\sqrt{3} + 1)x \text{ cm} \quad (1)$$

$$BC = 2x, AC = \sqrt{2}x$$

$$\begin{aligned} AC : BC : AB &= x : 2x : (\sqrt{3} + 1)x \\ &= \sqrt{2} : 2 : \sqrt{3} + 1 \end{aligned} \quad (1)$$



11. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ One side of a rhombus is 10 cm and one of its angles measures 120° .

- Find the area of the rhombus
- Find the length of diagonals of the rhombus

(Time : 5, Score : 7 minute)

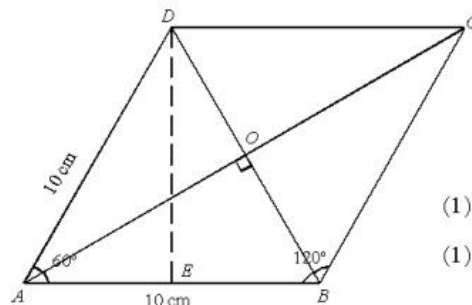
■ Scoring indicators

In $\triangle AED$, $\angle A = 60^\circ$, $\angle AED = 90^\circ$, $\angle ADE = 30^\circ$

$AD = 10 \text{ cm}$.

$$\begin{array}{ccc} 30^\circ & 60^\circ & 90^\circ \\ 1 & : & \sqrt{3} & : & 2 \\ \downarrow & & \downarrow & & \downarrow \\ 5 & & 5\sqrt{3} & & 10 \end{array}$$

$$AE = 5 \text{ cm}; DE = 5\sqrt{3} \text{ cm}$$



(a) Area = $10 \times 5\sqrt{3} = 50\sqrt{3}$ sq.cm (1)

Angles of $\triangle AOB$ are $30^\circ, 60^\circ, 90^\circ$

30°	60°	90°
1	$:\sqrt{3}$	$:2$
$\downarrow$	$\downarrow$	$\downarrow$
5	$5\sqrt{3}$	10

(1)

$OA = 5\sqrt{3}$ cm; $OB = 5$ cm

$AC = 10\sqrt{3}$ cm; $BD = 10$ cm (1)

12. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

? In $\triangle ABC$, $\angle A = 40^\circ$, $BC = 8$ cm
Find the circumdiameter of the triangle.

[$\sin 40^\circ = 0.64$]

(Score: 3, Time: 5 minute)

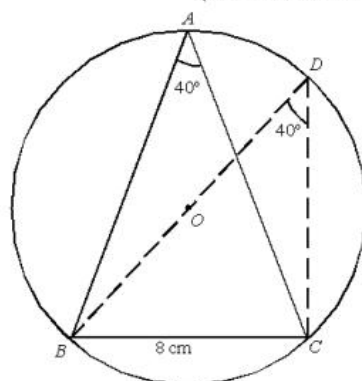
■ Scoring indicators

Draw Diameter BD and join CD .

In $\triangle BCD$, $\angle D = 40^\circ$

$$\sin 40^\circ = \frac{BC}{BD} = \frac{8}{BD} \quad (1)$$

$$BD = \frac{8}{\sin 40^\circ} = \frac{8}{0.64} = 12.5 \text{ cm} \quad (1)$$



13. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

? In $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 70^\circ$, $AB = 6$ cm.

(a) Find the diameter of its circum circle

(b) Find the lengths of other two sides AC and BC .

[$\sin 60^\circ = 0.87$; $\sin 50^\circ = 0.77$; $\sin 70^\circ = 0.94$]

(Score: 5, Time: 8 minute)

■ Scoring indicators

(a) Draw diameter AD and join BD . (1)

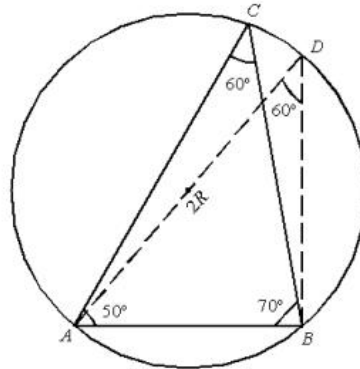
$$\sin 60^\circ = \frac{AB}{AD} = \frac{6}{AD}$$

$$2R = AD = \frac{6}{\sin 60^\circ} = \frac{6}{0.87} = 6.9$$

$$BC = 2R \sin 50^\circ = 6.9 \times 0.77$$

$$= 5.31 \text{ cm}$$

$$AC = 2R \sin 70^\circ = 6.9 \times 0.94 = 6.49 \text{ cm}$$



(1)

(1)

(1)

(1)

14. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $AB = 10 \text{ cm}$; $AC = 6 \text{ cm}$; $\angle A = 70^\circ$

(a) Find the area of the triangle

(b) Find BC

$[\cos 70^\circ = 0.34; \sin 70^\circ = 0.94]$

(Score: 4, Time: 6 minute)

■ Scoring indicators

$$\sin 70^\circ = \frac{CD}{6}$$

$$CD = 6 \times \sin 70^\circ$$

$$= 6 \times 0.94 = 5.64 \text{ cm}$$

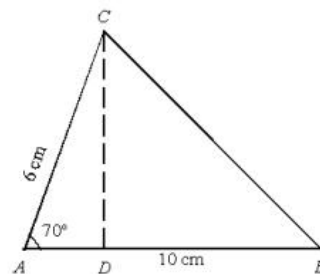
$$\cos 70^\circ = \frac{AD}{6}$$

$$AD = 6 \times \cos 70^\circ = 6 \times 0.34 = 2.04 \text{ cm}$$

$$BD = 10 - 2.04 = 7.96 \text{ cm}$$

$$BC = \sqrt{CD^2 + BD^2} = \sqrt{(5.64)^2 + (7.96)^2}$$

$$= \sqrt{31.81 + 63.36} = \sqrt{95.17} \text{ cm}$$



(1)

(1)

(1)

15. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $\angle A = 60^\circ$, $\angle B = 45^\circ$ and $AB = 10 \text{ cm}$

a) Find the perpendicular distance from C to AB ?

(b) Find the area of the triangle?

(Score: 4, Time: 6 minute)

■ Scoring indicators

- (a) Draw CD perpendicular to AB

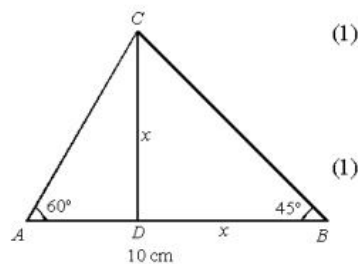
$$\text{If } CD = x \text{ then } BD = x; AD = \frac{\sqrt{3}x}{3}$$

$$\frac{\sqrt{3}x}{3} + x = 10$$

$$\sqrt{3}x + 3x = 30$$

$$x = \frac{30}{3 + \sqrt{3}} = \frac{30(3 - \sqrt{3})}{6}$$

$$= 5(3 - \sqrt{3}) \text{ cm}$$



(1)

(1)

(1)

(b) $\text{Area} = \frac{1}{2} \times 10 \times 5(3 - \sqrt{3})$

$$= 25(3 - \sqrt{3}) \text{ sq.cm}$$

(1)

16. Learning Outcome

- The relation between the chord of a circle and its central angle can be represented using the trigonometric measurement sine.

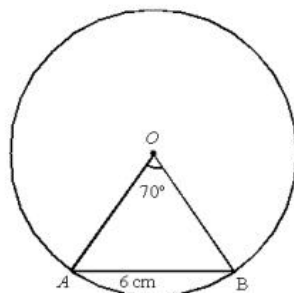


AB is a chord of a circle with centre 'O'.

$\angle AOB = 70^\circ$ and, $AB = 16 \text{ cm}$. Find the diameter of the circle

$[\sin 35^\circ = 0.57]$

(Score : 3, Time : 5minute)



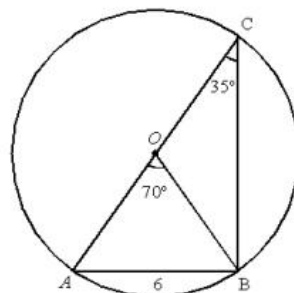
■ Scoring indicators

Draw diameter AC and join BC.

$$\sin 35^\circ = \frac{AB}{AC}$$

$$AC = \frac{AB}{\sin 35^\circ} = \frac{6}{0.57}$$

$$= 10.52 \text{ cm}$$



(1)

(1)

(1)

17. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $\angle A = \angle B = 30^\circ$, $AB = 12$ cm.

- Find the perimeter of the triangle
- Find the area of the triangle
- Find the ratio of the sides of a triangle having angles 30° , 30° , 120°

(Score: 5, Time: 8 minute)

■ Scoring indicators

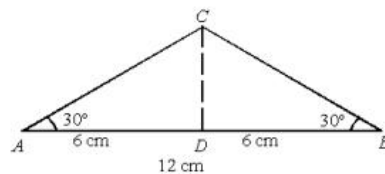
Angles of $\triangle ACD$ 30° , 60° , 90°

30°	60°	90°
1	$\sqrt{3}$	2
$\downarrow$	$\downarrow$	$\downarrow$
$2\sqrt{3}$	6	$4\sqrt{3}$

$$CD = 2\sqrt{3} \text{ cm}$$

$$AC = 4\sqrt{3} \text{ cm}$$

$$BC = 4\sqrt{3} \text{ cm}$$



(1)

$$(a) \text{ Perimeter of } \triangle ABC = 4\sqrt{3} + 4\sqrt{3} + 12 = (12 + 8\sqrt{3}) \text{ cm}$$

(1)

$$(b) \text{ Area of } \triangle ABC = \frac{1}{2} \times 12 \times 2\sqrt{3} = 12\sqrt{3} \text{ sq.cm}$$

(1)

$$(c) AC = BC = 4\sqrt{3}, AB = 12 \text{ cm}$$

$$\text{Ratio} = 4\sqrt{3} : 4\sqrt{3} : 12 = 1 : 1 : \sqrt{3}$$

(1)

18. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $AB = 8$ cm; $\angle A = 45^\circ$ and $\angle B = 60^\circ$

- Find the perpendicular distance from C to AB.
- Find the area of the triangle
- Find the ratio of the sides of a triangle having angles 45° , 60° , 75°

(Score: 5, Time: 8 minute)

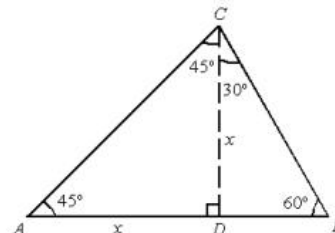
■ Scoring indicators

Draw CD perpendicular to AB

In $\triangle ADC$,

45°	45°	90°
1	1	$\sqrt{2}$
$\downarrow$	$\downarrow$	$\downarrow$
x	x	$\sqrt{2}x$

In $\triangle BDC$,



(1)

$$\begin{array}{ccc}
 30^\circ & 60^\circ & 90^\circ \\
 1 & : & \sqrt{3} & : & 2 \\
 \downarrow & & \downarrow & & \downarrow \\
 \frac{x}{3}\sqrt{3} & & x & & \frac{2x}{3}\sqrt{3}
 \end{array} \quad (1)$$

$$AB = x + \frac{x}{3}\sqrt{3} = 8$$

$$x(3 + \sqrt{3}) = 24$$

$$x = \frac{24}{3 + \sqrt{3}} = 4(3 - \sqrt{3}) \quad (1)$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 8 \times 4(3 - \sqrt{3}) = 16(3 - \sqrt{3}) \text{ sq.cm} \quad (1)$$

$$AC = \sqrt{2}x, BC = \frac{2\sqrt{3}}{3}x, AB = x + \frac{x\sqrt{3}}{3}$$

$$\begin{aligned}
 BC : AC : AB &= \frac{2\sqrt{3}}{3}x : \sqrt{2}x : \left(x + \frac{\sqrt{3}}{3}x\right) = \frac{2\sqrt{3}}{3} : \sqrt{2} : 1 + \frac{\sqrt{3}}{3} \\
 &= 2\sqrt{3} : 3\sqrt{2} : 3 + \sqrt{3} \\
 &= 2 : \sqrt{6} : \sqrt{3} + 1 \quad (1)
 \end{aligned}$$

19. Learning Outcome

- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

? If one side of a regular pentagon is 6cm, find the length of its diagonal?

[sin 18° = 0.31]

(Score: 3, Time: 5 minute)

■ Scoring indicators

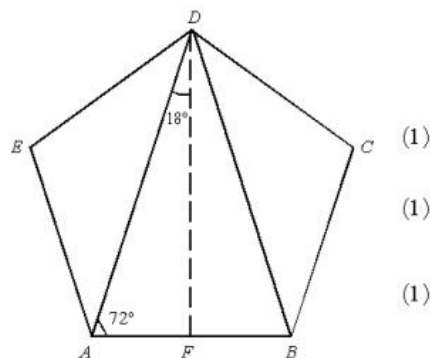
ABCDE is a regular pentagon

Angles of $\triangle ADF$

are 18°, 72°, 90°

$$\sin 18^\circ = \frac{3}{AD}$$

$$AD = \frac{3}{\sin 18^\circ} = \frac{3}{0.31} = 9.68 \text{ cm}$$



20. Learning Outcome

- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

- Two sides of a triangle are 9cm and 10cm and the angle between those sides is 105° . find the area of the triangle.

$$[\sin 75^\circ = 0.97]$$

(Score: 3, Time: 6 minute)

■ Scoring indicators

$$\text{In } \triangle BDC \quad \angle CBD = 180^\circ - 105^\circ = 75^\circ$$

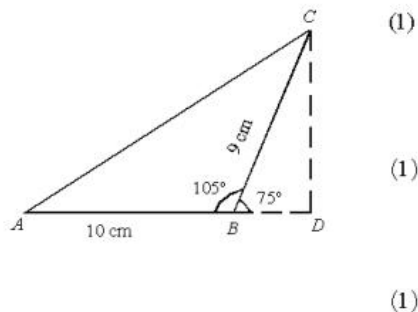
$$\sin 75^\circ = \frac{CD}{9}$$

$$0.97 = \frac{CD}{9}$$

$$CD = 0.97 \times 9 = 8.73 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times 10 \times 8.77$$

$$= 43.65 \text{ sq.cm}$$



21. Learning Outcome

- Interpreting sine, cosine and tangent as measures of an angle using straight lines.

- The diagonal of a rectangle is 12cm and it makes an angle 35° with one side. Find the perimeter of the rectangle.

$$[\sin 35^\circ = 0.57, \cos 35^\circ = 0.82]$$

(Score: 3, Time: 5 minute)

■ Scoring indicators

Let the breadth of the rectangle = x and length = y

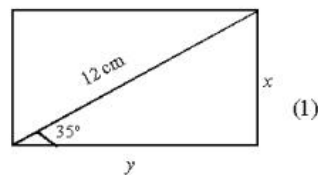
$$\sin 35^\circ = \frac{x}{12}$$

$$x = 12 \times \sin 35^\circ = 12 \times 0.57 = 6.84 \text{ cm}$$

$$\cos 35^\circ = \frac{y}{12}$$

$$y = 12 \times \cos 35^\circ = 12 \times 0.82 = 9.84 \text{ cm}$$

$$\text{Perimeter} = 2(6.84 + 9.84) = 2 \times 16.68 = 33.36 \text{ cm}$$



22. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

- ❓ In $\triangle ABC$, $\angle A = 125^\circ$, $BC = 8$ cm.
Find the diameter of the circumcircle. [$\sin 55^\circ = .82$]

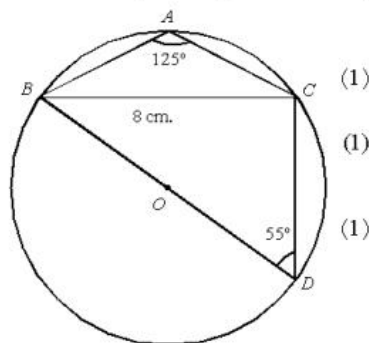
(Score: 2, Time: 6 minute)

■ Scoring indicators

Draw Diameter BD and join CD.

$$\text{In } \triangle BCD, \sin 55^\circ = \frac{BC}{BD}$$

$$BD = \frac{BC}{\sin 55^\circ} = \frac{8}{0.82} = 9.76 \text{ cm}$$



23. Learning Outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

- ❓ Can one cut out a triangle of one side 7 cm and its opposite angle 40° from a circular sheet of diameter 10 cm. Justify your answer. [$\sin 40^\circ = 0.64$]

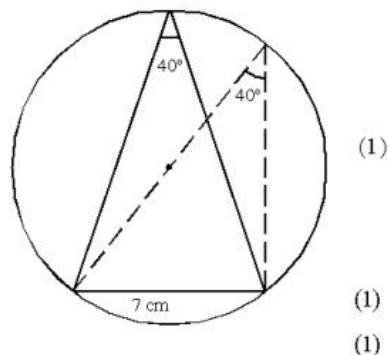
(Score: 4, Time: 7 minute)

■ Scoring indicators

The diameter of the circum circle of a triangle with one angle 40° and its opposite

$$\text{side } 7 \text{ cm} = \frac{7}{\sin 40^\circ}$$

$$= \frac{7}{0.64} = 10.93 \text{ cm}$$



Diameter of the paper is 10 cm, which is less than 10.93 cm. Hence triangle cannot be cut out. (1)

24. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

- ❓ Find the area of a triangle whose sides are a and b and the angle between those sides is C .

(Score: 2, Time: 3 minute)

■ Scoring indicators

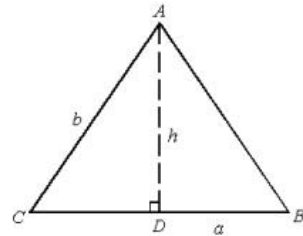
In $\triangle ABC$, Draw AD perpendicular to BC.

$$\sin C = \frac{h}{b}$$

$$h = b \sin C$$

$$\text{Area} = \frac{1}{2} ah$$

$$= \frac{1}{2} ab \sin C$$



(1)

(1)

25. Learning outcome

- Identifying that sides of a triangle are proportional to the sine values of the corresponding opposite angles.

❓ Find the sides of a triangle whose angles are A, B and C and its circumdiameter d.

(Score: 3, Time: 5 minute)

■ Scoring indicators

Draw a diameter BD and join CD

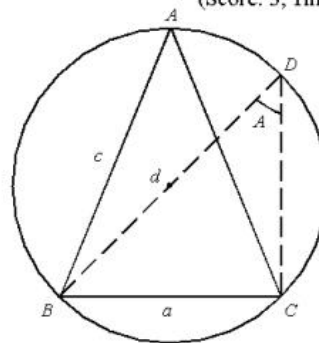
$$\angle BDC = A; BC = a$$

$$\sin A = \frac{a}{BD} = \frac{a}{d}$$

$$a = d \sin A$$

$$\text{Also } b = d \sin B \text{ and}$$

$$c = d \sin C$$



(1)

(1)

26. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

❓ The parallel sides of an isosceles trapezium are 10cm and 6 cm and the non parallel sides 8 cm. If one angle is 110° .

(a) Find the distance between the parallel sides

(b) Find the area of the isosceles trapezium

$$[\sin 70 = 0.94]$$

(Score: 3, Time: 6 minutes)

■ Scoring indicators

In trapezium ABCD

$$AB = 10 \text{ cm}$$

$$CD = 6 \text{ cm}$$

$$AD = 8 \text{ cm}$$

$$\angle ADC = 110^\circ, \angle A = 180 - 110 = 70^\circ$$

Draw DE perpendicular to AB

$$\sin 70^\circ = \frac{DE}{8}$$

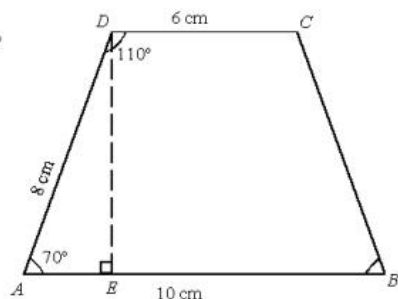
$$0.94 = \frac{DE}{8}$$

$$DE = 0.94 \times 8 = 7.52 \text{ cm}$$

$$\text{Area of the trapezium} = \frac{1}{2} \times DE \times (AB + CD)$$

$$= \frac{1}{2} \times 7.52 (10 + 6)$$

$$= 60.16 \text{ sq.cm}$$



(1)

(1)

27. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.



In $\triangle ABC$, $AB = 8 \text{ cm}$; $\angle A = 50^\circ$, $\angle B = 70^\circ$

- Find the perpendicular distance from C to AB .
 - Find the area of the triangle
- $[\tan 50^\circ = 1.2, \tan 70^\circ = 2.75]$

(Score: 5, Time: 8 minute)

■ Scoring indicators

Draw CD perpendicular to AB .

$$\tan 50^\circ = \frac{h}{x}$$

$$h = x \tan 50^\circ$$

$$\tan 70^\circ = \frac{h}{8-x}$$

$$h = (8-x) \tan 70^\circ$$

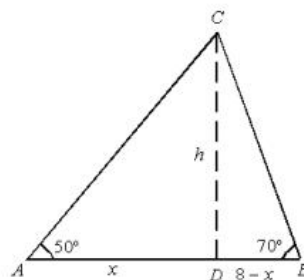
$$x \tan 50^\circ = (8-x) \tan 70^\circ$$

$$x \tan 50^\circ = 8 \times \tan 70^\circ - x \tan 70^\circ$$

$$x \tan 50^\circ + x \tan 70^\circ = 8 \times \tan 70^\circ$$

$$x = \frac{8 \times \tan 70^\circ}{\tan 50^\circ + \tan 70^\circ}$$

$$h = x \tan 50^\circ$$



(1)

(1)

(1)

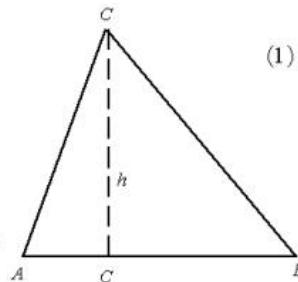
$$\begin{aligned}
 &= \frac{8 \times \tan 70^\circ \times \tan 50^\circ}{\tan 50^\circ + \tan 70^\circ} \\
 &= \frac{8 \times 2.75 \times 1.2}{2.75 + 1.2} \\
 &= 6.68 \text{ cm} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \times 8 \times 6.68 \\
 &= 26.72 \text{ sq.cm} \quad (1)
 \end{aligned}$$

let h be the height from C to AB ,

$$h = \frac{c \tan A \times \tan B}{\tan A + \tan B}$$

$$\text{Area} = \frac{1}{2} ch = \frac{1}{2} c^2 \frac{\tan A \cdot \tan B}{\tan A + \tan B}$$



28. Learning Outcome

- Finding the measurements of a triangle based on some given measurements using trigonometric measures.

? In $\triangle ABC$, $AB = 10\text{cm}$, $\angle A = 40^\circ$, $\angle B = 110^\circ$. Find the area of the triangle.
[$\tan 40^\circ = 0.84$, $\tan 70^\circ = 2.75$]

(Score: 5, Time: 8 minute)

■ Scoring indicators

In $\triangle BDC$, $\angle CBD = 70^\circ$

$$\tan 70^\circ = \frac{h}{x} \quad (1)$$

$$h = x \tan 70^\circ$$

$$\tan 40^\circ = \frac{h}{10+x}$$

$$h = (10+x) \tan 40^\circ$$

$$x \tan 70^\circ = (10+x) \tan 40^\circ$$

$$x (\tan 70^\circ - \tan 40^\circ) = 10 \tan 40^\circ \quad (1)$$

$$x = \frac{10 \tan 40^\circ}{\tan 70^\circ - \tan 40^\circ} \quad (1)$$

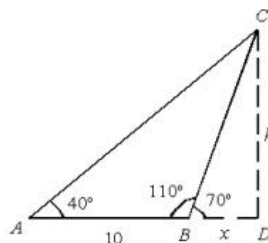
$$h = x \tan 70^\circ$$

$$= \frac{10 \tan 40^\circ \tan 70^\circ}{\tan 70^\circ - \tan 40^\circ} \quad (1)$$

$$\text{Area} = \frac{1}{2} \times 10 \times h$$

$$= \frac{1}{2} \times 10 \times \frac{10 \tan 40^\circ \tan 70^\circ}{\tan 70^\circ - \tan 40^\circ}$$

$$= \frac{1}{2} \times \frac{100 \times 0.84 \times 2.75}{2.75 - 0.84} = 60.47 \text{ sq.cm} \quad (1)$$



29. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

When the sun is at an elevation of 40° , the shadow of a flagpost is 15 metres.

- Find the height of the flagpost?
- What would be the length of the shadow, when the sun is at an elevation of 45°
[$\tan 40^\circ = 0.84$; $\sin 40^\circ = 0.64$]

(Score: 4, Time: 6 minute)

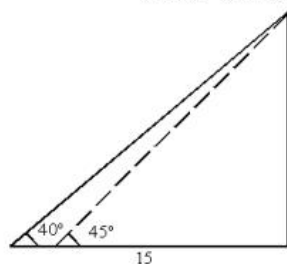
■ Scoring indicators

Let H be the height of the flagpost

$$\tan 40^\circ = \frac{H}{15}$$

$$H = 15 \times \tan 40^\circ$$

$$= 15 \times 0.84 = 12.60 \text{ metre}$$



(1)

(1)

(1)

length of shadow when the sun is at an elevation of $45^\circ = H = 12.60 \text{ metre}$ (1)

30. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

Two buildings in a plane ground are 20 metres apart. From the top of the smaller building, one sees the base of the building at a depression of 50° and its top at an elevation of 25°

- Draw a rough figure and mark the measurements
- Find the height of the smaller building
- Find the heights of the bigger building

$$[\tan 50^\circ = 1.2; \tan 25^\circ = 0.4]$$

(Score: 5, Time: 8 minute)

■ Scoring indicators

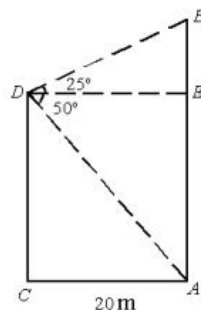
In $\triangle ADE$,

$$\tan 50^\circ = \frac{AE}{DE}$$

$$AE = 20 \times \tan 50^\circ$$

$$= 20 \times 1.2 = 24 \text{ m}$$

Height of the smaller building = 24 metre



(1)

(1)

(1)

In $\triangle BED$,

$$\tan 25^\circ = \frac{BE}{20}$$

$$\begin{aligned} BE &= 20^\circ \times \tan 25^\circ \\ &= 20 \times 0.47 = 9.4 \text{ metre} \end{aligned} \quad (1)$$

$$\begin{aligned} \text{Height of the bigger building} &= 24 + 9.4 \\ &= 33.4 \text{ metre} \end{aligned} \quad (1)$$

31. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.
- ?** One sees the top of a tree on the bank of a river at an elevation of 70° from the other bank. Stepping 20 metres back, he sees the top of the tree at an elevation of 55° . Height of the person is 1.4 metres.
- Draw a rough figure and mark the measurements
 - Find the height of the tree
 - Find the width of the river.
- [$\tan 70^\circ = 2.75$; $\tan 55^\circ = 1.43$]

(Score: 5, Time: 8 minute)

■ Scoring indicators

In $\triangle ABC$,

$$\tan 70^\circ = \frac{AB}{x}$$

$$AB = x \tan 70^\circ \quad (1)$$

In $\triangle ABD$,

$$\tan 55^\circ = \frac{AB}{x+20}$$

$$AB = (x+20) \tan 55^\circ$$

$$x \tan 70^\circ = (x+20) \tan 55^\circ$$

$$x \tan 70^\circ = x \tan 55^\circ + 20 \tan 55^\circ$$

$$x (\tan 70^\circ - \tan 55^\circ) = 20 \tan 55^\circ$$

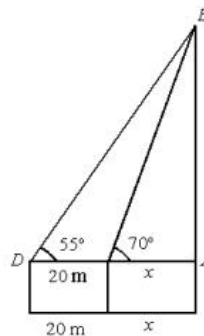
$$x = \frac{20 \times \tan 55^\circ}{\tan 70^\circ - \tan 55^\circ}$$

$$= \frac{20 \times 1.43}{2.75 - 1.43} = \frac{28.6}{1.32} = 21.67 \quad (1)$$

$$AB = x \tan 70^\circ = 21.67 \times 2.75 = 59.59 \text{ m}$$

$$\text{Height of the tree} = 59.59 + 1.4 = 60.99 \text{ m}$$

$$\text{Width of the river} = 21.67 \text{ m} \quad (1)$$



32. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

? A tower is built in a river of width 80 metres. One sees the top of the tower at an elevation of 55° and 65° from either banks of the river.

- Draw a figure using the given measurements
- Find the distance from the water level of river to the top of the tower.
- Find the distances to either banks from the foot of the tower.

$$[\tan 55^\circ = 1.43; \tan 65^\circ = 2.14]$$

(Score: 5, Time: 9 minute)

■ Scoring indicators

From the figure,

$$CD = x \tan 65^\circ$$

$$CD = (80 - x) \tan 55^\circ$$

$$x \tan 65^\circ = (80 - x) \tan 55^\circ$$

$$x \tan 65^\circ + x \tan 55^\circ = 80 \tan 55^\circ$$

$$x = \frac{80 \tan 55^\circ}{\tan 65^\circ + \tan 55^\circ}$$

$$= \frac{80 \times 1.43}{2.14 + 1.43} = \frac{114.4}{3.57} = 32 \text{ metre} \quad (1)$$

$$CD = x \tan 65^\circ$$

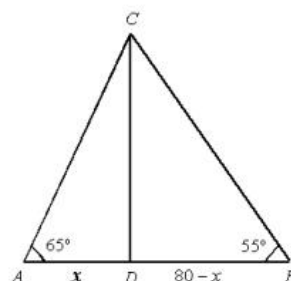
$$= 32 \times 2.14$$

$$= 68.48 \text{ metre} \quad (1)$$

$$\text{Height of the tower} = 68.48 \text{ metre} \quad (1)$$

$$\text{Distance from the foot of the tower to one bank of the river} = 32 \text{ metre}$$

$$\text{Distance from the foot of the tower to other bank of the river} = 80 - 32 = 48 \text{ metre} \quad (1)$$



33. Learning Outcome

- Explaining how distances and heights which cannot be directly measured can be computed using trigonometry.

? A man sees the bottom and top of a building at a depression of 55° and 35° respectively from the top of a 40 metres high tower.

- Draw a rough figure using the given data and mark the measurements
- Find the distance from tower to the building
- Find the height of the tower.

$$[\tan 55^\circ = 1.43; \tan 35^\circ = 0.7]$$

(Time : 5, Score : 8 minute)

■ Scoring indicators

$$\text{Tower} \Rightarrow CD = 40 + x$$

$$\text{building} \Rightarrow AB$$

In $\triangle BED$,

$$\tan 35^\circ = \frac{x}{BE}$$

$$BE = \frac{x}{\tan 35^\circ}$$

In $\triangle ACD$,

$$\tan 55^\circ = \frac{40 + x}{AC}$$

$$AC = \frac{40 + x}{\tan 55^\circ}$$

$$BE = AC \Rightarrow \frac{x}{\tan 35^\circ} = \frac{40 + x}{\tan 55^\circ} \quad (1)$$

$$x \tan 55^\circ = (40 + x) \tan 35^\circ$$

$$x (\tan 55^\circ - \tan 35^\circ) = 40 \tan 35^\circ$$

$$x = \frac{40 \times \tan 35^\circ}{\tan 55^\circ - \tan 35^\circ}$$

$$= \frac{40 \times 0.7}{1.43 - 0.7}$$

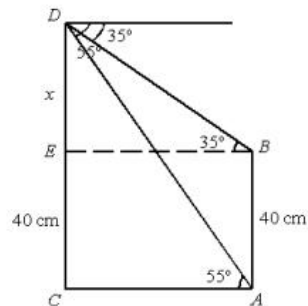
$$= \frac{28}{0.73} = 38.35 \quad (1)$$

$$\text{Height of the tower} = CD = 40 + x$$

$$= 40 + 38.35 = 78.35 \text{ metre} \quad (1)$$

$$\text{Distance from the tower to the building} = BE = \frac{x}{\tan 35^\circ} = \frac{38.35}{0.7}$$

$$= 54.8 \text{ metre} \quad (1)$$



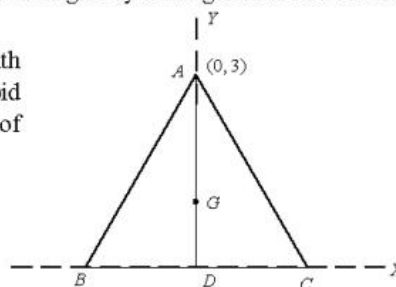
COORDINATES

1. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axes of coordinates.



Consider an equilateral triangle ABC with A(0, 3). AD is the height. If G is the centroid and D is the origin, find the coordinates of B, C, D and G



(Score: 4, Time: 9 minutes)

■ Scoring Indicators

- D is the origin $\therefore$ coordinates is (0, 0) (1)
- G divides AD in the ratio 2:1
- $\therefore$ coordinates of G is (0, 1) (1)
- Triangle ADB is a right triangle with $30^\circ, 60^\circ, 90^\circ$
- $\therefore BD = \frac{3}{\sqrt{3}} = \sqrt{3}$
- $\therefore$ coordinates of B is $(-\sqrt{3}, 0)$ (1)
- $\therefore$ coordinates of C is $(\sqrt{3}, 0)$ (1)

2. Learning Outcome

- Finding the distance between two points using their coordinates.



Find the coordinates of a point on the x-axis which is at a distance of 5 units from (4, -5). Find the coordinates on the y-axis

(Score: 3, Time: 6 minutes)

■ Scoring Indicators

- The point on x-axis which is at a distance of 5 units from (4, -5) is (4, 0). (1)
- The point on y-axis which is at a distance of 5 units from (4, -5) are (0, -2), (0, -8) (1)

3. Learning Outcome

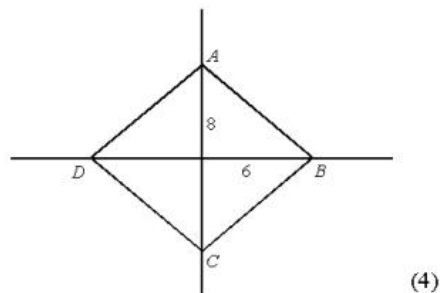
- Finding the co ordinates of various points of a figure by drawing axis of coordinates.

- ❓ Consider a rhombus ABCD whose diagonals meet at origin. The length of diagonals are 8 units and 6 units. Find the coordinates of the vertices.

(Score: 4, Time: 4 minutes)

■ Scoring Indicators

- Finding the coordinates of the vertices by drawing the figure.
(3, 0), (-3, 0), (0, 4), (0, -4)



4. Learning Outcome

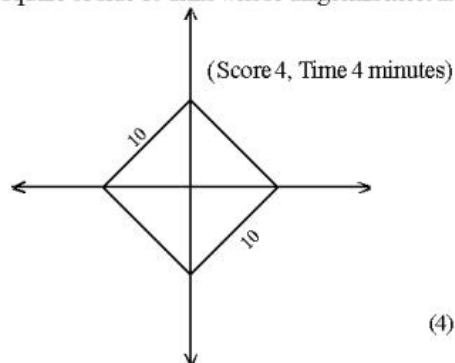
- Finding the coordinates of various points of a figure by drawing axis of coordinates.

- ❓ Find the coordinates of the vertices of a square of side 10 units whose diagonals meet at origin.

(Score 4, Time 4 minutes)

■ Scoring Indicators

- $(0, 5\sqrt{2})$, $(0, -5\sqrt{2})$,
 $(5\sqrt{2}, 0)$, $(-5\sqrt{2}, 0)$



5. Learning Outcome

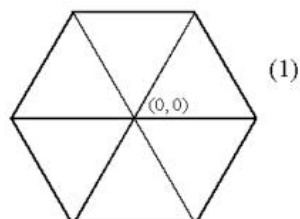
Finding the coordinates of various points of a figure by drawing axis of coordinates.

- ❓ Consider a regular hexagon of side 6 units whose diagonals meet at the origin. Find the coordinates of the vertices.

(Score 5, Time 8 minutes)

■ Scoring Indicators

- (6, 0) (0, 6)
Finding the coordinates of the vertices using the concept of right angle triangles 30° , 60° , 90°
 $(3, 3\sqrt{3})$, $(-3, 3\sqrt{3})$, $(3, -3\sqrt{3})$, $(-3, -3\sqrt{3})$



(4)

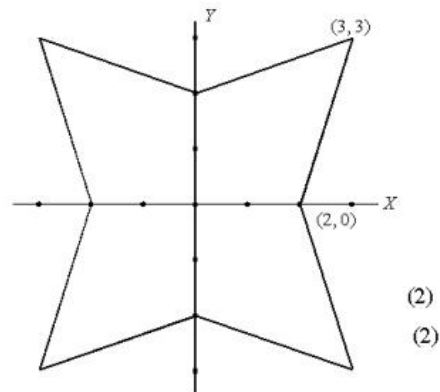
6. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axis of coordinates.
- ?** Find the coordinates of other points.

(Score 5, Time 8 minutes)

■ Scoring Indicators

- Finding $(-2, 0)$, $(0, 2)$, $(0, -2)$
- Finding $(3, -3)$, $(-3, -3)$, $(-3, 3)$



7. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axis of coordinates.

- ?** Find the coordinates of the fourth vertices of the parallelogram. Find the length of the sides of the parallelogram. Write the length of diagonals

■ Scoring Indicators

The difference of x-coordinate of the points A, C = 2

The difference of x-coordinate of the points B, D = 2

Hence the x-coordinate of the point

$$x = 11$$

The difference of y-coordinate of the points A, C = 7 - 5 = 2

The difference of y-coordinate of the points B, D = 2

Hence the y-coordinate of the point $x = 10 + 2 = 12$

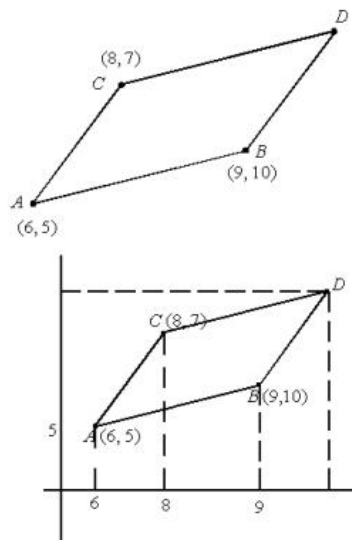
Therefore the coordinate of D is $(11, 12)$

$$AB = \sqrt{3^2 + 5^2} = \sqrt{24} = CD$$

$$AC = \sqrt{2^2 + 2^2} = \sqrt{8} = BD$$

$$AD = \sqrt{5^2 + 7^2} = \sqrt{74}$$

$$BC = \sqrt{1^2 + 3^2} = \sqrt{10}$$



(1)

(1)

(1)

(1)

(1)

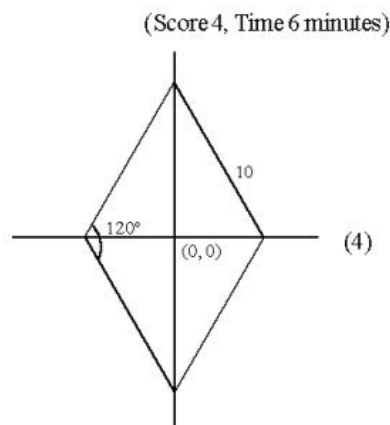
8. Learning Outcome

- Finding the coordinates of various points of a figure by drawing axis of coordinates.

? Consider a rhombus of side 10 units whose diagonals are coordinate axis. If an angle is 120° , find the coordinates of the other vertices.

■ Scoring Indicators

Understanding that a rhombus is formed by joining two equilateral triangle. Using the right triangle of 30° , 60° , 90° find the diagonals as 10 , $10\sqrt{3}$ units. Hence the coordinates of the vertices of the rhombus are $(5, 0)$, $(-5, 0)$, $(0, 5\sqrt{3})$, $(0, -5\sqrt{3})$



9. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Find the coordinates of the points on x-axis which are equidistant from the points $(-5, 8)$ and $(6, -4)$

(Score 4, Time 8 minutes)

■ Scoring Indicators

The y-coordinate of points on x-axis will be zero. Hence assume that a point on x-axis is $(x, 0)$

$$\text{Distance between } (x, 0) \text{ and } (-5, 8) = \sqrt{(x+5)^2 + 8^2} \quad (1)$$

$$\text{Distance between } (x, 0) \text{ and } (6, -4) = \sqrt{(x-6)^2 + (-4)^2} \quad (1)$$

$$(x+5)^2 + 8^2 = (x-6)^2 + (-4)^2$$

$$x^2 + 10x + 25 + 64 = x^2 - 6x + 36 + 16 \quad (1)$$

$$16x = 36 + 16 - 25 - 64 = 52 - 89 = -37$$

$$x = \frac{-37}{16}$$

$$\therefore \text{coordinate} \left(\frac{-37}{16}, 0 \right) \quad (1)$$

10. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Prove that $A(4, 5)$, $B(4, 2)$, $C(8, 2)$ represents the vertices of a right angled triangle. find the coordinate of the circumcentre. What is the radius of the circumcircle.

(Score 4, Time 8 minutes)

■ Scoring Indicators

Distance between (4, 5) and (4, 2) = $5 - 2 = 3$

Distance between (4, 2) and (8, 2) = $8 - 4 = 4$

Distance between (4, 5) and (8, 2) = $\sqrt{(4-8)^2 + (5-2)^2} = \sqrt{4^2 + 3^2} = 5$ (1)

3, 4, 5 are sides of a right angled triangle, since $5^2 = 3^2 + 4^2$ (1)

The coordinates of circumcentre is (6, 3.5) (1)

Radius of circumcircle = 2.5 units (1)

11. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

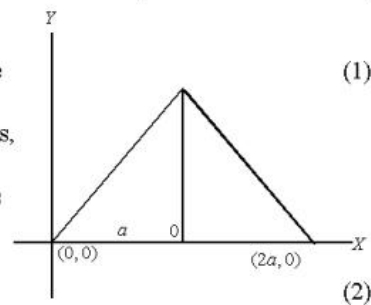
? Show that the coordinates of all vertices of an equilateral triangle can never be rational numbers at the same time.

(Score 3, Time 5 minutes)

■ Scoring Indicators

Let one of the side of an equilateral triangle be x-axis.

If one side of an equilateral triangle is '2a' units, then the coordinate of the vertices are (0, 0), (2a, 0), (0, $\sqrt{3}a$). Since the height is always $\sqrt{3}$ times the half of a side, the coordinates of vertices will always have an irrational number



12. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Consider a circle with centre at origin and radius 6 units. Let PA be a tangent to the circle from P(10, 0). Find the length of the tangent. Find the coordinates of A.

(Score 5, Time 8 minutes)

■ Scoring Indicators

Distance between (0, 0) and (10, 0) = 10

Given radius of circle = 6

Distance of the third side = $\sqrt{10^2 + 6^2}$

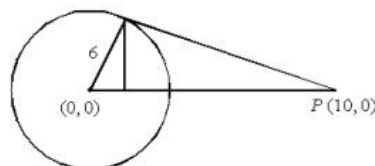
= $\sqrt{100 + 36} = \sqrt{136} = 11.66$ (1)

$OC \times OP = r^2$

$OC = \frac{r^2}{OP} = \frac{36}{10} = 3.6$ (2)

$AC = \sqrt{6^2 + (3.6)^2} = \sqrt{36 + 12.96} = \sqrt{48.96} = 6.99$ (1)

Coordinate of A is (3.6, 6.99)



13. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Consider a circle centered at origin with $(12, 5)$ a point on it. Find the coordinates of other four points on the circle. What is the radius of the circle.

(Score 5, Time 8 minutes)

■ Scoring Indicators

Given $(12, 5)$ a point on the circle

radius of the circle $= \sqrt{12^2 + 5^2} = 13$

Points of the circle

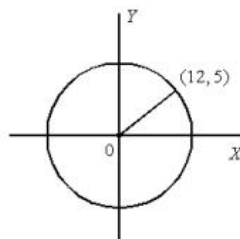
$(13, 0), (0, 13), (-13, 0), (0, -13)$

$(12, 5), (12, -5), (-12, -5), (-12, 5)$

$(5, 12), (5, -12), (-5, 12), (-5, -12)$

Writing any four of the points

Finding the radius



(4)

(1)

14. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Consider a circle centered at $(2, 3)$ and $(8, 11)$ be a point on it.

a) What is the radius of the circle?

b) Find the coordinates of four points on the circle.

(Score 5, Time 8 minutes)

■ Scoring Indicators

Radius = Distance between $(2, 3)$ and $(8, 11)$

$$= \sqrt{(8-2)^2 + (11-3)^2} \quad (1)$$

$$= \sqrt{6^2 + 8^2} = 10$$

Points 10 units away from $(2, 3)$ are $(12, 3), (-8, 3), (2, 13), (2, -7)$.

Again point obtained by shifting x coordinate by 6 units and y coordinate by 8 units are

$(2+6, 3+8); (2+6, 3-8); (2-6, 3+8); (2-6, 3-8)$

$\Rightarrow (8, 11), (8, -5), (-4, 11), (-4, -5)$

Again; $(2+8, 3+6); (2+8, 3-6); (2-8, 3+6); (2-8, 3-6)$

$\Rightarrow (10, 9), (10, -3), (-6, 9), (-6, -3)$

Writing any four points

(4)

15. Learning Outcome

- Formation of different geometric figures using number pairs

? Draw x, y axis and mark the points $A(2, 3)$, $B(-2, 3)$, $C(-2, -3)$, $D(2, -3)$. Draw the polygon connecting these points taken in order and name the polygon.

(Score 4 Time 8 minutes)

■ Scoring Indicators

- | | |
|--|-----|
| Drawing the coordinate axis | (1) |
| Marking the coordinates | (2) |
| Connecting the points and identifying the figure | (1) |

16. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.

? Show that (4, 2), (7, 5), (9, 7) lie in a line

(Score 4 Time 8 minutes)

■ Scoring Indicators

- | | |
|---|-----|
| Finding distance between (4, 2) and (7, 5) | (1) |
| Finding distance between (7, 5) and (9, 7) | (1) |
| Finding distance between (4, 2), (9, 7) | (1) |
| Showing the sum of smaller distance is equal to long distance | (1) |

17. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths

? P is a point on the perpendicular bisector of the line joining (2, 5) and (6, 5). If the x coordinate and y coordinate of P are equal write the coordinate of P

(Score 5 Time 8 minutes)

■ Scoring Indicators

Understanding the concept that the distance of a point on the perpendicular bisector is equidistant from the end points.

Assume that the coordinate of P is (a, a).

$$\text{Then ; Distance between (a, a) and (2, 3)} = \sqrt{(a-2)^2 + (a-3)^2} \quad (1)$$

$$\text{Distance between (a, a) and (6, 5)} = \sqrt{(a-6)^2 + (a-5)^2} \quad (1)$$

They are equal

$$\sqrt{(a-2)^2 + (a-3)^2} = \sqrt{(a-6)^2 + (a-5)^2}$$

$$(a-2)^2 + (a-3)^2 = (a-6)^2 + (a-5)^2 \quad (1)$$

$$a^2 - 4a + 4a^2 - 6a + 9 = a^2 - 12a + 36 + a^2 - 10a + 25 - 4a - 6a + 12a + 10a$$

$$= 36 + 25 - 9 - 4$$

$$12a = 48$$

$$a = \frac{48}{12} = 4$$

$$\text{Coordinates of P (4, 4)} \quad (1)$$

18. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths



Write the coordinate of the centre of a circle passing through the points (9, 3), (7, -1) (1, -1). Find the radius of the circle.

(Score 5 Time 8 minutes)

■ Scoring Indicators

Assume that the (x, y) be centre of the circle

$$\text{Distance between (9, 3) and (x, y)} = \sqrt{(x-9)^2 + (y-3)^2}$$

$$\text{Distance between (7, -1) and (x, y)} = \sqrt{(x-7)^2 + (y+1)^2} \quad (1)$$

They are equal

$$\therefore \sqrt{(x-7)^2 + (y+1)^2} = \sqrt{(x-9)^2 + (y-3)^2} \quad (1)$$

$$(x-7)^2 = (x-9)^2$$

$$x^2 - 14x + 49 = x^2 - 18x + 81$$

$$12x = 32$$

$$\therefore x = \frac{32}{12} = \frac{8}{3} \quad (1)$$

$$\sqrt{(x-9)^2 + (y-3)^2} = \sqrt{(x-1)^2 + (y+1)^2}$$

$$(x-9)^2 + (y-3)^2 = (x-1)^2 + (y+1)^2$$

$$\text{when } x = \frac{8}{3}$$

$$(x-9)^2 + y^2 - 6y + 9 = (x-1)^2 + y^2 + 2y + 1 \quad (1)$$

$$25 + y^2 - 6y + 9 = 9 + y^2 + 2y + 1$$

$$8y = 24$$

$$y = \frac{24}{8} = 3$$

The coordinate of the centre of the circle is $(\frac{8}{3}, 3)$

$$\therefore \text{Radius} = \text{Distance between } (\frac{8}{3}, 3) \text{ and } (1, -1)$$

$$= \sqrt{(4-\frac{8}{3})^2 + (3+1)^2} = \sqrt{3^2 + 4^2} = 5 \quad (1)$$

19. Learning Outcome

- Explaining the method of finding the distance between two points when their coordinates are given.



If (x, y) be a point equi distant from the points (7, 5), (4, 3), then show that $6x + 4y = 49$

(Score 4 Time 7 minutes)

■ Scoring Indicators

$$\text{Distance between (x, y) and (7, 5)} = \sqrt{(x-7)^2 + (y-5)^2} \quad (1)$$

$$\text{Distance between (x, y) and (4, 3)} = \sqrt{(x-4)^2 + (y-3)^2} \quad (1)$$

They are equal

$$(x - 7)^2 + (y - 5)^2 = (x - 4)^2 + (y - 3)^2$$

Solving and concluding

$$6x + 4y = 49$$

(2)

20. Learning Outcome

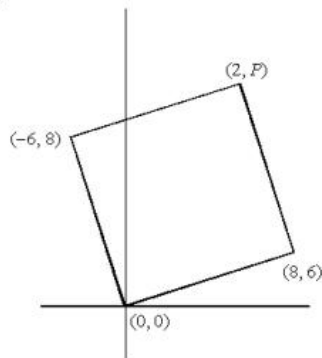
- Explaining the method of finding the distance between two points when their coordinates are given.



Three vertices of a square are given. If the fourth vertex is $(2, P)$ find the ratio of P . Find the area of the square.

(Score 4, Time 7 minutes)

■ Scoring Indicators



$$\text{Distance between } (-6, 8) \text{ and } (2, P) = \sqrt{(-6-2)^2 + (8-P)^2} \quad (1)$$

$$\text{Distance between } (8, 6) \text{ and } (2, P) = \sqrt{(8-2)^2 + (6-P)^2} \quad (1)$$

They are equal

$$8^2 + (8-P)^2 = 6^2 + (6-P)^2$$

$$64 + 64 - 16P + P^2 = 36 + 36 - 12P + P^2 \quad (1)$$

$$4P = 56$$

$$P = \frac{56}{4} = 14$$

$$\text{Distance between } (0, 0) \text{ and } (8, 6) = \sqrt{8^2 + 6^2} = 10 \quad (1)$$

$$\text{Area} = 10^2 = 100$$

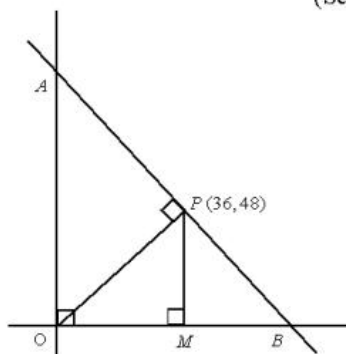
21. Learning Outcome

- Using the coordinates of vertices of geometric figures to determine various lengths



In the figure if P is (36, 48), then find the coordinates of A, B, M

(Score 4 Time 7 minutes)



■ Scoring Indicators

$\triangle OMP$ is a right triangle

$$\therefore OP = \sqrt{48^2 + 36^2} = 60$$

$\triangle PAO, \triangle OMP$ are similar

(1)

$$= \frac{OA}{OP} = \frac{OP}{PM} \therefore OA = \frac{OP^2}{PM} = \frac{60^2}{48} = 75$$

$\triangle OMP, \triangle OPB$ are similar

(1)

$$= \frac{OB}{OP} = \frac{OP}{OM} \therefore OB = \frac{OP^2}{OM} = \frac{60^2}{36} = 100$$

(1)

Coordinates of M (36, 0)

(1)

Coordinates of A (0, 75)

(1)

Coordinates of B (100, 0)

(1)

