NCERT SOLUTIONS

CLASS - 12th



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Class : 12th
Subject : Maths
Chapter : 7
Chapter Name : Integrals

Q1 Find an anti-derivative (or integral) of the following functions by the method of inspection.
sin 2x

Answer. The anti derivative of sin 2x is a function of x whose derivative is sin 2x. It is known that,

$\frac{d}{dx}(\cos 2x) = -2\sin 2x$

$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$

$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

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Q2 Find an anti derivative (or integral) of the following functions by the method of inspection.
cos 3x

Answer. The anti derivative of cos 3x is a function of x whose derivative is cos 3x. It is known that,

$\frac{d}{dx}(\sin 3x) = 3\cos 3x$

$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$

$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$.

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Q3 Find an anti derivative (or integral) of the following functions by the method of inspection.
e^{2x}

Answer. The anti derivative of e^{2x} is the function of x whose derivative is e^{2x}. It is known that,

$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$

$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$

$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

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Q4 Find an anti derivative (or integral) of the following functions by the method of inspection.
$(ax + b)^2$

Answer. The anti derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$. It is known that

$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$

$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$

$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$

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Q5 Find an anti derivative (or integral) of the following functions by the method of inspection: $\sin 2x - 4e^{3x}$

Answer. The anti derivative of $\sin 2x - 4e^{3x}$ is the function of x whose derivative is $\sin 2x - 4e^{3x}$.
It is known that,

$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$

Therefore, the anti derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

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Q6 Find the following integral $\int\left(4e^{3x}+1\right)dx$

Answer.

$\int\left(4e^{3x}+1\right)dx$

$= 4\int e^{3x}dx + \int 1dx$

$= 4\left(\frac{e^{3x}}{3}\right) + x + C$

$= \frac{4}{3}e^{3x} + x + C$

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Q7 Find the following integral $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer.

$\int x^2\left(1-\frac{1}{x^2}\right)dx$

$= \int\left(x^2-1\right)dx$

$= \int x^2dx - \int 1dx$

$= \frac{x^3}{3} - x + C$

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Q8 Find the following integral $\int\left(ax^2+bx+c\right)dx$

Answer.

$$\int\left(ax^2+bx+c\right)dx$$

$$=a\int x^2dx+b\int xdx+c\int 1.\,dx$$

$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$

$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

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Q9 Find the following integral $\int\left(2x^2+e^x\right)dx$

Answer.

$$\int\left(2x^2+e^x\right)dx$$

$$=2\int x^2dx+\int e^xdx$$

$$=2\left(\frac{x^3}{3}\right)+e^x+C$$

$$=\frac{2}{3}x^3+e^x+C$$

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Q10 Find the following integral $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$

Answer.

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$

$$=\int(x+\frac{1}{x}-2)dx$$

$$=\int xdx+\int\frac{1}{x}dx-2\int 1dx$$

$$=\frac{x^2}{2}+\log|x|-2x+\mathrm{C}$$

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Q11 Find the following integral $\int\frac{x^3+5x^2-4}{x^2}dx$

Answer.

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$=\int\left(x+5-4x^{-2}\right)dx$$

$$=\int xdx+5\int 1\cdot dx-4\int x^{-2}dx$$

$$=\frac{x^2}{2}+5x-4\left(\frac{x^{-1}}{-1}\right)+C$$

$$=\frac{x^2}{2}+5x+\frac{4}{x}+C$$

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Q12 Find the following integral $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Answer.

$\int \frac{x^3+3x+4}{\sqrt{x}}dx$

$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{\frac{1}{2}}\right)dx$

$= \frac{x^7}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(\frac{1}{x^{\frac{1}{2}}}\right)}{\frac{1}{2}} + C$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$

$= \frac{2}{7}x^2 + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$

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Q13 Find the following integral $\int \frac{x^3-x^2+x-1}{x-1}dx$

Answer.

$\int \frac{x^3-x^2+x-1}{x-1}dx$

On dividing, we obtain

$= \int \left(x^2 + 1\right)dx$

$= \int x^2 dx + \int 1 dx$

$= \frac{x^3}{3} + x + C$

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Q14 Find the following integral $\int (1-x)\sqrt{x}dx$

Answer.

$\int (1-x)\sqrt{x}dx$

$= \int (\sqrt{x} - x^{\frac{3}{2}})dx$

$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$

$= \frac{x^{\frac{3}{2}}}{2} - \frac{x^2}{2} + C$

$= \frac{2}{3}x^2 - \frac{2}{5}x^2 + C$

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Q15 Find the following integral $\int \sqrt{x}\left(3x^2 + 2x + 3\right)dx$

Answer.

$\int\sqrt{x}\left(3x^2+2x+3\right)dx$

$=\int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$=3\int x^{\frac{5}{2}}dx+2\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$

$=3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right)+2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)+3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$

$=\frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+C$

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Q16 Find the following integral $\int\left(2x-3\cos x+e^x\right)dx$

Answer.

$\int\left(2x-3\cos x+e^x\right)dx$

$=2\int xdx-3\int\cos xdx+\int e^xdx$

$=\frac{2x^2}{2}-3(\sin x)+e^x+C$

$=x^2-3\sin x+e^x+C$

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Q17 Find the following integral $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

Answer.

$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

$=2\int x^2dx-3\int\sin xdx+5\int x^{\frac{1}{2}}dx$

$=\frac{2x^3}{3}-3(-\cos x)+5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$

$=\frac{2}{3}x^3+3\cos x+\frac{10}{3}x^2+C$

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Q18 Find the following integral $\int\sec x(\sec x+\tan x)dx$

Answer.

$\int\sec x(\sec x+\tan x)dx$

$=\int\left(\sec^2x+\sec x\tan x\right)dx$

$=\int\sec^2xdx+\int\sec x\tan xdx$

$=\tan x+\sec x+C$

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Q19 Find the following integral $\int \frac{\sec^2 x}{\csc^2 x} dx$

Answer. $\int \frac{\sec^2 x}{\csc^2 x} dx$

$= \int \frac{\cos^2 x}{\frac{1}{\sin^2 x}} dx$

$= \int \frac{\sin^2 x}{\cos^2 x} dx$

$= \int \tan^2 x$

$= \int \left(\sec^2 x - 1 \right) dx$

$= \int \sec^2 x dx$

$= \tan x - x + C$

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Q20 Find the following integral $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

Answer.

$\int \frac{2 - 3\sin x}{\cos^2 x} dx$

$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$

$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$

$= 2\tan x - 3\sec x + C$

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Q21 The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$ equals.

(A) $\frac{1}{3} x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3} x^{\frac{2}{3}} + \frac{1}{2} x^2 + C$

(C) $\frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2} x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{1}{2}} + C$

Answer.

$\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

$= \int x^{\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

Hence, the correct Answer is C.

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Q22 If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Answer.

It is given that,

$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

$\therefore$ Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}dx$

$f(x) = 4\int x^3 dx - 3\int\left(x^{-4}\right)dx$

$f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$

$f(x) = x^4 + \frac{1}{x^3} + C$

Also,

$f(2) = 0$

$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$

$\Rightarrow 16 + \frac{1}{8} + C = 0$

$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$

$\Rightarrow C = \frac{-129}{8}$

$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

Hence, the correct Answer is A.

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Q1 Integrate the function $\frac{2x}{1+x^2}$.

Answer.

Let $1 + x^2 = t$

$\therefore 2xdx = dt$

$\Rightarrow \int\frac{2x}{1+x^2}dx = \int\frac{1}{1}dt$

$= \log|t| + C$

$= \log\left|1 + x^2\right| + C$

$= \log\left(1 + x^2\right) + C$

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Q2 Integrate the function $\frac{(\log x)^2}{x}$

Answer. Let log |x| = t ,

$\therefore \frac{1}{x}dx = dt$

$$\Rightarrow \int \frac{(\log|x|)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(\log|x|)^3}{3} + C$$

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Q3 Integrate the function $\frac{1}{x + x\log x}$

Answer.
$\frac{1}{x + x\log x} = \frac{1}{x(1 + \log x)}$
Let $1 + \log x = t$
$\frac{1}{x} dx = dt$
$\Rightarrow \int \frac{1}{x(1 + \log x)} dx = \int \frac{1}{t} dt$
$= \log|t| + C$
$= \log|1 + \log x| + C$

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Q4 Integrate the function: sin x · sin (cos x)

Answer. sin x · sin (cos x)
Let cos x = t
∴ −sin x dx = dt
$\Rightarrow \int \sin x \cdot \sin(\cos x) dx = -\int \sin t dt$
$= -[-\cos t] + C$
$= \cos t + C$
$= \cos(\cos x) + C$

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Q5 Integrate the function sin (ax + b) cos (ax + b)

Answer. $\sin(ax + b)\cos(ax + b) = \frac{2\sin(ax + b)\cos(ax + b)}{2} = \frac{\sin 2(ax + b)}{2}$
Let $2(ax + b) = t$
∴ $2a dx = dt$
$\Rightarrow \int \frac{\sin 2(ax + b)}{2} dx = \frac{1}{2}\int \frac{\sin t dt}{2a}$
$= \frac{1}{4a}[-\cos t] + C$
$= \frac{-1}{4a}\cos 2(ax + b) + C$

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Q6 Integrate the function $\sqrt{ax + b}$

Answer.
Let $ax + b = t$
$\Rightarrow a dx = dt$
$\therefore dx = \frac{1}{a} dt$

$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$

$= \frac{1}{a}\left(\frac{t^2}{\frac{3}{2}}\right) + C$

$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

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Q7 Integrate the function $x\sqrt{x+2}$

Answer. Let (x + 2) = t
$\therefore$ dx = dt

$\Rightarrow \int x\sqrt{x+2}dx = \int (t-2)\sqrt{t}dt$

$= \int t^{\frac{3}{2}} - 2t^{\frac{1}{2}})dt$

$= \int t^{\frac{3}{2}} dt - 2\int t^{\frac{1}{2}} dt$

$= \frac{t^2}{5} - 2\left(\frac{t^{\frac{1}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

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Q8 Integrate the function $x\sqrt{1+2x^2}$

Answer.
Let $1 + 2x^2 = t$
$\therefore 4xdx = dt$

$\Rightarrow \int x\sqrt{1+2x^2}dx = \int_4^{\sqrt{t}} dt$

$= \frac{1}{4}\int t^{\frac{1}{2}} dt$

$= \frac{1}{4}\left(\frac{t^3}{2}\right) + C$

$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$

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Q9 Integrate the function: $(4x+2)\sqrt{x^2+x+1}$

Answer.
Let $x^2 + x + 1 = t$
$\therefore (2x+1)dx = dt$
$\int (4x+2)\sqrt{x^2+x+1}dx$

$\int (4x+2)\sqrt{x^2+x+1}dx$

$= \int 2\sqrt{t}dt$

$= 2\int \sqrt{t}dt$

$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$

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Q10 Integrate the function : $\frac{1}{x-\sqrt{x}}$

Answer.

$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x}-1)}$

Let $(\sqrt{x}-1) = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{x}(\sqrt{x}-1)^{dx}} = \int \frac{2}{t}dt$

$= 2\log|t| + C$

$= 2\log|\sqrt{x}-1| + C$

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Q11 Integrate the function : $\frac{x}{\sqrt{x+4}}, x > 0$

Answer. Let x + 4 = t

$\therefore$ dx = dt

$\int \frac{x}{\sqrt{x+4}}dx = \int \frac{(t-4)}{\sqrt{t}}dt$

$= \int \left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) + C$

$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$

$= \frac{2}{3}t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$

$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C$

$= \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$

$= \frac{2}{3}\sqrt{x+4}(x-8) + C$

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Q12 Integrate the function:

$\left(x^3-1\right)^{\frac{1}{3}}x^5$

Answer.

Let $x^3-1=t$

$\therefore 3x^2dx=dt$

$\Rightarrow \int\left(x^3-1\right)^{\frac{1}{3}}x^5dx=\int\left(x^3-1\right)^{\frac{1}{3}}x^3\cdot x^2dx$

$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$

$=\frac{1}{3}\int t^{\frac{4}{3}}+t^{\frac{1}{3}})dt$

$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{3}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$

$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{1}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$

$=\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+C$

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Q13 Integrate the function : $\frac{x^2}{\left(2+3x^3\right)^3}$

Answer.

Let $2+3x^3=t$

$\therefore 9x^2dx=dt$

$\Rightarrow \int\frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$

$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+\mathrm{C}$

$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+\mathrm{C}$

$=\frac{-1}{18\left(2+3x^3\right)^2}+\mathrm{C}$

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Q14 Integrate the function : $\frac{1}{x(\log x)^m}, x>0, m\neq 1$

Answer. Let log x = t

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{1}{x(\log x)^m}dx = \int \frac{dt}{(t)^m}$

$= \frac{(\log x)^{1-\pi}}{(1-m)} + C$

$= \frac{(\log x)^{1-\pi}}{(1-m)} + C$

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Q15 Integrate the function : $\frac{x}{9-4x^2}$

Answer.

Let $9 - 4x^2 = t$

$\therefore \ -8xdx = dt$

$\Rightarrow \int \frac{x}{9-4x^2}dx = \frac{-1}{8}\int \frac{1}{t}dt$

$= \frac{-1}{8}\log|t| + C$

$= \frac{-1}{8}\log\left|9 - 4x^2\right| + C$

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Q16 Integrate the function : e^{2x+3}

Answer. Let 2x + 3 = t

$\therefore$ 2dx = dt

$\Rightarrow \int e^{2x+3}dx = \frac{1}{2}\int e^{'}dt$

$= \frac{1}{2}\left(e^t\right) + C$

$= \frac{1}{2}e^{(2x+3)} + C$

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Q17 Integrate the function : $\frac{x}{e^{x^2}}$

Answer.

Let $x^2 = t$

$\therefore 2xdx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^{'}}dt$

$= \frac{1}{2}\int e^{-t}dt$

$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$

$= -\frac{1}{2}e^{-x^2} + C$

$= \frac{-1}{2e^{x^2}} + C$

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Q18 Integrate the function : $\frac{e^{\tan^{-1}x}}{1+x^2}$

Answer.

Let $\tan^{-1}x = t$

$\frac{1}{1+x^2}dx = dt$

$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt$

$= e^t + C$

$= e^{\ln n^{-1}x} + C$

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Q19 Integrate the function : $\frac{e^{2x}-1}{e^{2x}+1}$

Answer.

$\frac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^x , we obtain

$\frac{\left(e^{2x}-1\right)}{e^{2x}+1)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\frac{e^x+1)}{e^x}$

Let $e^x + e^{-x} = t$

$\left(e^x - e^{-x}\right)dx = dt$

$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

$= \int \frac{dt}{t}$

$= \log|t| + C$

$= \log\left|e^x + e^{-x}\right| + C$

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Q20 Integrate the function : $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Answer.

Let $e^{2x} + e^{-2x} = t$

$\therefore \left(2e^{2x} - 2e^{-2x}\right)dx = dt$

$\Rightarrow 2\left(e^{2x} - e^{-2x}\right)dx = dt$

$$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx = dt$$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx = \int\frac{dt}{2t}$$

$$= \frac{1}{2}\int\frac{1}{t}dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log\left|e^{2x}+e^{-2x}\right| + C$$

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Q21 Integrate the function : $\tan^2(2x-3)$

Answer.

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let $2x-3 = t$

$\therefore 2dx = dt$

$$\Rightarrow \int\tan^2(2x-3)dx = \int\left(\sec^2(2x-3)\right) - 1dx$$

$$= \frac{1}{2}\int\left(\sec^2 t\right)dt - \int 1dx$$

$$= \frac{1}{2}\int\sec^2 t dt - \int 1dx$$

$$= \frac{1}{2}\tan t - x + C$$

$$= \frac{1}{2}\tan(2x-3) - x + C$$

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Q22 Integrate the function : $\sec^2(7-4x)$

Answer. Let $7-4x = t$

$\therefore -4dx = dt$

$$\therefore \int\sec^2(7-4x)dx = \frac{-1}{4}\int\sec^2 t dt$$

$$= \frac{-1}{4}(\tan t) + C$$

$$= \frac{-1}{4}\tan(7-4x) + C$$

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Q23 Integrate the function : $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Answer. Let $\sin^{-1}x = t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx = \int t dt$

$= \frac{\left(\sin^{-1} x\right)^2}{2} + C$

$= \frac{\left(\sin^{-1} x\right)^2}{2} + C$

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Q24 Integrate the function : $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Answer.

$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$

Let $3\cos x + 2\sin x = t$

$(-3\sin x + 2\cos x)dx = dt$

$$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2}\int_t^1 dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$$

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Q25 Integrate the function : $\frac{1}{\cos^2 x(1-\tan x)^2}$

Answer.

$\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$

Let$(1 - \tan x) = t$

$z - \sec^2 x dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{(1 - \tan x)^2}dx = \int \frac{-dt}{t^2}$$

$$= -\int t^{-2}dt$$

$$= +\frac{1}{t} + C$$

$$= \frac{1}{(1 - \tan x)} + C$$

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Q26 Integrate the function : $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Answer.

Let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$$\Rightarrow \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int \cos t dt$$

$$= 2\sin t + C$$

$$= 2\sin\sqrt{x} + C$$

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Q27 Integrate the function : $\sqrt{\sin 2x}\cos 2x$

Answer.

Let $\sin 2x = t$

$\therefore 2\cos 2x dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\cos 2x dx = \frac{1}{2}\int \sqrt{t} dt$$

$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{3}t^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

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Q28 Integrate the function : $\frac{\cos x}{\sqrt{1+\sin x}}$

Answer. Let $1 + \sin x = t$

$\therefore \cos x dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}}dx = \int \frac{dt}{\sqrt{t}}$$

$$\Rightarrow = \frac{t^{\frac{1}{2}}}{2} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1+\sin x} + C$$

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Q29 Integrate the function : cot x log sin x

Answer. Let log sin x = t

$$\Rightarrow \frac{1}{\sin x}\cdot \cos x dx = dt$$

$\therefore \cot x dx = dt$

$$\Rightarrow \cot x \log\sin x dx = \int t dt$$

$$\Rightarrow = \frac{t^2}{2} + C$$

$$= \frac{1}{2}(\log\sin x)^2 + C$$

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Q30 Integrate the function : $\frac{\sin x}{1+\cos x}$

Answer. Let 1 + cos x = t

$\therefore$ −sin x dx = dt

$$\Rightarrow \int \frac{\sin x}{1+\cos x}dx = \int -\frac{dt}{t}$$

$$= -\log|t| + C$$

$$= -\log|1+\cos x| + C$$

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Q31 Integrate the function : $\frac{\sin x}{(1+\cos x)^2}$

Answer. Let 1 + cos x = t

$\therefore$ −sin x dx = dt

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}dx = \int -\frac{dt}{t^2}$$

$$= -\int t^{-2}dt$$

$$= \frac{1}{t} + C$$

$$= \frac{1}{1+\cos x} + C$$

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Q32 Integrate the function : $\frac{1}{1+\cot x}$

Answer.

$$\text{Let } I = \int \frac{1}{1+\cot x}dx$$

$$= \int \frac{1}{1+\frac{\cos x}{\sin x}}dx$$

$$= \int \frac{\sin x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x+\cos x)}dx$$

$$= \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}(x) + \frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx$$

Let sin x + cos x = t ⇒ (cos x − sin x) dx = dt

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2}|\log \sin x+\cos x| + C$$

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Q33 Integrate the function : $\frac{1}{1-\tan x}$

Answer.

Let $I = \int \frac{1}{1-\tan x}dx$

$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$

$= \int \frac{\cos x}{\cos x - \sin x}dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$

$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$

$= \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$

$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$

Put cos x − sin x = t ⇒ (−sin x − cos x) dx = dt

$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$

$= \frac{x}{2} - \frac{1}{2}\log|t| + C$

$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

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Q34 Integrate the function : $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Answer.

Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$

$= \int \frac{\sqrt{\tan x}x\cos x}{\sin x \cos x \times \cos x}dx$

$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x}dx$

$= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t} + C$

$= 2\sqrt{\tan x} + C$

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Q35 Integrate the function : $\frac{(1+\log x)^2}{x}$

Answer. Let 1 + log x = t

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(1+\log x)^3}{3} + C$$

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Q36 Integrate the function : $\frac{(x+1)\,(x+\log x)^2}{x}$

Answer.

$$\frac{(x+1)\,(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\ldots \left(1+\frac{1}{x}\right)dx = dt$$

$$\Rightarrow \left(1+\frac{1}{x}\right)(x+\log x)^2 dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{1}{3}(x+\log x)^3 + C$$

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Q37 Integrate the function : $\frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8}$

Answer.

Let $x^4 = t$

$$\therefore 4x^3 dx = dt$$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8}dx = \frac{1}{4}\int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2}dt \ldots (1)$$

Let $\tan^{-1} t = u$

$$\frac{1}{1+t^2}dt = du$$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)dx}{1+x^8} = \frac{1}{4}\int \sin u\,du$$

$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} t\right) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$$

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Q38 Choose the correct answer : $\int \frac{10x^9 + 10^x \log_e 10 dx}{x^{10} + 10^x}$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer.

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right)dx = dt$

$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct Answer is D.

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Q39 Choose the correct answer : $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

(A) $\tan x + \cot x + C$ (B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$ (D) $\tan x - \cot 2x + C$

Answer.

Let $I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{1}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec^2 x dx + \int \csc^2 x dx$

$= \tan x - \cot x + C$

Hence, the correct Answer is B .

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Q1 Find the integrals of the functions : $\sin^2(2x + 5)$

Answer.

$\sin^2(2x + 5) = \frac{1 - \cos 2(2x + 5)}{2} = \frac{1 - \cos(4x + 10)}{2}$

$\Rightarrow \int \sin^2(2x + 5) dx = \int_2^{1 - \cos\ (4x + 10)} dx$

$= \frac{1}{2}\int 1 dx - \frac{1}{2}\int \cos(4x + 10)$

$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x + 10)}{4}\right) + C$

$= \frac{1}{2}x - \frac{1}{8}\sin(4x + 10) + C$

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Q2 Find the integrals of the functions : $\sin 3x \cos 4x$

Answer. It is known that,

$$\sin A\cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$$

$$\therefore \int \sin 3x\cos 4x\,dx = \frac{1}{2}\int\{\sin(3x+4x) + \sin(3x-4x)\}dx$$

$$= \frac{1}{2}\int\{\sin 7x + \sin(-x)\}dx$$

$$= \frac{1}{2}\int\{\sin 7x - \sin x\}dx$$

$$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

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Q3 Find the integrals of the functions : cos 2x cos 4x cos 6x

Answer.

It is known that, $\cos A\cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}$

$$\therefore \int \cos 2x(\cos 4x\cos 6x)dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x) + \cos(4x-6x)\}\right]dx$$

$$= \frac{1}{2}\int\{\cos 2x\cos 10x + \cos 2x\cos(-2x)\}dx$$

$$= \frac{1}{2}\int\left\{\cos 2x\cos 10x + \cos^2 2x\right\}dx$$

$$= \frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x) + \cos(2x-10x)\right\} + \left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4}\right] + C$$

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Q4 Find the integrals of the function : $\sin^3(2x+1)$

Answer.

Let $I = \int \sin^3(2x+1)$

$\Rightarrow \int \sin^3(2x+1)dx = \int \sin^2(2x+1)\cdot\sin(2x+1)dx$

$= \int\left(1 - \cos^2(2x+1)\right)\sin(2x+1)dx$

Let $\cos(2x+1) = t$

$\Rightarrow -2\sin(2x+1)dx = dt$

$\Rightarrow \sin(2x+1)dx = \frac{-dt}{2}$

$$\Rightarrow I = \frac{-1}{2}\int\left(1 - t^2\right)dt$$

$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos(2x + 1) - \frac{\cos^3(2x + 1)}{3}\right\}$$

$$= \frac{-\cos(2x + 1)}{2} + \frac{\cos^3(2x + 1)}{6} + C$$

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Q5 Find the integrals of the function : $\sin^3 x\cos^3 x$

Answer.

$$\text{Let } I = \int \sin^3 x\cos^3 x \cdot dx$$

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$

$$= \int \cos^3 x\left(1 - \cos^2 x\right)\sin x \cdot dx$$

$$\text{Let} \cos x = t$$

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\Rightarrow I = -\int t^3\left(1 - t^2\right)dt$$

$$= -\int\left(t^3 - t^5\right)dt$$

$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$

$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

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Q6 Find the integrals of the function : sin x sin 2x sin 3x

Answer. It is known that,

$$\sin A\sin B = \frac{1}{2}\{\cos(A - B) - \cos(A + B)\}$$

$$\therefore \int \sin x\sin 2x\sin 3x dx = \int\left[\sin x \cdot \frac{1}{2}\{\cos(2x - 3x) - \cos(2x + 3x)\}\right]dx$$

$$= \frac{1}{2}\int(\sin x\cos(-x) - \sin x\cos 5x)dx$$

$$= \frac{1}{2}\int(\sin x\cos x - \sin x\cos 5x)dx$$

$$= \frac{1}{2}\int \frac{\sin 2x}{2}dx - \frac{1}{2}\int \sin x\cos 5x dx$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int\left\{\frac{1}{2}\sin(x + 5x) + \sin(x - 5x)\right\}dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int(\sin 6x + \sin(-4x))dx$$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

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Q7 Find the integrals of the function : sin 4x sin 8x

Answer. It is known that, $\sin A \sin B = \frac{1}{2}\cos(A - B) - \cos(A + B)$

$$\therefore \int \sin 4x \sin 8x dx = \int\left\{\frac{1}{2}\cos(4x - 8x) - \cos(4x + 8x)\right\}dx$$

$$= \frac{1}{2}\int(\cos(-4x) - \cos 12x)dx$$

$$= \frac{1}{2}\int(\cos 4x - \cos 12x)dx$$

$$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right]$$

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Q8 Find the integrals of the function : $\frac{1-\cos x}{1+\cos x}$

Answer. $\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 2}{2\cos^2 x}$ $\left[2\sin^2\frac{x}{2} = 1 - \cos x \text{ and } 2\cos^2\frac{x}{2} = 1 + \cos x\right]$

$$= \tan^2\frac{x}{2}$$

$$= \left(\sec^2\frac{x}{2} - 1\right)$$

$$\therefore \int\frac{1 - \cos x}{1 + \cos x}dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$= \begin{bmatrix} x \\ -\frac{2}{1} - x \end{bmatrix} + C$$

$$= 2\tan\frac{x}{2} - x + C$$

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Q9 Find the integrals of the function : $\frac{\cos x}{1+\cos x}$

Answer. $\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$ $\left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$

$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\therefore \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx$$

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan\frac{x}{2} + C$$

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Q10 Find the integrals of the function : $\sin^4 x$

Answer.

$$\sin^4 x = \sin^2 x \sin^2 x$$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{4}(1-\cos 2x)^2$$

$$= \frac{1}{4}\left[1 + \cos^2 2x - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$\therefore \int \sin^4 x dx = \frac{1}{4}\int\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C$$

$$= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} - 2\sin 2x\right] + C$$

$$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

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Q11 Find the integrals of the function : $\cos^4 2x$

Answer.

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$

$$= \left(\frac{1+\cos 4x}{2}\right)^2$$

$$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$$

$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$

$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$\therefore \int \cos^4 2x\,dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$

$$= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

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Q12 Find the integrals of the function : $\frac{\sin^2 x}{1+\cos x}$

Answer. $\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$ $\left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$

$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= 2\sin^2\frac{x}{2}$$

$$= 1-\cos x$$

$$\therefore \int \frac{\sin^2 x}{1+\cos x}dx = \int(1-\cos x)dx$$

$$= x-\sin x+C$$

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Q13 Find the integrals of the function : $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

Answer. $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}$ $\left[\cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$

$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$

$= 2[\cos(x)+\cos\alpha]$

$= 2\cos x + 2\cos\alpha$

$\therefore \int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx = \int 2\cos x + 2\cos\alpha$

$= 2[\sin x + x\cos\alpha] + C$

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Q14 Find the integrals of the function : $\frac{\cos x-\sin x}{1+\sin 2x}$

Answer. $\frac{\cos x-\sin x}{1+\sin 2x} = \frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}\left[\sin^2 x+\cos^2 x=1;\ \sin 2x=2\sin x\cos x\right]$

$= \frac{\cos x-\sin x}{(\sin x+\cos x)^2}$

Let $\sin x + \cos x = t$

$\therefore (\cos x-\sin x)dx = dt$

$\Rightarrow \int\frac{\cos x-\sin x}{1+\sin 2x}dx = \int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$

$= \int\frac{dt}{t^2}$

$= \int t^{-2}dt$

$= -t^{-1}+C$

$= -t^{-1}+C$

$= -\frac{1}{\sin x+\cos x}+C$

$= \frac{1}{\sin x+\cos x}+C$

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Q15 Find the integrals of the function : $\tan^3 2x\sec 2x$

Answer.

$\tan^3 2x\sec 2x = \tan^2 2x\tan 2x\sec 2x$

$= \left(\sec^2 2x-1\right)\tan 2x\sec 2x$

$= \sec^2 2x\cdot\tan 2x\sec 2x - \tan 2x\sec 2x$

$\therefore \int\tan^3 2x\sec 2x\,dx = \int\sec^2 2x\tan 2x\sec 2x\,dx - \int\tan 2x\sec 2x\,dx$

$= \int\sec^2 2x\tan 2x\sec 2x\,dx - \frac{\sec 2x}{2}+C$

Let $\sec 2x = t$

$\therefore 2\sec 2x\tan 2x\,dx = dt$

$\therefore \int\tan^3 2x\sec 2x\,dx = \frac{1}{2}\int t^2dt - \frac{\sec 2x}{2}+C$

$= \frac{t^3}{6}-\frac{\sec 2x}{2}+C$

$= \frac{(\sec 2x)^3}{6}-\frac{\sec 2x}{2}+C$

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Q16 Find the integrals of the function : $\tan^4 x$

Answer.

$\tan^4 x$

$= \tan^2 x \cdot \tan^2 x$

$= \left(\sec^2 x - 1\right)\tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$\therefore \int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx + \int 1 \cdot dx$

$= \int \sec^2 x \tan^2 x dx - \tan x + x + C \ldots (1)$

Consider $\int \sec^2 x \tan^2 x dx$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$

From equation (1), we obtain

$\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \tan x + x + C$

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Q17 Find the integrals of the function : $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Answer.

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \csc x$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \csc x) dx$$

$$= \sec x - \csc x + C$$

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Q18 Find the integrals of the function : $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Answer. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \quad \left[\cos 2x = 1 - 2\sin^2 x\right]$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$

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Q19 Find the integrals of the function : $\frac{1}{\sin x \cos^3 x}$

Answer.

$$\frac{1}{\sin x\cos^3 x} = \frac{\sin^2 x + \cos^2 x}{\sin x\cos^3 x}$$

$$= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x\cos x}$$

$$= \tan x\sec^2 x + \frac{1\cos^2 x}{\cos^2 x}$$

$$= \tan x\sec^2 x + \frac{\sec^2 x}{\tan x}$$

$$\therefore \int \frac{1}{\sin x\cos^3 x}dx = \int \tan x\sec^2 x dx + \int \frac{\sec^2 x}{\tan x}dx$$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\Rightarrow \int \frac{1}{\sin x\cos^3 x}dx = \int t dt + \int_t^1 dt$$

$$= \frac{t^2}{2} + |\log t| + C$$

$$= \frac{1}{2}\tan^2 x + |\log|\tan x| + C$$

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Q20 Find the integrals of the function : $\frac{\cos 2x}{(\cos x + \sin x)^2}$

Answer.

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x\cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}dx = \int \frac{\cos 2x}{(1 + \sin 2x)}dx$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}dx = \int \frac{\cos 2x}{(1 + \sin 2x)}dx$$

Let $1 + \sin 2x = t$

$$\Rightarrow 2\cos 2x dx = dt$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2}dx = \frac{1}{2}\int_t^1 dt$$

$$= \frac{1}{2}\log|t| + C$$

$$= \frac{1}{2}\log|1 + \sin 2x| + C$$

$$= \frac{1}{2}\log\left|(\sin x + \cos x)^2\right| + C$$

$$= \log|\sin x + \cos x| + C$$

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Q21 Find the integrals of the function : $\sin^{-1}(\cos x)$

Answer.

$\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1 - t^2}$

$$\Rightarrow (-\sin x)dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1 - t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1 - t^2}}\right)$$

$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}} dt$

Let $\sin^{-1} t = u$

$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$

$\therefore \int \sin^{-1}(\cos x) dx = \int 4 du$

$= -\frac{u^2}{2} + C$

$= \frac{-\left(\sin^1 t\right)^2}{2} + C$

$= \frac{-\left[\sin^{-1}(\cos x)\right]^2}{2} + C$

It is known that,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x\right)$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x) dx = \frac{-\left[\frac{\pi}{2} - x\right]^2}{2} + C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$

$$= \frac{\pi x}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

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Q22 Find the integrals of the function : $\frac{1}{\cos(x-a)\cos(x-b)}$

Answer.

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)]}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)]}{\sin(a-b)}$$

$$= \frac{1}{\sin(a-b)}[\tan(x-b) - \tan(x-a)]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)}\int [\tan(x-b) - \tan(x-a)] dx$$

$$= \frac{1}{\sin(a-b)}[-\log|\cos(x-b)| + \log\cos(x-a)|]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right] + C$$

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Q23 Choose the correct answer

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

(A) tan x + cot x + C

(B) tan x + cosec x + C

(C) − tan x + cot x + C

(D) tan x + sec x + C

Answer.

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x}) dx$$

$$= \int \left(\sec^2 x - \csc^2 x\right) dx$$

$$= \tan x + \cot x + C$$

Hence, the correct Answer is A

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Q24 $\int \frac{e^x (1+x)}{\cos^2 \left(e^x x\right)}$ equals

(A) $-\cot\left(ex^x\right) + C$ (B)$\tan\left(xe^x\right) + C$

(C) $\tan\left(e^x\right) + C$ (D)$\cot\left(e^x\right) + C$

Answer. $\int \frac{e^x (1+x)}{\cos^2 \left(e^x x\right)}$

Let exx = t

$$\Rightarrow \left(e^x \cdot x + e^x \cdot 1\right) dx = dt$$

$$e^x (x + 1) dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2 \left(e^x x\right)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t dt$$

$$= \tan t + C$$

$$= \tan\left(e^x \cdot x\right) + C$$

Hence, the correct Answer is B.

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Q1 Integrate the function : $\frac{3x^2}{x^6 + 1}$

Answer.

Let $x^3 = t$

$$\therefore 3x^2 dx = dt$$

$$\Rightarrow 3x^2 + 1$$

$$\Rightarrow \int \frac{3x^2}{x^6 + 1} dx = \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1}(t) + C = \tan^{-1}\left(x^3\right) + C$$

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Q2 Integrate the function : $\frac{1}{\sqrt{1+4x^2}}$

Answer. Let 2x = t

∴ 2dx = dt

$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$ $\left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$

$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C$

$= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + C$

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Q3 Integrate the function : $\frac{1}{\sqrt{(2-x)^2+1}}$

Answer. Let 2 − x = t

⇒ −dx = dt

$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$ $\left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$

$= -\log\left|t+\sqrt{t^2+1}\right| + C$

$= -\log|2-x+\sqrt{(2-x)^2+1}| + C$

$= \log\frac{1}{(2-x)+\sqrt{x^2-4x+5}} + C$

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Q4 Integrate the function : $\frac{1}{\sqrt{9-25x^2}}$

Answer. Let 5x = t

∴ 5dx = dt

$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}dx = \frac{1}{5}\int \frac{1}{9-t^2}dt$

$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}dt$

$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C$

$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$

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Q5 Integrate the function : $\frac{3x}{1+2x^4}$

Answer.

Let $\sqrt{2}x^2 = t$

$\therefore 2\sqrt{2}xdx = dt$

$\Rightarrow \int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$

$= \frac{3}{2\sqrt{2}}\left[\tan^{-1}t\right] + C$

$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right) + C$

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Q6 Integrate the function : $\frac{x^2}{1-x^6}$

Answer. Let $x^3 = t$

$\therefore 3x^2dx = dt$

$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$

$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$

$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$

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Q7 Integrate the function : $\frac{x-1}{\sqrt{x^2-1}}$

Answer. $\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx. \ldots . (1)$

For $\int \frac{x}{\sqrt{x^2-1}}dx,$ let $x^2 - 1 = t \Rightarrow 2xdx = dt$

$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$

$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$

$= \sqrt{t}$

$= \sqrt{x^2 - 1}$

From (1), we obtain

$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \quad \left[\int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x + \sqrt{x^2 - a^2}\right|\right]$

$= \sqrt{x^2 - 1} - \log\left|x + \sqrt{x^2 - 1}\right| + C$

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Q8 Integrate the function : $\frac{x^2}{\sqrt{x^6+a^6}}$

Answer.

Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$

$= \frac{1}{3}\log\left|t + \sqrt{t^2+a^6}\right| + C$

$= \frac{1}{3}\log\left|x^3 + \sqrt{x^6+a^6}\right| + C$

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Q9 Integrate the function : $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

Answer.

Let $\tan x = t$

$\therefore \sec^2 x dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx = \int \frac{dt}{\sqrt{t^2+2^2}}$

$= \log\left|t + \sqrt{t^2+4}\right| + C$

$= \log\left|\tan x + \sqrt{\tan^2 x+4}\right| + C$

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Q10 Integrate the function : $\frac{1}{\sqrt{x^2+2x+2}}$

Answer.

$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$

Let $x + 1 = t$

$\therefore dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$

$= \log\left|t + \sqrt{t^2+1}\right| + C$

$= \log\left|(x+1) + \sqrt{(x+1)^2+1}\right| + C$

$= \log\left|(x+1) + \sqrt{x^2+2x+2}\right| + C$

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Q11 Integrate the function : $\frac{1}{\sqrt{9x^2+6x+5}}$

Answer.

$\int \frac{1}{9x^2+6x+5}dx = \int \frac{1}{(3x+1)^2+(2)^2}dx$

Let $(3x + 1) = t$

$\therefore 3dx = dt$

$\Rightarrow \int \frac{1}{(3x+1)^2+(2)^2}dx = \frac{1}{3}\int \frac{1}{t^2+2^2}dt$

$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right] + C$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$

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Q12 Integrate the function : $\frac{1}{\sqrt{7-6x-x^2}}$

Answer.

$7 - 6x - x^2$ can be written as $7 - \left(x^2 + 6x + 9 - 9\right)$

Therefore,

$7 - \left(x^2 + 6x + 9 - 9\right)$

$= 16 - \left(x^2 + 6x + 9\right)$

$= 16 - (x + 3)^2$

$= (4)^2 - (x + 3)^2$

$\therefore \int \frac{1}{\sqrt{7-6x-x^2}}dx = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$

Let $x + 3 = t$

$\Rightarrow dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx = \int \frac{1}{\sqrt{(4)^2-(t)^2}}dt$

$= \sin^{-1}\left(\frac{t}{4}\right) + C$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$

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Q13 Integrate the function : $\frac{1}{\sqrt{(x-1)(x-2)}}$

Answer.

$(x - 1)(x - 2)$ can be written as $x^2 - 3x + 2$

Therefore,

$x^2 - 3x + 2$

$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2$

$= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$

$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}}dx = \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

Let $x - \frac{3}{2} = t$

$\therefore dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx = \int \frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}dt$$

$$= \log\left|t + \sqrt{t^2 - \left(\frac{1}{2}\right)^2}\right| + C$$

$$= \log\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}| + C$$

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Q14 Integrate the function : $\frac{1}{\sqrt{8+3x-x^2}}$

Answer.

$8 + 3x - x^2$ can be written as $8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$

Therefore,

$$8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$$

$$= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}dx = \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}dx}$$

Let $x - \frac{3}{2} = t$

$\therefore dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{\sqrt{41}}\right) + \mathrm{C}$$

$$= \sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + \mathrm{C}$$

$$= \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + \mathrm{C}$$

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Q15 Integrate the function : $\frac{1}{\sqrt{(x-a)(x-b)}}$

Answer.

$(x - a)(x - b)$ can be written as $x^2 - (a + b)x + ab$

Therefore,

$x^2 - (a + b)x + ab$

$= x^2 - (a + b)x + \frac{(a+b)^2}{4} - \frac{(a+b)^2}{4} + ab$

$= \left[x - \left(\frac{a+b}{2}\right)\right]^2 - \frac{(a-b)^2}{4}$

$\Rightarrow \int \frac{1}{\sqrt{(x-a)(x-b)}dx} = \int \frac{1}{\sqrt{\left\{x - \left(\frac{a+b}{2}\right)\right\}^2 - \left(\frac{a-b}{2}\right)^2}}dx$

Let $x - \left(\frac{a+b}{2}\right) = t$

$\therefore dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\left\{x - \left(\frac{a+b}{2}\right)\right\}^2 - \left(\frac{a-b}{2}\right)^2}}dx = \int \frac{1}{\sqrt{t^2 - \left(\frac{a-b}{2}\right)^2}}dt$

$= \log\left|t + \sqrt{t^2 - \left(\frac{a-b}{2}\right)^2}\right| + C$

$= \log\left|\left\{x - \left(\frac{a+b}{2}\right)\right\} + \sqrt{(x - a)(x - b)}\right| + C$

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Q16 Integrate the function : $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Answer.

Let $4x + 1 = A\frac{d}{dx}\left(2x^2 + x - 3\right) + B$

$\Rightarrow 4x + 1 = A(4x + 1) + B$

$\Rightarrow 4x + 1 = 4Ax + A + B$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)dx = dt$

$\Rightarrow \int \frac{4x + 1}{\sqrt{2x^2 + x - 3}}dx = \int \frac{1}{\sqrt{t}}dt$

$= 2\sqrt{t} + C$

$= 2\sqrt{2x^2 + x - 3} + C$

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Q17 Integrate the function : $\frac{x+2}{\sqrt{x^2-1}}$

Answer.

Let $x + 2 = A\frac{d}{dx}\left(x^2 - 1\right) + B \quad \ldots(1)$

$\Rightarrow x + 2 = A(2x) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \frac{1}{2}$

$B = 2$

From (1), we obtain

$(x + 2) = \frac{1}{2}(2x) + 2$

Then, $\int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \ldots (2)$

In $\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t \Rightarrow 2xdx = dt$

$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}[2\sqrt{t}]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

Then, $\int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x + \sqrt{x^2-1}\right|$

From equation (2), we obtain

$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + C$

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Q18 Integrate the function : $\frac{5x-2}{1+2x+3x^2}$

Answer.

Let $5x - 2 = A\frac{d}{dx}\left(1 + 2x + 3x^2\right) + B$

$\Rightarrow 5x - 2 = A(2 + 6x) + B$

Equating the coefficient of x and constant term on both sides, we obtain

$5 = 6A \Rightarrow A = \frac{5}{6}$

$2A + B = -2 \Rightarrow B = -\frac{11}{3}$

$\therefore 5x - 2 = \frac{5}{6}(2 + 6x) + \left(-\frac{11}{3}\right)$

$\Rightarrow \int \frac{5x-2}{1+2x+3x^2}dx = \int \frac{56\left(2+6x-\frac{11}{3}\right)}{1+2x+3x^2}dx$

$= \frac{5}{6}\int \frac{2+6x}{1+2x+3x^2}dx - \frac{11}{3}\int \frac{1}{1+2x+3x^2}dx$

Let $I_1 = \int \frac{2+6x}{1+2x+3x^2}dx$ and $I_2 = \int \frac{1}{1+2x+3x^2}dx$

$\therefore \int \frac{5x-2}{1+2x+3x^2}dx = \frac{5}{6}I_1 - \frac{11}{3}I_2 \ldots\ldots (1)$

$I_1 = \int \frac{2+6x}{1+2x+3x^2} dx$

Let $1 + 2x + 3x^2$

$\Rightarrow (2 + 6x)dx = dt$

$\therefore I_1 = \int \frac{dt}{t}$

$I_1 = \log|t|$

$I_1 = \log\left|1 + 2x + 3x^2\right|$

$I_2 = \int \frac{1}{1+2x+3x^2} dx$

$1 + 2x + 3x^2$ can be written as $1 + 3\left(x^2 + \frac{2}{3}x\right)$

Therefore,

$1 + 3\left(x^2 + \frac{2}{3}x\right)$

$= 1 + 3\left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9}\right)$

$= 1 + 3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3}$

$= \frac{2}{3} + 3\left(x + \frac{1}{3}\right)^2$

$= 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{2}{9}\right]$

$= 3\left[\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2\right]$

$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2\right]} dx$

$= \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1}\left(\frac{x + \frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]$

$= \frac{1}{3}\left[\frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$

Substituting equations (2) and (3) in equation (1), we obtain

$$\int \frac{5x-2}{1+2x+3x^2} dx$$

$= \frac{5}{6}\left[\log|1 + 2x + 3x^2|\right] - \frac{11}{3}\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right] + C$

$$= \frac{5}{6}\log\left|1 + 2x + 3x^2\right| - \frac{11}{3\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$$

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Q19 Integrate the function : $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Answer.

$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$

Let $6x + 7 = A\frac{d}{dx}\left(x^2 - 9x + 20\right) + B$

$\Rightarrow 6x + 7 = A(2x - 9) + B$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

$-9A + B = 7 \Rightarrow B = 34$

$\therefore 6x + 7 = 3\,(2x - 9) + 34$

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$

$$= 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int \frac{1}{\sqrt{x^2-9x+20}}dx$$

Let $I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$\therefore \int \frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2 \ldots . (1)$

Then,

$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$

Let $x^2 - 9x + 20 = t$

$\Rightarrow (2x - 9)dx = dt$

$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$

$I_1 = 2\sqrt{t}$

$I_1 = 2\sqrt{x^2 - 9x + 20}$

and $I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$

$x^2 - 9x + 20$ can be written as $x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$

Therefore,

$x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$

$= \left(x - \frac{9}{2}\right)^2 - \frac{1}{4}$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$\Rightarrow I_2 = \int \sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$

$I_2 = \log\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\,| \ldots \ldots \ldots (3)$

Substituting equations (2) and (3) in (1), we obtain

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3\left[2\sqrt{x^2-9x+20}\right] + 34\log\left[\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right] + C$$

$= 6\sqrt{x^2 - 9x + 20} + 34\log\left[\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\right] + C$

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Q20 Integrate the function : $\frac{x+2}{\sqrt{4x-x^2}}$

Answer.

Let $x + 2 = A\frac{d}{dx}\left(4x - x^2\right) + B$

$\Rightarrow x + 2 = A(4 - 2x) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$-2A = 1 \Rightarrow A = -\frac{1}{2}$

$4A + B = 2 \Rightarrow B = 4$

$\Rightarrow (x + 2) = -\frac{1}{2}(4 - 2x) + 4$

$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$

$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int \frac{1}{\sqrt{4x-x^2}}dx$

Let $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2\int \frac{1}{\sqrt{4x-x^2}}dx$

$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}I_1 + 4I_2$

Then, $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x - x^2 = t$

$\Rightarrow (4 - 2x)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x - x^2}$

$I_2 = \int \frac{1}{\sqrt{4x - x^2}}dx$

$\Rightarrow 4x - x^2 = -\left(-4x + x^2\right)$

$= \left(-4x + x^2 + 4 - 4\right)$

$= 4 - (x - 2)^2$

$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}}dx = \sin^{-1}\left(\frac{x-2}{2}\right)$

Using equations (2) and (3) in (1), we obtain

$\int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}\left(2\sqrt{4x - x^2}\right) + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$

$= -\sqrt{4x - x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$

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Q21 Integrate the function : $\frac{x+2}{\sqrt{x^2+2x+3}}$

Answer.

$$\int \frac{(x+2)}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2 \ldots\ldots (1)$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x + 2)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \ldots (2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$\Rightarrow x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x + 1)^2 + (\sqrt{2})^2$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx = \log|(x+1) + \sqrt{x^2+2x+3}| \ldots (3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log|(x+1) + \sqrt{x^2+2x+3}| + C$$

$$= \sqrt{x^2+2x+3} + \log|(x+1) + \sqrt{x^2+2x+3}| + C$$

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Q22 Integrate the function : $\frac{x+3}{x^2-2x-5}$

Answer.

Let $(x + 3) = A\frac{d}{dx}\left(x^2 - 2x - 5\right) + B$

$(x + 3) = A(2x - 2) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \frac{1}{2}$

$-2A + B = 3 \Rightarrow B = 4$

$\therefore (x + 3) = \frac{1}{2}(2x - 2) + 4$

$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$

$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

Let $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$ and $I_2 = \int \frac{1}{x^2-2x-5}dx$

$\therefore \int \frac{x+3}{\left(x^2-2x-5\right)}dx = \frac{1}{2}I_1 + 4I_2$

Then , $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x - 2)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log\left|x^2 - 2x - 5\right|$

$I_2 = \int x^2-2x-5 dx$

$= \int \frac{1}{\left(x^2 - 2x + 1\right) - 6}dx$

$= \int \frac{1}{(x-1)^2 + (\sqrt{6})^2}dx$

$= \frac{1}{2\sqrt{6}}\log\left(\frac{x - 1 - \sqrt{6}}{x - 1 + \sqrt{6}}\right)$

Substituting (2) and (3) in (1), we obtain

$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log\left|x^2 - 2x - 5\right| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$

$= \frac{1}{2}\log\left|x^2 - 2x - 5\right| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$

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Q23 Integrate the function : $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Answer.

Let $5x + 3 = A\frac{d}{dx}\left(x^2 + 4x + 10\right) + B$

$\Rightarrow 5x + 3 = A(2x + 4) + B$

Equating the coefficients of x and constant term, we obtain

$2A = 5 \Rightarrow A = \frac{5}{2}$

$4A + B = 3 \Rightarrow B = -7$

$\therefore 5x + 3 = \frac{5}{2}(2x + 4) - 7$

$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$

$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$

Let $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$

$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2$

Then $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Let $x^2 + 4x + 10 = t$

$\therefore (2x + 4)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2 + 4x + 10}$

$I_2 = \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx$

$= \int \frac{1}{\sqrt{\left(x^2 + 4x + 4\right) + 6}} dx$

$= \int \frac{1}{(x + 2)^2 + (\sqrt{6})^2} dx$

$= \log\left|(x + 2)\sqrt{x^2 + 4x + 10}\right|$

Using equations (2) and (3) in (1), we obtain

$\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx = \frac{5}{2}\left[2\sqrt{x^2 + 4x + 10}\right] - 7\log(x + 2) + \sqrt{x^2 + 4x + 10} + C$

$= 5\sqrt{x^2 + 4x + 10} - 7\log(x + 2) + \sqrt{x^2 + 4x + 10}\,| + C$

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Q24 Choose the correct answer $\int \frac{dx}{x^2 + 2x + 2}$ equals

(A) $x\tan^{-1}(x + 1) + C$ (B) $\tan^{-1}(x + 1) + C$

(C) $(x + 1)\tan^{-1}x + C$ (D) $\tan^{-1}x + C$

Answer.

$\int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{\left(x^2 + 2x + 1\right) + 1}$

$= \int \frac{1}{(x + 1)^2 + (1)^2} dx$

$= \left[\tan^{-1}(x + 1)\right] + C$

Hence, the correct Answer is B.

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Q25 Choose the correct answer $\int \frac{dx}{\sqrt{9x - 4x^2}}$ equals

(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x - 8}{8}\right) + C$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{8x - 9}{9}\right) + C$

(C) $\frac{1}{3}\sin^{-1}\left(\frac{9x - 8}{8}\right) + C$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{9x - 8}{9}\right) + C$

Answer.

$\int \frac{dx}{\sqrt{9x - 4x^2}}$

$= \int \frac{1}{\sqrt{-4\left(x^2 - \frac{9}{4}x\right)}} dx$

$= \int \frac{1}{-4\left(x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64}\right)} dx$

$$= \int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1}\frac{y}{a} + \mathrm{C}\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Hence, the correct Answer is B.

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Q1 Integrate the rational function : $\frac{x}{(x+1)(x+2)}$

Answer.

$\frac{x}{\text{Let}}\ \frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$

$\Rightarrow \int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$

$= -\log|x+1| + 2\log|x+2| + C$

$= \log(x+2)^2 - \log|x+1| + C$

$= \log\frac{(x+2)^2}{(x+1)} + C$

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Q2 Integrate the rational function : $\frac{1}{x^2-9}$

Answer.

$\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A = -\frac{1}{6}$ and $B = \frac{1}{6}$

$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$

$$\Rightarrow \int \frac{1}{\left(x^2-9\right)}dx = \int \frac{-1}{6(x+3)} + \frac{1}{6(x-3)})dx$$

$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + C$$

$$= \frac{1}{6}\log\frac{|(x-3)}{(x+3)}| + \mathrm{C}$$

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Q3 Integrate the rational function : $\frac{3x-1}{(x-1)(x-2)(x-3)}$

Answer.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$3x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$

Substituting $x = 1, 2,$ and 3 respectively in equation (1), we obtain

$A = 1, B = -5,$ and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)}dx = \int\left\{\frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}\right\}dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

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Q4 Integrate the rational function : $\frac{x}{(x-1)(x-2)(x-3)}$

Answer.

Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$x = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2) \ldots (1)$

Substituting $x = 1, 2,$ and 3 respectively in equation (1), we obtain

$$A = \frac{1}{2}, B = -2, \text{ and } C = \frac{3}{2}$$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)}dx = \int\left\{\frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}dx\right.$$

$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + \mathrm{C}$$

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Q5 Integrate the rational function : $\frac{2x}{x^2+3x+2}$

Answer.

$$\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$2x = A(x + 2) + B(x + 1)$

Substituting $x = -1$ and -2 in equation (1), we obtain

$A = -2$ and $B = 4$

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)}dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4\log|x+2| - 2\log|x+1| + C$$

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Q6 Integrate the rational function : $\frac{1-x^2}{x(1-2x)}$

Answer.

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $\left(1 - x^2\right)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

$$\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

$$\Rightarrow (2 - x) = A(1 - 2x) + Bx$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (1), we obtain

$A = 2$ and $B = 3$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left\{\frac{2}{x} + \frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int \frac{1-x^2}{x(1-2x)}dx = \int \left\{\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{1-2x}\right)\right\} dx$$

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)}\log|1 - 2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4}\log|1 - 2x| + C$$

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Q7 Integrate the rational function : $\frac{x}{\left(x^2+1\right)(x-1)}$

Answer.

$$\frac{x}{\left(x^2+1\right)(x-1)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{C}{(x-1)}$$

$$x = (Ax + B)(x - 1) + C\left(x^2 + 1\right)$$

$$x = Ax^2 - Ax + Bx - B + Cx^2 + C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}$, and $C = \frac{1}{2}$

From equation (1), we obtain

$$\therefore \frac{x}{\left(x^2+1\right)(x-1)} = \frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int \frac{x}{\left(x^2+1\right)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + \mathrm{C}$$

Consider $\int \frac{2x}{x^2+1}dx$, let $\left(x^2+1\right) = t \Rightarrow 2xdx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log\left|x^2+1\right|$$

$$\therefore \int \left(x^2+1\right)(x-1) = -\frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1 + \frac{1}{2}\tan^{-1}x| + C$$

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Q8 Integrate the rational function : $\frac{x}{(x-1)^2(x+2)}$

Answer.

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Substituting $x = 1$, we obtain
Substituting $x = 1$, we obtain

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain On solving, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{(x+2)}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

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Q9 Integrate the rational function : $\frac{3x+5}{x^3-x^2-x+1}$

Answer.

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$3x + 5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$3x + 5 = A\left(x^2-1\right) + B(x+1) + C\left(x^2+1-2x\right)$

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

$A = -\frac{1}{2}$ and $C = \frac{1}{2}$

$\therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$

$\Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int \frac{1}{x-1}dx + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x+1)}dx$

$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C$

$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + C$

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Q10 Integrate the rational function : $\frac{2x-3}{\left(x^2-1\right)(2x+3)}$

Answer.

$\frac{2x-3}{\left(x^2-1\right)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$

$Let \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$

$\Rightarrow (2x-3) = A\left(2x^2 + x - 3\right) + B\left(2x^2 + 5x + 3\right) + C\left(x^2 - 1\right)$

$\Rightarrow (2x-3) = (2A + 2B + C)x^2 + (A + 5B)x + (-3A + 3B - C)$

Equating the coefficients of x^2 and x, we obtain

$B = -\frac{1}{10}, A = \frac{5}{2}$, and $C = -\frac{24}{5}$

$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$

$\Rightarrow \int \frac{2x-3}{\left(x^2-1\right)(2x+3)}dx = \frac{5}{2}\int \frac{1}{(x+1)}dx - \frac{1}{10}\int \frac{1}{x-1}dx - \frac{24}{5}\int \frac{1}{(2x+3)}dx$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3|$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$

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Q11 Integrate the rational function : $\frac{5x}{(x+1)\left(x^2-4\right)}$

Answer.

$\frac{5x}{(x+1)\left(x^2-4\right)} = \frac{5x}{(x+1)(x+2)(x-2)}$

$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2). \ldots (1)$

Substituting $x = -1, -2,$ and 2 respectively in equation (1), we obtain

$A = \frac{5}{3}, B = -\frac{5}{2},$ and $C = \frac{5}{6}$

$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$

$\Rightarrow \int \frac{5x}{(x+1)\left(x^2-4\right)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$

$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$

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Q12 Integrate the rational function : $\frac{x^3+x+1}{x^2-1}$

Answer.

At can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $\left(x^3 + x + 1\right)$ by $x^2 - 1,$ we obtain

$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$

$\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$2x + 1 = A(x - 1) + B(x + 1)$

Substituting $x = 1$ and -1 in equation (1), we obtain

$A = \frac{1}{2}$ and $B = \frac{3}{2}$

$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$

$\Rightarrow \int \frac{x^3+x+1}{x^2-1} dx = \int x dx + \frac{1}{2}\int \frac{1}{(x+1)} dx + \frac{3}{2}\int \frac{1}{(x-1)} dx$

$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$

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Q13 Integrate the rational function : $\frac{2}{(1-x)\left(1+x^2\right)}$

Answer.

Let $\frac{2}{(1-x)\left(1+x^2\right)} = \frac{A}{(1-x)} + \frac{Bx+C}{\left(1+x^2\right)}$

$2 = A\left(1 + x^2\right) + (Bx + C)(1 - x)$

$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

Equating the coefficient of $x^2, x,$ and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1, B = 1,$ and $C = 1$

$\therefore \frac{2}{(1-x)\left(1+x^2\right)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$

$\Rightarrow \int \frac{2}{(1-x)\left(1+x^2\right)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$= -\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{2x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$= -\log|x-1| + \frac{1}{2}\log\left|1+x^2\right| + \tan^{-1}x + C$

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Q14 Integrate the rational function : $\frac{3x-1}{(x+2)^2}$

Answer.

Let $\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$

$\Rightarrow 3x - 1 = A(x+2) + B$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}dx = 3\int \frac{1}{(x+2)}dx - 7\int \frac{x}{(x+2)^2}dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{7}{(x+2)} + C$$

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Q15 Integrate the rational function : $\frac{1}{x^4-1}$

Answer.

$$\frac{1}{\left(x^4-1\right)} = \frac{1}{\left(x^2-1\right)\left(x^2+1\right)} = \frac{1}{(x+1)(x-1)\left(1+x^2\right)}$$

Let $\frac{1}{(x+1)(x-1)\left(1+x^2\right)} = \frac{A}{(x+1)^+} + \frac{B}{(x-1)} + \frac{Cx+D}{\left(x^2+1\right)}$

$1 = A(x-1)\left(x^2+1\right) + B(x+1)\left(x^2+1\right) + (Cx+D)\left(x^2-1\right)$

$1 = A\left(x^3 + x - x^2 - 1\right) + B\left(x^3 + x + x^2 + 1\right) + Cx^3 + Dx^2 - Cx - D$

$1 = (A+B+C)x^3 + (-A+B+D)x^2 + (A+B-C)x + (-A+B-D)$

Equating the coefficient of $x^3, x^2, x,$ and constant term, we obtain

$A + B + C = 0$

$-A + B + D = 0$

$A + B - C = 0$

$-A + B - D = 1$

On solving these equations, we obtain

$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0,$ and $D = -\frac{1}{2}$

$$\therefore \frac{1}{x^4-1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2\left(x^2+1\right)}$$

$$\Rightarrow \int \frac{1}{x^4-1}dx = -\frac{1}{4}\log|x-1| + \frac{1}{4}\log|x-1| - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$$

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Q16 Integrate the rational function :

$\frac{1}{x\left(x^n+1\right)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Answer.

$\frac{1}{x\left(x^n+1\right)}$

Multiplying numerator and denominator by x^{n-1}, we obtain

$\frac{1}{x\left(x^n+1\right)} = \frac{x^{n-1}}{x^{n-1}x\left(x^n+1\right)} = \frac{x^{n-1}}{x^n\left(x^n+1\right)}$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$\therefore \int \frac{1}{x\left(x^n+1\right)}dx = \int \frac{x^{n-1}}{x^n\left(x^n+1\right)}^{dx} = \frac{1}{n}\int \frac{1}{t(t+1)}dt$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$1 = A(1 + t) + Bt$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$

$\Rightarrow \int \frac{1}{x\left(x^n+1\right)}dx = \frac{1}{n}\int\left\{\frac{1}{t} - \frac{1}{(t+1)}\right\}dx$

$= \frac{1}{n}[\log|t| - \log|t+1|] + C$

$= -\frac{1}{n}\left[\log\left|x^n\right| - \log|x^n+1\right] + C$

$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

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Q17 Integrate the rational function :

$\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint : Put $\sin x = t$]

Answer.

$\frac{\cos x}{(1-\sin x)(2-\sin x)}$

Let $\sin x = t \Rightarrow \cos x dx = dt$

$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int \frac{dt}{(1-t)(2-t)}$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$1 = A(2 - t) + B(1 - t)$

Substituting $t = 2$ and then $t = 1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$

$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int\left\{\frac{1}{1-t} - \frac{1}{(2-t)}\right\}dt$

$= -\log|1-t| + \log|2-t| + C$

$= \log\left|\frac{2-t}{1-t}\right| + C$

$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$

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Q18 Integrate the rational function : $\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}$

Answer.

$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)} = 1 - \frac{\left(4x^2+10\right)}{\left(x^2+3\right)\left(x^2+4\right)}$

Let $\frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+3\right)} + \frac{Cx+D}{\left(x^2+4\right)}$

$4x^2 + 10 = (Ax + B)\left(x^2 + 4\right) + (Cx + D)\left(x^2 + 3\right)$

$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$

$4x^2 + 10 = (A + C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$

Equating the coefficients of $x^3, x^2, x,$ and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0, B = -2, C = 0,$ and $D = 6$

$\therefore \frac{4x^2+10}{\left(x^2+3\right)\left(x^2+4\right)} = \frac{-2}{\left(x^2+3\right)} + \frac{6}{\left(x^2+4\right)}$

$\frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)} = 1 - \left(\frac{-2}{\left(x^2+3\right)} + \frac{6}{\left(x^2+4\right)}\right)$

$\Rightarrow \int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)} dx = \int \left\{1 + \frac{2}{\left(x^2+3\right)} - \frac{6}{\left(x^2+4\right)}\right\} dx$

$= \int \left\{1 + \frac{2}{x^2 + (\sqrt{3})^2} - \frac{6}{x^2 + 2^2}\right\}$

$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$

$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$

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Q19 Integrate the rational function : $\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$

$$\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$$

Answer. Let $x^2 = t \Rightarrow 2xdx = dt$

$$\therefore \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \int \frac{dt}{(t+1)(t+3)} \quad \ldots(1)$$

Let $\frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$

$1 = A(t + 3) + B(t + 1)$

substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$A = \frac{1}{2}$ and $B = -\frac{1}{2}$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \int \{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\}dt$$

$$= \frac{1}{2}\log|(t + 1)| - \frac{1}{2}\log|t + 3| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

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Q20 Integrate the rational function : $\frac{1}{x\left(x^4-1\right)}$

Answer.

$$\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$$

Let $x^2 = t \Rightarrow 2xdx = dt$

$$\therefore \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \int \frac{dt}{(t+1)(t+3)}$$

Let $\frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$

$1 = A(t + 3) + B(t + 1)$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$A = \frac{1}{2}$ and $B = -\frac{1}{2}$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \int \left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\}dt$$

$$= \frac{1}{2}\log|(t + 1)| - \frac{1}{2}\log|t + 3| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

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Q21 Integrate the rational function : $\frac{1}{\left(e^x-1\right)}$ [Hint : Put $e^x = t$]

Answer.

$\frac{1}{x\left(x^4-1\right)}$

Multiplying numerator and denominator by x^3, we obtain

$\frac{1}{x\left(x^4-1\right)} = \frac{x^3}{x^4\left(x^4-1\right)}$

$\therefore \int \frac{1}{x\left(x^4-1\right)}dx = \int \frac{x^3}{x^4\left(x^4-1\right)}dx$

Let $x^4 = t \Rightarrow 4x^3dx = dt$

$\therefore \int \frac{1}{x\left(x^4-1\right)}dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$

$1 = A(t-1) + Bt. \ldots (1)$

Substituting $t = 0$ and 1 in (1), we obtain

$A = -1$ and $B = 1$

$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$

$\Rightarrow \int \frac{1}{x\left(x^4-1\right)}dx = \frac{1}{4}\int \left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$

$= \frac{1}{4}[-\log|t| + \log|t-1|] + C$

$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$

$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$

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Q22 Choose the correct answer : $\int \frac{xdx}{(x-1)(x-2)}$ equals

(A) $\log\left|\frac{(x-1)^2}{x-2}\right| + C$ (B) $\log\left|\frac{(x-2)^2}{x-1}\right| + C$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$ (D) $\log|(x-1)(x-2)| + C$

Answer.

$\frac{1}{\left(e^x-1\right)}$

Let $e^x = t \Rightarrow e^xdx = dt$

$\Rightarrow \int \frac{1}{e^x-1}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$1 = A(t-1) + Bt$

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)}dx = \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

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Q23 Choose the correct answer : $\int \frac{dx}{x\left(x^2+1\right)}$ equals

(A) $\log|x| - \frac{1}{2}\log\left(x^2+1\right) + C$ (B) $\log|x| + \frac{1}{2}\log\left(x^2+1\right) + C$

(C) $-\log|x| + \frac{1}{2}\log\left(x^2+1\right) + C$ (D) $\frac{1}{2}\log|x| + \log\left(x^2+1\right) + C$

Answer.

Let $\frac{1}{x\left(x^2+1\right)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = A\left(x^2+1\right) + (Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

A + B = 0

C = 0

A = 1

On solving these equations, we obtain

A = 1, B = −1, and C = 0

$\therefore \frac{1}{x\left(x^2+1\right)} = \frac{1}{x} + \frac{-x}{x^2+1}$

$\Rightarrow \int \frac{1}{x\left(x^2+1\right)}dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$

$= \log|x| - \frac{1}{2}\log\left|x^2+1\right| + C$

Hence, the correct Answer is A

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Q1 Integrate the function : $x\sin x$

Answer. Let $I = \int x\sin x dx$

Taking x as first function and sin x as second function and integrating by parts, we obtain

$I = x\int \sin x dx - \int \left\{ \left(\frac{d}{dx}x \right) \int \sin x dx \right\} dx$

$= x(-\cos x) - \int 1 \cdot (-\cos x) dx$

$= -x\cos x + \sin x + C$

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Q2 Integrate the function : $x\sin 3x$

Answer. Let $I = \int x\sin 3x dx$

Taking x as first function and sin 3x as second function and integrating by parts, we obtain

$$I = x\int \sin 3x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x dx \right\}$$

$$= x\left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

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Q3 Integrate the function :$x^2 e^x$

Answer. Let $I = \int x^2 e^x dx$
Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

$$= x^2 e^x - \int 2x \cdot e^x dx$$

$$= x^2 e^x - 2\int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2\left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right]$$

$$= x^2 e^x - 2\left[xe^x - \int e^x dx \right]$$

$$= x^2 e^x - 2\left[xe^x - e^x \right]$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x\left(x^2 - 2x + 2 \right) + C$$

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Q4 Integrate the function :$x\log x$

Answer. Let $I = \int x\log x dx$
Taking log x as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

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Q5 Integrate the function :$x\log 2x$

Answer. Let $I = \int x\log 2x dx$
Taking log 2x as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x dx - \int \left\{ \left(\frac{d}{dx} 2\log x \right) \int x dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

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Q6 Integrate the function :$x^2 \log x$

Answer. Let $I = \int x^2 \log x dx$
Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

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Q7 Integrate the function :$x \sin^{-1} x$

Answer. Let $I = \int x \sin^{-1} x dx$
Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + \mathrm{C}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} \left(2x^2 - 1 \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C$$

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Q8 Integrate the function :$x\tan^{-1}x$

Answer. Let $I = \int x\tan^{-1}x dx$
Taking $\tan^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1}x\int x dx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int x dx\right\}dx$$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \int\frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left(x - \tan^{-1}x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C$$

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Q9 Integrate the function :$x\cos^{-1}x$

Answer. Let $I = \int x\cos^{-1}x dx$
Taking as first function and x as second function and integrating by parts, we obtain

$$I = \cos^{-1}x\int x dx - \int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x dx\right\}dx$$

$$= \cos^{-1}x\frac{x^2}{2} - \int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\sqrt{1-x^2}dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1}x$$

where, $I_1 = \int\sqrt{1-x^2}dx$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{d}{dx}\sqrt{1-x^2}\int x dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{-2x}{2\sqrt{1-x^2}}\cdot x dx$$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\sqrt{1-x^2}dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1}x$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\sqrt{1-x^2}dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$

$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1}x$

where, $I_1 = \int\sqrt{1-x^2}dx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{d}{dx}\sqrt{1-x^2}\int xdx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{-2x}{2\sqrt{1-x^2}}\cdot xdx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{-x^2}{\sqrt{1-x^2}}dx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{1-x-1}{\sqrt{1-x^2}}dx$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{\int\sqrt{1-x^2}dx + \int\frac{-dx}{\sqrt{1-x^2}}\right\}$

$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{I_1 + \cos^{-1}x\right\}$

$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1}x$

$\therefore I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x$

Substituting in (1) , we obtain

$I = \frac{x\cos^{-1}x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x\right) - \frac{1}{2}\cos^{-1}x$

$= \frac{\left(2x^2-1\right)}{4}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + C$

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Q10 Integrate the function :$\left(\sin^{-1}x\right)^2$

Answer. Let $I = \int\left(\sin^{-1}x\right)^2\cdot 1dx$

Taking $\left(\sin^{-1}x\right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \left(\sin^{-1}x\right)\int 1dx - \int\left\{\frac{d}{dx}\left(\sin^{-1}x\right)^2 \cdot \int 1 \cdot dx\right\}dx$$

$$= \left(\sin^{-1}x\right)^2 \cdot x - \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot xdx$$

$$= x\left(\sin^{-1}x\right)^2 + \int \sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$

$$= x\left(\sin^{-1}x\right)^2 + \left[\sin^{-1}x\int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$= x\left(\sin^{-1}x\right)^2 + \left[\sin^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}dx\right]$$

$$= x\left(\sin^{-1}x\right)^2 + 2\sqrt{1-x^2}\sin^{-1}x - \int 2dx$$

$$= x\left(\sin^{-1}x\right)^2 + 2\sqrt{1-x^2}\sin^{-1}x - 2x + C$$

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Q11 Integrate the function : $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

Answer. Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

$$I = \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}xdx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1}x\int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$= \frac{-1}{2}\left[\cos^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + \int 2dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + 2x\right] + C$$

$$= -\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + C$$

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Q12 Integrate the function : $x\sec^2 x$

Answer. Let $I = \int x\sec^2 xdx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x\int \sec^2 x dx - \int \left\{ \left\{ \frac{d}{dx} x \right\} \int \sec^2 x dx \right\} dx$$

$$= x\tan x - \int 1 \cdot \tan x dx$$

$$= x\tan x + \log|\cos x| + C$$

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Q13 Integrate the function :$\tan^{-1} x$

Answer. Let $I = \int 1 \cdot \tan^{-1} x dx$
Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx$$

$$= x\tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x\tan^{-1} x - \frac{1}{2} \log \left| 1 + x^2 \right| + C$$

$$= x\tan^{-1} x - \frac{1}{2} \log \left(1 + x^2 \right) + C$$

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Q14 Integrate the function :$x(\log x)^2$

Answer. $I = \int x(\log x)^2 dx$
Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\log x)^2 \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right)^{-} \right\} \int x dx] dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[2\log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x dx$$

Again integrating by parts , we obtain

$$I = \frac{x^2}{2} (\log x)^2 - \left[\log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C$$

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Q15 Integrate the function :$\left(x^2 + 1 \right) \log x$

Answer. Let $I = \int \left(x^2 + 1 \right) \log x dx = \int x^2 \log x dx + \int \log x dx$

Let $I = I_1 + I_2 \ldots (1)$

Where $I_1 = \int x^2 \log x dx$ and $I_2 = \int \log x dx$

$I_1 = \int x^2 \log x dx$

Taking log x as first function and x^2 as second function and integrating by parts , we obtain

$$I_1 = \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right)$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1$$

$I_2 = \int \log x dx$

Taking log x as first function and 1 as second function and integrating by parts , we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2$$

Using equations (2) and (3) in (1) , we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + \left(C_1 + C_2 \right)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

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Q16 Integrate the function :$e^x(\sin x + \cos x)$

Answer. Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$f'(x) = \cos x$

$$I = \int e^x \left\{ f(x) + f'(x) \right\} dx$$

It is known that , $\int e^x \left\{ f(x) + f'(x) \right\} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

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Q17 Integrate the function : $\frac{xe^x}{(1+x)^2}$

Answer. Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$

Let $f(x) = \frac{1}{1+x}$ $f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ f(x) + f'(x) \right\} dx$

It is known that, $\int e^x \left\{ f(x) + f'(x) \right\} dx = e^x f(x) + C$

$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$

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Q18 Integrate the function :$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$

Answer.

$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$

$= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2})}{2\cos^2 \frac{x}{2}} \right)$

$= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2\cos^2 \frac{x}{2}}$

$= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$

$= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2$

$= \frac{1}{2} e^2 \left(1 + \tan \frac{x}{2} \right)^2$

$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2\tan \frac{x}{2} \right]$

$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2\tan \frac{x}{2} \right]$

$\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right]$

Let $\tan \frac{x}{2} = f(x)$ $f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

It is known that $\int e^x \left\{ f(x) + f'(x) \right\} dx = e^x f(x) + C$

From equation (1) , we obtain

$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$

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Q19 Integrate the function :$e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Answer. Let $I=\int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Also let $\frac{1}{x}=f(x)\quad f'(x)=\frac{-1}{x^2}$

It is known that, $\int e^x\left\{f(x)+f'(x)\right\}dx=e^xf(x)+C$

$\therefore I=\frac{e^x}{x}+C$

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Q20 Integrate the function : $\frac{(x-3)e^x}{(x-1)^3}$

Answer.

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

$f(x)=\frac{1}{(x-1)^2}\quad f'(x)=\frac{-2}{(x-1)^3}$

It is known that, $\int e^x\left\{f(x)+f'(x)\right\}dx=e^xf(x)+C$

$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx=\frac{e^x}{(x-1)^2}+C$

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Q21 Integrate the function :$e^{2x}\sin x$

Answer. Let $I=\int e^{2x}\sin x dx$ ...(1)
Integrating by parts , we obtain

$$I=\sin x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\cdot\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x}\cdot \sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\cdot \frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\cdot \sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I$$

$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot \sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$$

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Q22 Integrate the function :$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Answer. Let $x = \tan\theta \quad dx = \sec^2\theta d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta d\theta = 2\int\theta\cdot\sec^2\theta d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$$

$$= 2[\theta\cdot\tan\theta - \int\tan\theta d\theta]$$

$$= 2[\theta\tan\theta + \log|\cos\theta|] + C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{\frac{1}{2}} + C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$$

$$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$$

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Q23 Choose the correct answer : $\int x^2 e^{x^3}dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$

(B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3}$ + C

(D) $\frac{1}{3}e^{x^2}$ + C

Answer. Let $I = \int x^2 e^{x^3} dx$

Also , let $x^3 = t \quad 3x^2 dx = dt$

$\Rightarrow I = \frac{1}{3}\int e^{t} dt$

$= \frac{1}{3}\left(e^{t}\right)$ + C

$= \frac{1}{3}e^{x^3}$ + C

Hence , the correct answer is A .

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Q24 Choose the correct answer : $\int e^x \sec x(1 + \tan x)dx$ equals

(A) $e^x \cos x + C$

(B) $e^x \sec x + C$

(C) $e^x \sin x$ + C

(D) $e^x \tan x$ + C

Answer. $\int e^x \sec x(1 + \tan x)dx$

Let $I = \int e^x \sec x(1 + \tan x)dx = \int e^x(\sec x + \sec x \tan x)dx$

Also, let $\sec x = f(x) \quad \sec x \tan x = f'(x)$

It is known that , $\int e^x \left\{ f(x) + f'(x) \right\} dx = e^x f(x)$ + C

$\therefore I = e^x \sec x + C$

Hence, the correct Answer is B.

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Q1 Integrate the function :$\sqrt{4 - x^2}$

Answer.

Let $I = \int \sqrt{4 - x^2} dx = \int \sqrt{(2)^2 - (x)^2} dx$

It is known that, $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

$\therefore I = \frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1} \frac{x}{2} + C$

$= \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1} \frac{x}{2} + C$

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Q2 Integrate the function : $\sqrt{1 - 4x^2}$

Answer.

Let $I = \int \sqrt{1 - 4x^2} dx = \int \sqrt{(1)^2 - (2x)^2} dx$

Let $2x = t \Rightarrow 2dx = dt$

$\therefore I = \frac{1}{2}\int \sqrt{(1)^2 - (t)^2} dt$

It is known that, $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$ + C

$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1 - t^2} + \frac{1}{2}\sin^{-1} t \right]$ + C

$= \frac{t}{4}\sqrt{1-t^2} + \frac{1}{4}\sin^{-1}t + C$

$= \frac{2x}{4}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$

$= \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$

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Q3 Integrate the function : $\sqrt{x^2+4x+6}$

Answer.

Let $I = \int\sqrt{x^2+4x+6}dx$

$= \int\sqrt{x^2+4x+4+2}dx$

$= \int\sqrt{\left(x^2+4x+4\right)+2}dx$

$= \int\sqrt{(x+2)^2+(\sqrt{2})^2}dx$

It is known that,$\int\sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \frac{2}{2}\log(x+2) + \sqrt{x^2+4x+6}| + \text{C}$

$= \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log(x+2) + \sqrt{x^2+4x+6}| + \text{C}$

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Q4 Integrate the function :$\sqrt{x^2+4x+1}$

Answer.

Let $I = \int\sqrt{x^2+4x+1}dx$

$= \int\sqrt{\left(x^2+4x+4\right)-3}dx$

$= \int\sqrt{(x+2)^2-(\sqrt{3})^2}dx$

It is known that,$\int\sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + \text{C}$

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Q5 Integrate the function : $\sqrt{1-4x-x^2}$

Answer.

Let $I = \int\sqrt{1-4x-x^2}dx$

$= \int\sqrt{1-\left(x^2+4x+4-4\right)}dx$

$= \int\sqrt{1+4-(x+2)^2}dx$

$= \int\sqrt{(\sqrt{5})^2-(x+2)^2}dx$

It is a known that,$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x-5} - \frac{9}{2}\log(x+2) + \sqrt{x^2+4x-5}| + \text{C}$

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Q6 Integrate the function : $\sqrt{x^2 + 4x - 5}$

Answer.

Let $I = \int\sqrt{x^2 + 4x - 5}dx = \int\sqrt{\left(x^2 + 4x + 4\right) - 9}dx = \int\sqrt{(x + 2)^2 - (3)^2}dx$ It is known that, $\int\sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log(x + 2) + \sqrt{x^2 + 4x - 5}| + C$

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Q7 Integrate the function :$\sqrt{1 + 3x - x^2}$

Answer.

Let $I = \int\sqrt{1 + 3x - x^2}dx$

$= \int\sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}dx$

$= \int\sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2}dx$

$= \int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}dx$

It is known that, $\int\sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore I = \frac{3}{2}\sqrt{1 + 3x - x^2} + \frac{13}{4\times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$

$= \frac{2x-3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C$

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Q8 Integrate the function :8. $\sqrt{x^2 + 3x}$

Answer.

Let $I = \int\sqrt{x^2 + 3x}dx$

$= \int\sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}}dx$

$= \int\sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}dx$

It is known that, $\int\sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$\therefore I = \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x} - \frac{\frac{9}{4}}{2}\log\left(x+\frac{3}{2}\right) + \sqrt{x^2+3x}| + C$

$= \frac{(2x+3)}{4}\sqrt{x^2+3x} - \frac{9}{8}\log\left(x+\frac{3}{2}\right) + \sqrt{x^2+3x}| + C$

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Q9 Integrate the function : $\sqrt{1+\frac{x^2}{9}}$

Answer.

Let $I = \int\sqrt{1+\frac{x^2}{9}}dx = \frac{1}{3}\int\sqrt{9+x^2}dx = \frac{1}{3}\int\sqrt{(3)^2+x^2}dx$

It is known that, $\int\sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$\therefore I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9} + \frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right] + C$

$= \frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x+\sqrt{x^2+9}\right| + C$

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Q10 Choose the correct answer : $\int\sqrt{1+x^2}$ equals to

(A) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)| + C$

(B) $\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}} + C$ (C) $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}} + C$

(D) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right| + C$

Answer.

It is known that, $\int\sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$\therefore \int\sqrt{1+x^2}dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$

Hence, the correct Answer is A .

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Q11 Choose the correct answer : $\int\sqrt{x^2-8x+7}$ equal to

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7} + 9\log\left|x+4+\sqrt{x^2-8x+7}\right| + C$

(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

Answer.

Let $I = \int \sqrt{x^2 - 8x + 7}dx$

$= \int \sqrt{\left(x^2 - 8x + 16\right) - 9}dx$

$= \int \sqrt{(x-4)^2 - (3)^2}dx$

It is known that, $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$\therefore I = \frac{(x-4)}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|(x-4) + \sqrt{x^2 - 8x + 7}\right| + C$

Hence, the correct Answer is D.

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Q1 Evaluate the following definite integral as limit of sums : $\int_a^b x dx$

Answer.

It is known that,

$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a) + f(a+h) + \ldots + f(a+(n-1)h)]$, where $h = \frac{b-a}{n}$

Here, $a = a$, $b = b$, and $f(x) = x$

$\therefore \int_a^b x dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[a + (a+h)\ldots(a+2h), a + (n-1)h]$

$= (b-a)\lim_{n\to\infty}\frac{1}{n}[(a + a + a + \ldots + a) + (h + 2h + 3h + \ldots + (n-1)h)]$

$= (b-a)\lim_{n\to\infty}\frac{1}{n}[na + h(1 + 2 + 3 + \ldots + (n-1))]$

$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h\left\{\frac{(n-1)(n)}{2}\right\}\right]$

$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + \frac{n(n-1)h}{2}\right]$

$= (b-a)\lim_{n\to\infty}\frac{n}{n}\left[a + \frac{(n-1)h}{2}\right]$

$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)(b-a)}{2n}\right]$

$= (b-a)\lim_{n\to\infty}\left[a + \frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$

$= (b-a)\left[a + \frac{(b-a)}{2}\right]$

$= (b-a)\left[\frac{2a + b - a}{2}\right]$

$= \frac{(b-a)(b+a)}{2}$

$= \frac{1}{2}\left(b^2 - a^2\right)$

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Q2 Evaluate the following definite integral as limit of sums : $\int_0^5 (x+1)dx$

Answer.

Let $I = \int_0^5 (x + 1)dx$

It is known that,

$\int_a^b f(x)dx = (b - a)\lim_{n \to \infty} \frac{1}{n}[f(a) + f(a + h)\ldots f(a + (n - 1)h)]$, where $h = \frac{b-a}{n}$

Here, $a = 0$, $b = 5$, and $f(x) = (x + 1)$

$\Rightarrow h = \frac{5-0}{n} = \frac{5}{n}$

$\therefore \int_0^5 (x + 1)dx = (5 - 0)\lim_{n \to \infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + \ldots + f\left((n - 1)\frac{5}{n}\right)\right]$

$= 5\lim_{n \to \infty} \frac{1}{n}\left[1 + \left(\frac{5}{n} + 1\right) + \ldots\left\{1 + \left(\frac{5(n - 1)}{n}\right)\right\}\right]$

$= 5\lim_{n \to \infty} \frac{1}{n}\left[(1 + 1 + 1\ldots 1) + \left[\frac{5}{n} + 2 \cdot \frac{5}{n} + 3 \cdot \frac{5}{n} + \ldots(n - 1)\frac{5}{n}\right]\right]$

$= 5\lim_{n \to \infty} \frac{1}{n}\left[n + \frac{5}{n}\{1 + 2 + 3\ldots(n - 1)\}\right]$

$= 5\lim_{n \to \infty} \frac{1}{n}\left[n + \frac{5}{n} \cdot \frac{(n - 1)n}{2}\right]$

$= 5\lim_{n \to \infty} \frac{1}{n}\left[n + \frac{5(n - 1)}{2}\right]$

$= 5\lim_{n \to \infty}\left[1 + \frac{5}{2}\left(1 - \frac{1}{n}\right)\right]$

$= 5\left[1 + \frac{5}{2}\right]$

$= 5\left[\frac{7}{2}\right]$

$= \frac{35}{2}$

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Q3 Evaluate the following definite integral as limit of sums : $\int_2^3 x^2 dx$

Answer.

$\int_a^b f(x)dx = (b - a)\lim_{n \to \infty} \frac{1}{n}[f(a) + f(a + h) + f(a + 2h)\ldots f\{a + (n - 1)h\}]$, where $h = \frac{b-a}{n}$

Here, $a = 2$, $b = 3$, and $f(x) = x^2$

$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$

$\therefore \int_2^3 x^2 dx = (3 - 2)\lim_{n \to \infty} \frac{1}{n}\left[f(2) + f\left(2 + \frac{1}{n}\right) + f\left(2 + \frac{2}{n}\right)\ldots f\left\{2 + (n - 1)\frac{1}{n}\right\}\right]$

$= \lim_{n \to \infty} \frac{1}{n}\left[(2)^2 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots\left(2 + \frac{(n-1)}{n}\right)^2\right]$

$= \lim_{n \to \infty} \frac{1}{n}\left[2^2 + \left\{2^2 + \left(\frac{1}{n}\right)^2 + 2 \cdot 2 \cdot \frac{1}{n}\right\} + \ldots + \left\{(2)^2 + \frac{(n-1)^2}{n^2} + 2 \cdot 2 \cdot \frac{(n-1)}{n}\right\}\right]$

$$= \lim_{n \to \infty} \frac{1}{n}\left[\left(2^2 + \ldots + 2^2\right) + \left\{\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{n-1}{n}\right)^2\right\} + 2 \cdot 2 \cdot \left\{\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + \frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[4n + \frac{1}{n^2}\left\{1^2 + 2^2 + 3^2 \ldots + (n-1)^2\right\} + \frac{4}{n}\{1 + 2 + \ldots + (n-1)\}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[4n + \frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\} + \frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[4n + \frac{n\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right)}{6} + \frac{4n-4}{2}\right]$$

$$= \lim_{n \to \infty}\left[4 + \frac{1}{6}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right) + 2 - \frac{2}{n}\right]$$

$$= 4 + \frac{2}{6} + 2$$

$$= \frac{19}{3}$$

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Q4 Evaluate the following definite integral as limit of sums : $\int_1^4 \left(x^2 - x\right)dx$

Answer.

Let $I = \int_1^4 \left(x^2 - x\right)dx$

$= \int_1^4 x^2 dx - \int_1^4 x dx$

Let $I = I_1 - I_2$, where $I_1 = \int x^2 dx$ and $I_2 = \int x dx$

It is known that,

$\int_a^b f(x)dx = (b-a)\lim_{n \to \infty} \frac{1}{n}[f(a) + f(a+h) + f(a+(n-1)h)]$, where $h = \frac{b-a}{n}$

For $I_1 = \int_1^4 x^2 dx$

$a = 1, b = 4$, and $f(x) = x^2$

$\therefore h = \frac{4-1}{n} = \frac{3}{n}$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{n \to \infty} \frac{1}{n}[f(1) + f(1+h) + \ldots + f(1+(n-1)h)]$$

$$= 3\lim_{n \to \infty} \frac{1}{n}\left[1^2 + \left(1 + \frac{3}{n}\right)^2 + \left(1 + 2 \cdot \frac{3}{n}\right)^2 + \ldots\left(1 + \frac{(n-1)3}{n}\right)^2\right]$$

$$= 3\lim_{n \to \infty} \frac{1}{n}\left[1^2 + \left\{1^2 + \left(\frac{3}{n}\right)^2 + 2 \cdot \frac{3}{n}\right\} + \ldots + \left\{1^2 + \left(\frac{(n-1)3}{n}\right)^2 + \frac{2 \cdot (n-1) \cdot 3}{n}\right\}\right]$$

$$= 3\lim_{n \to \infty} \frac{1}{n}\left[\left(1^2 + \ldots + 1^2\right) + \left(\frac{3}{n}\right)^2\left\{1^2 + 2^2 + \ldots + (n-1)^2\right\} + 2 \cdot \frac{3}{n}\{1 + 2 + \ldots + (n-1)\}\right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[n + \frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\} + \frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[n + \frac{9n}{6}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right) + \frac{6n-6}{2}\right]$$

$$= 3 \lim_{n \to \infty}\left[1 + \frac{9}{6}\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right) + 3 - \frac{3}{n}\right]$$

$= 3[1 + 3 + 3]$

$= 3[7]$

$I_1 = 21$...(2)

For $I_2 = \int x dx$

$a = 1, b = 4,$ and $f(x) = x$

$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$

$\therefore I_2 = (4-1) \lim_{n \to \infty} \frac{1}{n}[f(1) + f(1+h) + \ldots f(a + (n-1)h)]$

$= 3 \lim_{n \to 1} \frac{1}{n}[1 + (1+h) + \ldots + (1 + (n-1)h)]$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[1 + \left(1 + \frac{3}{n}\right) + \ldots + \left\{1 + (n-1)\frac{3}{n}\right\}\right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[(1 + 1 + \ldots + 1) + \frac{3}{n}(1 + 2 + \ldots + (n-1))\right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[n + \frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$= 3 \lim_{n \to \infty} \frac{1}{n}\left[1 + \frac{3}{2}\left(1 - \frac{1}{n}\right)\right]$$

$$= 3\left[1 + \frac{3}{2}\right]$$

$$= 3\left[\frac{5}{2}\right]$$

$I_2 = \frac{15}{2}$

From equations (2) and (3), we obtain

$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$

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Q5 Evaluate the following definite integral as limit of sums : $\int_{-1}^{1} e^x dx$

Answer.

Let $I = \int_1 e^x dx$...(1)

It is known that,

$\int_a^b f(x)dx = (b-a) \lim_{n \to \infty} \frac{1}{n}[f(a) + f(a+h)\ldots f(a + (n-1)h)],$ where $h = \frac{b-a}{n}$

Here, $a = -1, b = 1,$ and $f(x) = e^x$

$\therefore h = \frac{1+1}{n} = \frac{2}{n}$

$$\therefore I = (1+1) \lim_{n \to \infty} \frac{1}{n}\left[f(-1) + f\left(-1 + \frac{2}{n}\right) + f\left(-1 + 2 \cdot \frac{2}{n}\right) + \ldots + f\left(-1 + \frac{(n-1)2}{n}\right)\right]$$

$\therefore I = (1+1)\lim_{n\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2\cdot\frac{2}{n}\right)+\ldots+f\left(-1+\frac{(n-1)2}{n}\right)\right]$

$= 2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}+e\left(-1\frac{2}{n}\right)+e\left(-1+2\frac{2}{n}\right)+\ldots e\left(-1+(n-1)\frac{2}{n}\right)\right]$

$= 2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$

$= 2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}}-1}{e^{\frac{2}{n}}-1}\right]$

$= e^{-1}\times 2\lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2-1}{e^{\frac{2}{n}}-1}\right]$

$= \frac{e^{-1}\times 2\left(e^2-1\right)}{\lim_{\frac{2}{n}\to 0}\left(\frac{e^{\frac{2}{n}}-1}{n}\right)\times 2}$

$= e^{-1}\left[\frac{2\left(e^2-1\right)}{2}\right] \qquad \left[\lim_{h\to 0}\left(\frac{e^A-1}{h}\right)=1\right]$

$= \frac{e^2-1}{c}$

$= \left(e-\frac{1}{e}\right)$

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Q6 Evaluate the following definite integral as limit of sums : $\int_0^4\left(x+e^{2x}\right)dx$

Answer.

It is known that,

$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a)+f(a+h)+\ldots+f(a+(n-1)h)],$ where $h = \frac{b-a}{n}$

Here, $a = 0, b = 4,$ and $f(x) = x + e^{2x}$

$\therefore h = \frac{4-0}{n} = \frac{4}{n}$

$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx = (4-0)\lim_{n\to\infty}\frac{1}{n}[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)]$

$= 4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^0\right)+\left(h+e^{2h}\right)+\left(2h+e^{22h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$

$= 4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)k}\right\}\right]$

$= 4\lim_{n\to\infty}\frac{1}{n}\left[\{h+2h+3h+\ldots+(n-1)h\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$

$= 4\lim_{n\to\infty}\frac{1}{n}\left[\frac{(h(n-1)n)}{2}+\left(\frac{e^{2hin}-1}{e^{2h}-1}\right)\right]$

$= 4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$

$= 4(2) + 4 \lim_{n \to \infty} \frac{\left(e^8 - 1\right)}{\frac{e^{\frac{8}{n}} - 1}{\frac{8}{n}}) 8}$

$= 8 + \frac{4 \cdot \left(e^8 - 1\right)}{8} \qquad \left(\lim_{x \to 0} \frac{e^x - 1}{x} = 1\right)$

$= 8 + \frac{e^8 - 1}{2}$

$= \frac{15 + e^8}{2}$

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Q1 Evaluate the definite integral : $\int_{-1}^{1}(x + 1)dx$

Answer.

Let $I = \int_{-1}^{4}(x + 1)dx$

$\int(x + 1)dx = \frac{x^2}{2} + x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(1) - \mathrm{F}(-1)$

$= \left(\frac{1}{2} + 1\right) - \left(\frac{1}{2} - 1\right)$

$= \frac{1}{2} + 1 - \frac{1}{2} + 1$

$= 2$

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Q2 Evaluate the definite integral : $\int_{x}^{1}\frac{1}{x}dx$

Answer.

Let $I = \int_{2}^{3}\frac{1}{x}dx$

$\int\frac{1}{x}dx = \log|x| = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(3) - \mathrm{F}(2)$

$= \log|3| - \log|2| = \log\frac{3}{2}$

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Q3 Evaluate the definite integral : $\int^{2}\left(4x^3 - 5x^2 + 6x + 9\right)dx$

Answer.

Let $I = \int\left(4x^3 - 5x^2 + 6x + 9\right)dx$

$$\int\left(4x^3 - 5x^2 + 6x + 9\right)dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = \mathrm{F}(x)$$

By second fundamental theorem of calculus , we obtain

$I = F(2) - F(1)$

$$I = \left\{2^4 - \frac{5 \cdot (2)^3}{3} + 3(2)^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3}$$

$$= \frac{64}{3}$$

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Q4 Evaluate the definite integral : $\int_0^x \sin 2x dx$

Answer.

Let $I = \int_0^\pi \sin 2x dx$

$$\int \sin 2x dx = \left(\frac{-\cos 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus , we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{1}{4}\right) - \cos 0\right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{-}{2}\right) - \cos 0\right]$$

$$= -\frac{1}{2}[0 - 1]$$

$$= \frac{1}{2}$$

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Q5 Evaluate the definite integral : $\int_{\frac{x}{2}} \cos 2x dx$

Answer.

Let $I = \int_{\frac{\pi}{\theta}} \cos 2x dx$

$$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)$

$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$

$= \frac{1}{2}[\sin\pi - \sin 0]$

$= \frac{1}{2}[0 - 0] = 0$

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Q6 Evaluate the definite integral : $\int_1^6 e^x dx$

Answer.

Let $I = \int_4^6 e^x dx$

$\int e^x dx = e^x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(5) - \mathrm{F}(4)$

$= e^5 - e^4$

$= e^4(e - 1)$

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Q7 Evaluate the definite integral : $\int_0^x \tan x dx$

Answer.

Let $I = \int_5^\pi \tan x dx$

$\int \tan x dx = -\log|\cos x| = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$

$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$

$= -\log\left|\frac{1}{\sqrt{2}|}\right| + \log|1|$

$= -\log(2)^{-\frac{1}{2}}$

$= \frac{1}{2}\log 2$

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Q8 Evaluate the definite integral : $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \mathrm{cosec} x dx$

Answer.

$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \mathrm{cosec} x dx$

$\int \csc x dx = \log|\csc x - \cot x| = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathbf{F}\left(\frac{\pi}{4}\right) - \mathrm{F}\left(\frac{\pi}{6}\right)$

$= \log\left|\csc\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\csc\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$

$= \log|\sqrt{2} - 1| - \log|2 - \sqrt{3}|$

$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$

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Q9 Evaluate the definite integral : $\int_0^4 \frac{dx}{\sqrt{1-x^2}}$

Answer.

Let $I = \int_0^4 \frac{dx}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$

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Q10 Evaluate the definite integral : $\int \frac{dx}{1+x^2}$

Answer.

Let $I = \int \frac{dx}{1+x^2}$

$\int \frac{dx}{1+x^2} = \tan^{-1}x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4}$

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Q11 Evaluate the definite integral : $\int_2^3 \frac{dx}{x^2-1}$

Answer.

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = F(3) - F(2)$

$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right.\right]$

$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\frac{1}{3}\right]$

$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$

$= \frac{1}{2}\left[\log\frac{3}{2}\right]$

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Q12 Evaluate the definite integral : $\int_{\theta}^{\frac{\pi}{\theta}} \cos^2 x dx$

Answer.

Let $I = \int_{\theta}^{\frac{\pi}{\theta}} \cos^2 x dx$

$\int \cos^2 x dx = \int \left(\frac{1+\cos 2x}{2}\right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus , we obtain

$I = \left[F\left(\frac{\pi}{2}\right) - F(0)\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= \frac{\pi}{4}$

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Q13 Evaluate the definite integral : $\int_{2}^{3} \frac{x dx}{x^2+1}$

Answer.

Let $I = \int_{2}^{3} \frac{x}{x^2+1} dx$

$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \frac{1}{2}\log\left(1 + x^2\right) = F(x)$

By second fundamental theorem of calculus , we obtain

$I = F(3) - F(2)$

$= \frac{1}{2}\left[\log\left(1 + (3)^2\right) - \log\left(1 + (2)^2\right)\right]$

$= \frac{1}{2}[\log(10) - \log(5)]$

$= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$

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Q14 Evaluate the definite integral : $\int \frac{2x+3}{5x^2+1} dx$

Answer.

$I = \int_0^4 \frac{2x+3}{5x^2+1} dx$

$$\int \frac{2x+3}{5x^2+1} dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1} dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1} dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1} dx + 3\int \frac{1}{5x^2+1} dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1} dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5} \cdot \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5}x)$$

$= F(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

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Q15 Evaluate the definite integral : $\int_0^1 xe^{x^2} dx$

Answer.

Let $I = \int_0 xe^{x^2} dx$

Put $x^2 = t \Rightarrow 2xdx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(1) - \mathrm{F}(0)$

$$= \frac{1}{2}e - \frac{1}{2}e^0$$

$$= \frac{1}{2}(e-1)$$

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Q16 Evaluate the definite integral : $\int \frac{5x^2}{x^2+4x+3}$

Answer. Let $I = \int_i^2 \frac{5x^2}{x^2+4x+3} dx$

Dividing $5x^2$ by $x^2 + 4x + 3$ we obtain

$$I = \int_1^2 \left\{ 5 - \frac{20x + 15}{x^2 + 4x + 3} \right\} dx$$

$$= \int_1^2 \left\{ 5dx - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \right.$$

$$= [5x]_1^2 - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$$

$I = 5 - I_1$, where $I = \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$

Consider $I_1 = \int_i^2 \frac{20x + 15}{x^2 + 4x + 8} dx$

Consider $I_1 = \int_1^2 \frac{20x + 15}{x^2 + 4x + 8} dx$

Let $20x + 15 = A\frac{d}{dx}\left(x^2 + 4x + 3\right) + B$

$= 2Ax + (4A + B)$

Equating the coefficients of x and constant term, we obtain

$A = 10$ and $B = -25$

$\Rightarrow I_1 = 10\int \frac{2x + 4}{x^2 + 4x + 3} dx - 25\int_1^2 \frac{dx}{x^2 + 4x + 3}$

Let $x^2 + 4x + 3 = t$

$\Rightarrow (2x + 4)dx = dt$

$\Rightarrow I_1 = 10\int \frac{dt}{t} - 25\int \frac{dx}{(x + 2)^2 - 1^2}$

$= 10\log t - 25\left[\frac{1}{2}\log\left(\frac{x + 2 - 1}{x + 2 + 1}\right)\right]$

$= \left[10\log\left(x^2 + 4x + 3\right)\right]_1^2 - 25\left[\frac{1}{2}\log\left(\frac{x + 1}{x + 3}\right)\right]_1^2$

$= [10\log 15 - 10\log 8] - 25\left[\frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\log\frac{2}{4}\right]$

$= [10\log(5 \times 3) - 10\log(4 \times 2)] - \frac{25}{2}[\log 3 - \log 5 - \log 2 + \log 4]$

$= [10\log 5 + 10\log 3 - 10\log 4 - 10\log 2] - \frac{25}{2}[\log 3 - \log 5 - \log 2 + \log 4]$

$= \left[10 + \frac{25}{2}\right]\log 5 + \left[-10 - \frac{25}{2}\right]\log 4 + \left[10 - \frac{25}{2}\right]\log 3 + \left[-10 + \frac{25}{2}\right]\log 2$

$= \frac{45}{2}\log 5 - \frac{45}{2}\log 4 - \frac{5}{2}\log 3 + \frac{5}{2}\log 2$

$= \frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$

Substituting the value of I1 in (1), we obtain

$I = 5 - \left[\frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}\right]$

$= 5 - \frac{5}{2}\left[9\log\frac{5}{4} - \log\frac{3}{2}\right]$

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Q17 Evaluate the definite integral : $\int_0^{\pi}\left(2\sec^2 x + x^3 + 2\right)dx$

Answer.

Let $I = \int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right) dx$

$\int \left(2\sec^2 x + x^3 + 2\right) dx = 2\tan x + \frac{x^4}{4} + 2x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}\left(\frac{\pi}{4}\right) - \mathrm{F}(0)$

$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - (2\tan 0 + 0 + 0)\right\}$

$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$

$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$

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Q18 Evaluate the definite integral : $\int_0^a \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$

Answer.

Let $I = \int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$

$= -\int_0^{\pi} \left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right) dx$

$= -\int_0^{\pi} \cos x dx$

$\int \cos x dx = \sin x = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = \mathrm{F}(\pi) - \mathrm{F}(0)$

$= \sin\pi - \sin 0$

$= 0$

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Q19 Evaluate the definite integral : $\int_0^2 \frac{6x+3}{x^2+4} dx$

Answer.

Let $I = \int_0^2 \frac{6x+3}{x^2+4} dx$

$\int \frac{6x+3}{x^2+4} dx = 3\int \frac{2x+1}{x^2+4} dx$

$= 3\int \frac{2x}{x^2+4} dx + 3\int \frac{1}{x^2+4} dx$

$= 3\log\left(x^2+4\right) + \frac{3}{2}\tan^{-1}\frac{x}{2} = \mathrm{F}(x)$

By second fundamental theorem of calculus , we obtain

$I = F(2) - F(0)$

$= \left\{3\log\left(2^2 + 4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0 + 4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$

$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$

$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$

$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$

$= 3\log 2 + \frac{3\pi}{8}$

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Q20 Evaluate the definite integral : $\int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

Answer. $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

$\int \left(xe^x + \sin\frac{\pi x}{4}\right)dx = x\int e^x dx - \int \left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx + \left\{\begin{matrix} -\cos\frac{\pi x}{4} \\ \\ \frac{\pi}{4} \end{matrix}\right\}$

$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$

$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$

$= F(x)$

By second fundamental theorem of calculus , we obtain

$I = F(1) - F(0)$

$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0\right)$

$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$

$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$

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Q21 Choose the correct answer : $\int^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{12}$

Answer. $\int \frac{dx}{1+x^2} = \tan^{-1}x = F(x)$

By second fundamental theorem of calculus , we obtain

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = F(\sqrt{3}) - F(1)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence, the correct Answer is D.

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Q22 Choose the correct answer : $\int_{0}^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$

(D) $\frac{\pi}{4}$

Answer.

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2 + (3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2 + t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$

$$= F(x)$$

By second fundamental theorem of calculus , we obtain

$$\int_{0}^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1}0$$

$$= \frac{1}{6}\tan^{-1}1 - 0$$

$$= \frac{1}{6}\times\frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

Hence the correct answer is C

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Q1 Evaluate the definite integral : $\int_{0}^{1} \frac{x}{x^2+1}dx$ using substitution

Answer. $\int_{0}^{1} \frac{x}{x^2+1}dx$

Let $x^2 + 1 = t \Rightarrow 2xdx = dt$

when $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$\therefore \int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$

$= \frac{1}{2}[\log|t]_1^2$

$= \frac{1}{2}[\log 2 - \log 1]$

$= \frac{1}{2}\log 2$

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Q2 Evaluate the definite integral : $\int_0^{\pi}\sqrt{\sin\phi}\cos^5\phi d\phi$ using substitution

Answer. Let $I = \int_{\theta}^{\pi}\sqrt{\sin\phi\cos^5\phi d\phi} = \int_b^{\pi}\sqrt{\sin\phi}\cos^4\phi\cos\phi d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$\therefore I = \int_0^1 \sqrt{t}\left(1 - t^2\right)^2 dt$

$= \int_0^1 t^{\frac{1}{2}}\left(1 + t^4 - 2t^2\right)dt$

$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$

$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$

$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$

$= \frac{154 + 42 - 132}{231}$

$= \frac{64}{231}$

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Q3 Evaluate the definite integral : $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ using substitution

Answer.

Let $I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Also, let $x = \tan\theta \quad dx = \sec^2\theta d\theta$

when $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{4}$

$I = \int_{\theta}^{\pi} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta d\theta$

$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta)\sec^2\theta d\theta$

$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta d\theta$

$= 2\int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta d\theta$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I = 2\left[\theta\int\theta\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{dx}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}}$$

$$I = 2\left[\theta\int\theta\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{dx}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]_0^{\frac{x}{4}}$$

$= 2\left[\theta\tan\theta - \int\tan\theta d\theta\right]_0^{\frac{\pi}{4}}$

$= 2[\theta\tan\theta + \log\cos\theta]_0^{\frac{\pi}{4}}$

$= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} + \log\cos\frac{\pi}{4}| - \log|\cos 0|\right]$

$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$

$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$

$= \frac{\pi}{2} - \log 2$

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Q4 Evaluate the definite integral : $\int_0^2 x\sqrt{x+2}$ (Put $x + 2 = t^2$) using substitution

Answer.

$\int_0^2 x\sqrt{x+2}dx$

Let $x + 2 = t^2 \quad dx = 2tdt$

when $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$\therefore \int_0^2 x\sqrt{x+2}dx = \int_{\sqrt{2}}^2 \left(t^2 - 2\right)\sqrt{t^2}2tdt$

$= 2\int_{\sqrt{2}}^2 \left(t^2 - 2\right)^2 dt$

$= 2\int_{\sqrt{2}}^2 \left(t^4 - 2t^2\right)dt$

$= 2\left[\frac{t^5}{5} - \frac{2t^3}{3}\right]_{\sqrt{2}}^2$

$$= 2\left[\frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16 + 8\sqrt{2}}{15}\right]$$

$$= \frac{16(2 + \sqrt{2})}{15}$$

$$= \frac{16(2 + \sqrt{2})}{15}$$

$$= \frac{16\sqrt{2}(\sqrt{2} + 1)}{15}$$

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Q5 Evaluate the definite integral : $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$ using substitution

Answer.

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$

Let $\cos x = t \quad -\sin x dx = dt$

when $x = 0$, $t = 1$ and when $x = \frac{\pi}{2}$, $t = 0$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx = -\int \frac{dt}{1 + t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

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Q6 Evaluate the definite integral : $\int_0^2 \frac{dx}{x+4-x^2}$ using substitution

Answer.

$$\int_0^2 \frac{dx}{x + 4 - x^2} = \int_0^2 \frac{dx}{-\left(x^2 - x - 4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2 - x + \frac{1}{4} - \frac{1}{4} - 4\right.}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2 - \frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}$$

Let $x - \frac{1}{2} = t \quad dx = dt$

When $x = 0, t = -\frac{1}{2}$ and when $x = 2, t = \frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{(5+\sqrt{17})(5+\sqrt{17})}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

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Q7 Evaluate the definite integral : $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$ using substitution

Answer.

$\int_{-1}^{1}\frac{dx}{x^2+2x+5}=\int_{-1}^{1}\frac{dx}{\left(x^2+2x+1\right)+4}=\int_{-1}^{2}\frac{dx}{(x+1)^2+(2)^2}$

Let x + 1 = t ⇒ dx = dt

When x = −1, t = 0 and when x = 1, t = 2

$\therefore \int_{-1}^{4}\frac{dx}{(x+1)^2+(2)^2}=\int_{3}^{2}\frac{dt}{t^2+2^2}$

$=\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$

$=\frac{1}{2}\tan^{-1}1-\frac{1}{2}\tan^{-1}0$

$=\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{8}$

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Q8 Evaluate the definite integral : $\int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$ using substitution

Answer.

$\int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$

Let $2x = t \quad 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$\therefore \int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx=\frac{1}{2}\int_2^1\left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$

$=\int_2^4\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$\Rightarrow \int_2^1\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt=\int_2^t e^t\left[f(t)+f'(t)\right]dt$

$=\left[e^t f(t)\right]_2^4$

$=\left[e^t\cdot\frac{2}{t}\right]_2^4$

$=\left[\frac{e^t}{t}\right]_2^4$

$=\frac{e^4}{4}-\frac{e^2}{2}$

$=\frac{e^2\left(e^2-2\right)}{4}$

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Q9 The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

(A) 6
(B) 0
(C) 3
(D) 4

Answer. Let $I = \int_{\frac{1}{3}}^{1} \left(x-x^3\right)^{\frac{1}{3}} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$\Rightarrow I = \int_{\lim^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta d\theta$

$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{x}{2}} \frac{(\sin\theta)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta d\theta$

$= \int_{\pi}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta d\theta$

$= \int_{\ln^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \sin^2\theta} \cos\theta d\theta$

$= \int_{\ln^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \csc^2\theta d\theta$

$= \int_{\ln^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \csc^2\theta d\theta$

$= \int_{\ln^{-1}\left(\frac{1}{3}\right)}^{\frac{x}{2}} (\cot\theta)^{\frac{5}{3}} \csc^2\theta d\theta$

$\cot\theta = t \quad -\csc 2\theta d\theta = dt$

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$\therefore I = -\int_{\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$

$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$

$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{9}$

$= -\frac{3}{8}\left[-(2\sqrt{2})^{\frac{8}{3}}\right]$

$= \frac{3}{8}\left[(\sqrt{8})^{\frac{8}{3}}\right]$

$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$

$= \frac{3}{8}[16]$

$= 3 \times 2$

$= 6$

Hence, the correct Answer is A.

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Q10 If $f(x) = \int_0^{\pi} t\sin t dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

Answer. $f(x) = \int_0^x t\sin t dt$

Integrating by parts, we obtain

$$f(x) = t\int_b^{\pi}\sin t dt - \int_0^r\left\{\left(\frac{d}{dt}t\right)\int\sin t dt\right\}dt$$

$$= [t(-\cos t)]_0^x - \int_0^x(-\cos t)dt$$

$$= [-t\cos t + \sin t]_0^x$$

$$= -x\cos x + \sin x$$

$$\Rightarrow f'(x) = -[\{x(-\sin x)\} + \cos x] + \cos x$$

$$= x\sin x - \cos x + \cos x$$

$$= x\sin x$$

Hence, the correct answer is B.

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Q1 By using the properties of definite integrals, evaluate the integral.

$\int_{\theta}^{\pi}\cos^2 x dx$

Answer.

$I = \int_{\theta}^{\pi}\cos^2 x dx$ .....(1)

$$\Rightarrow I = \int_{\theta}^{\pi}\cos^2\left(\frac{\pi}{2} - x\right)dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$\Rightarrow I = \int_{\theta}^{\pi}\sin^2 x dx$ .....(2)

Adding (1) and (2), we obtain

$$2I = \int_{\theta}^{\pi}\left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_{\theta}^{\pi} 1 dx$$

$$\Rightarrow 2I = [x]_{\theta}^{\pi}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

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Q2 By using the properties of definite integrals, evaluate the integral.

$$\int_{\theta}^{\pi}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Answer.

$$\int_{\theta}^{\pi}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Let $I = \int_{\theta}^{\pi}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ..... (1)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}}dx \qquad \ldots\ldots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

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Q3 By using the properties of definite integrals, evaluate the integral.

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}$$

Answer. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}dx \qquad \ldots\ldots\ldots (1)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}dx \qquad \ldots\ldots (2)$$

Adding (1) and (2), we obtain

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

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Q4 By using the properties of definite integrals, evaluate the integral.

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x+\cos^5 x}$$

Answer. *Let* $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx \qquad \ldots\ldots (1)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots\ldots (2)$

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx$

$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

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Q5 By using the properties of definite integrals, evaluate the integral.
$\int_{-5}^{5} |x + 2| dx$

Answer. $I = \int_{-5}^{5} |x + 2| dx$

It can be seen that (x+2) ≤ 0 on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$

$\therefore I = \int_{-5}^{-2} -(x + 2) dx + \int_{-2}^{5} (x + 2) dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$

$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$

$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$

$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$

$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$

$= 29$

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Q6 By using the properties of definite integrals, evaluate the integral.
$\int_2^8 |x - 5| dx$

Answer.

Let $I = \int_2^6 |x - 5| dx$

It can be seen that $(x - 5) \leq 0$ on $[2, 5]$ and $(x - 5) \geq 0$ on $[5, 8]$

$I = \int_2^5 -(x - 5) dx + \int_2^5 (x - 5) dx \qquad \left(\int_a^b f(x) = \int_a^t f(x) + \int_c^b f(x)\right)$

$= -\left[\frac{x^2}{2} - 5x\right]^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$

$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$

$= 9$

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Q7 By using the properties of definite integrals, evaluate the integral.
$\int_0^1 x(1 - x)^n dx$

Answer.

Let $I = \int_0^1 x(1-x)^n dx$

$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$

$= \int_0^1 (1-x)(x)^n dx$

$= \int_0^1 \left(x^n - x^{n+1}\right) dx$

$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 \qquad \left(\int_1^0 f(x)dx = \int_1^0 f(a-x)dx\right)$

$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$

$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$

$= \frac{1}{(n+1)(n+2)}$

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Q8 By using the properties of definite integrals, evaluate the integral.

$\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

Answer. $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$. . . . . (1)

$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I$

$\Rightarrow 2I = [x\log 2]_0^{\frac{\pi}{4}}$

$\Rightarrow 2I = \frac{\pi}{4}\log 2$

$\Rightarrow I = \frac{\pi}{8}\log 2$

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Q9 By using the properties of definite integrals, evaluate the integral.

$\int_0^2 x\sqrt{2-x}dx$

Answer.

Let $I = \int_0^2 x\sqrt{2-x}dx$

$I = \int_0^2 (2-x)\sqrt{x}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$= \int_0^2 \left\{ 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right\} dx$

$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} \right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \frac{16\sqrt{2}}{15}$

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Q10 By using the properties of definite integrals, evaluate the integral.

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2\log\sin x - \log\sin 2x) dx$

Answer.

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2\log\sin x - \log\sin 2x) dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \{2\log\sin x - \log(2\sin x\cos x)\} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \{2\log\sin x - \log\sin x - \log\cos x - \log 2\} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \{\log\sin x - \log\cos x - \log 2\} dx. \ldots . (1)$

It is known that, $\left(\int_0^a f(x) dx = \int_0^a f(a - x) dx \right)$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \{\log\cos x - \log\sin x - \log 2\} dx \quad \ldots (2)$

Adding (1) and (2), we obtain

$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (- \log 2 - \log 2) dx$

$\Rightarrow 2I = - 2\log 2 \int_0^{\pi} 1 dx$

$\Rightarrow I = - \log 2 \left[\frac{\pi}{2} \right]$

$\Rightarrow I = \frac{\pi}{2} \left[\log \frac{1}{2} \right]$

$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$

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Q11 By using the properties of definite integrals, evaluate the integral.

$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

Answer.

Let $I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos 2x) dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

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Q12 By using the properties of definite integrals, evaluate the integral.

$\int_0^{\pi} \frac{x dx}{1+\sin x}$

Answer. $I = \int_0^{\pi} \frac{x dx}{1+\sin x} \ldots\ldots (1)$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x} dx \ldots (2)$

Adding (1) and (2), we obtain

$2I = \int_0^{=} \frac{\pi}{1+\sin x} dx$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)} dx$

$\Rightarrow 2I = \pi \int_0^{=} \frac{1-\sin x}{\cos^2 x} dx$

$\Rightarrow 2I = \pi[\tan x - \sec x]_0^{\pi}$

$\Rightarrow 2I = \pi[2]$

$\Rightarrow I = \pi$

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Q13 By using the properties of definite integrals, evaluate the integral.

$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

Answer. Let $I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx \ldots (1)$

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$\therefore I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$

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Q14 By using the properties of definite integrals, evaluate the integral.

$\int_0^{2\pi} \cos^5 x dx$

Answer.

Let $I = \int_0^{2\pi} \cos^5 x dx$ …(1)

$\cos^5(2\pi - x) = \cos^5 x$

It is known that,

$\int_0^{2a} f(x)dx = 2\int_1^a f(x)dx$, if $f(2a - x) = f(x)$

$= 0$ if $f(2a - x) = -f(x)$

$\therefore I = 2\int_0^{\pi} \cos^5 x dx$

$\Rightarrow I = 2(0) = 0 \quad \left[\cos^5(\pi - x) = -\cos^5 x\right]$

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Q15 By using the properties of definite integrals, evaluate the integral.

$\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

Answer. Let I $= \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$. . . . (1)

$\Rightarrow I = \int_0^{\pi} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a - x)dx\right)$

$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$. . . . (2)

Adding (1) and (2), we obtain

$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{0}{1 + \sin x \cos x} dx$

$\Rightarrow I = 0$

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Q16 By using the properties of definite integrals, evaluate the integral.

$\int_0^{\pi} \log(1 + \cos x)dx$

Answer. $I = \int_0^{\pi} \log(1 + \cos x)dx$. . . . . (1)

$I = \int_0^{\pi} \log(1 + \cos x) \quad \left(\int_1^0 f(x)dx = \int_1^0 f(a - x)dx\right)$

$\Rightarrow I = \int_1^{\pi} \log(1 - \cos x)dx$. . . . (2)

Adding (1) and (2), we obtain

$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\}dx$

$\Rightarrow 2I = \int_0^{\pi} \log\left(1 - \cos^2 x\right)dx$

$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x dx$

$\Rightarrow 2I = 2\int_0^{\pi} \log \sin x dx$

$\Rightarrow I = \int_0^{\pi} \log \sin x dx$

$\sin(\pi - x) = \sin x$. . . . . (3)

$\therefore I = 2\int_{\frac{\pi}{\theta}}^{\pi} \log \sin x dx$. . . (4)

$\Rightarrow I = 2\int_0^x \log \sin\left(\frac{\pi}{2} - x\right)dx = 2\int_{\frac{\pi}{\theta}}^{\pi} \log \cos x dx$. . . . (5)

Adding (4) and (5), we obtain

$2I = 2\int_{\frac{\pi}{\theta}}^{\pi} (\log \sin x + \log \cos x)dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\log\sin x + \log\cos x + \log 2 - \log 2)dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\log 2\sin x\cos x - \log 2)dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log\sin 2x dx - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log 2 dx$

Let $2x = t \quad 2dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, \pi =$

$\therefore I = \frac{1}{2}\int_0^{\pi} \log\sin t dt - \frac{\log 2}{2}\log 2$

$\Rightarrow I = \frac{1}{2}I - \frac{1}{2}\log 2$

$\Rightarrow \frac{I}{2} = -\frac{\pi}{2}\log 2$

$\Rightarrow I = -\pi\log 2$

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Q17 By using the properties of definite integrals, evaluate the integral.
$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$

Answer. Let $I = \int_0^0 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$. . . . (1)

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$. . . . . (2)

Adding (1) and (2), we obtain

$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$

$\Rightarrow 2I = \int_0^a 1 dx$

$\Rightarrow 2I = [x]_0^a$

$\Rightarrow 2I = a$

$\Rightarrow I = \frac{a}{2}$

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Q18 By using the properties of definite integrals, evaluate the integral.
$\int_0^4 |x-1|dx$

Answer. $I = \int_0^4 |x-1|dx$

It can be seen that, $(x-1) \le 0$ when $0 \le x \le 1$ and $(x-1) \ge 0$ when $1 \le x \le 4$

$I = \int_0^4 |x-1|dx + \int^4 |x-1|dx \quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_t^b f(x)\right)$

$= \int_0^1 -(x-1)dx + \int_0^1 (x-1)dx$

$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$

$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$

$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$

$= 5$

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Q19 By using the properties of definite integrals, evaluate the integral.
Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, if f and g are defined as f(x) = f(a-x) and g(x) + g(a-x) = 4

Answer. $I = \int_0^a f(x)g(x)dx. \ldots (1)$

$\Rightarrow I = \int_1^0 f(a-x)g(a-x)dx \quad \left(\int_1^0 f(x)dx = \int_0^0 f(a-x)dx\right)$

$\Rightarrow I = \int_0^a f(x)g(a-x)dx. \ldots . (2)$

Adding (1) and (2), we obtain

$2I = \int_0^a \{f(x)g(x) + f(x)g(a-x)\}dx$

$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a-x)\}dx$

$\Rightarrow 2I = \int_0^\pi f(x) \times 4dx \quad [g(x) + g(a-x) = 4]$

$\Rightarrow I = 2\int_0^a f(x)dx$

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Q20 Choose the correct answer The value of

$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right)dx$is

(A) 0
(B) 2
(C) π
(D) 1

Answer.

Let $I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right)dx$

$\Rightarrow I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx$

It is known that if f(x) is an even function, then

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$I = 0 + 0 + 0 + 2\int_0^{\frac{\pi}{2}} 1 \cdot dx$

$= 2[x]_0^{\frac{\pi}{2}}$

$= \frac{2\pi}{2}$

$\pi =$

Hence, the correct Answer is C.

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Q21 Choose the correct answer The value of

$\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$is

(A) 2 (B) $\frac{3}{4}$ (C) 0 (D) −2

Answer. Let

$I = \int_0^2 \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$.....(1)

$$\Rightarrow I = \int_0^{\pi} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_{\frac{\pi}{\theta}}^{\pi} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \ldots\ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_{\frac{\pi}{\theta}}^{\pi} \left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right\}dx$$

$$\Rightarrow 2I = \int_{\frac{\pi}{\theta}}^{\pi} \log\left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow 2I = \int_{\frac{\pi}{\theta}}^{\pi} \log 1 dx$$

$$\Rightarrow 2I = \int_{\frac{\pi}{\theta}}^{\pi} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.

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Q1 Integrate the function

$$\frac{1}{x-x^3}$$

Answer.

$$\frac{1}{x-x^3} = \frac{1}{x\left(1-x^2\right)} = \frac{1}{x(1-x)(1+x)}$$

Let $\frac{1}{x(1-x)(1+x)} = \frac{A}{x} + \frac{B}{(1-x)} + \frac{C}{1+x}$

$$\Rightarrow 1 = A\left(1-x^2\right) + Bx(1+x) + Cx(1-x)$$

$$\Rightarrow 1 = A - Ax^2 + Bx + Bx^2 + Cx - Cx^2$$

Equating the coefficients of x^2 , x, and constant term, we obtain

$$-A + B - C = 0$$

$$B + C = 0$$

$$A = 1$$

On solving these equations, we obtain

$A = 1, B = \frac{1}{2}$, and $C = -\frac{1}{2}$

From equation (1), we obtain

$$\frac{1}{x(1-x)(1+x)} = \frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

$$\frac{1}{x(1-x)(1+x)} = \frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

$$\Rightarrow \int \frac{1}{x(1-x)(1+x)}dx = \int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{1-x}dx - \frac{1}{2}\int \frac{1}{1+x}dx$$

$$= \log|x| - \frac{1}{2}\log|(1-x)| - \frac{1}{2}\log|(1+x)|$$

$$= \log|x| - \log\left|(1-x)^{\frac{1}{2}}\right| - \log\left|(1+x)^{\frac{1}{2}}\right|$$

$$= \log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right| + C$$

$$= \log\left|\frac{x^2}{1-x^2}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right| + C$$

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Q2 Integrate the function

$\frac{1}{\sqrt{x+a}+\sqrt{(x+b)}}$

Answer.

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}} = \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$= \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}$$

$$= \frac{(\sqrt{x+a}-\sqrt{x+b})}{a-b}$$

$$\Rightarrow \int \frac{1}{\sqrt{x+a}-\sqrt{x+b}}dx = \frac{1}{a-b}\int(\sqrt{x+a}-\sqrt{x+b})dx$$

$$= \frac{1}{(a-b)}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + C$$

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Q3 Integrate the function

$\frac{1}{x\sqrt{ax-x^2}}$ [Hint: Put $x=\frac{a}{t}$]

Answer.

$\frac{1}{x\sqrt{ax-x^2}}$

Let $x = \frac{a}{t} \Rightarrow dx = -\frac{a}{t^2}dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax - x^2}}dx = \int \frac{1}{\frac{a}{t}\sqrt{a \cdot \frac{a}{t} - \left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}dt\right)$$

$$= -\int \frac{1}{at} \cdot \frac{1}{\sqrt{\frac{1}{t} - \frac{1}{t^2}}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t} - \frac{t^2}{t^2}}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$$

$$= -\frac{1}{a}[2\sqrt{t-1}] + C$$

$$= -\frac{1}{a}\left[2\sqrt{\frac{a}{x} - 1}\right] + C$$

$$= -\frac{2}{a}\left(\frac{\sqrt{a-x}}{\sqrt{x}}\right) + C$$

$$= -\frac{2}{a}\left(\sqrt{\frac{a-x}{x}}\right) + C$$

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Q4 Integrate the function

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Answer.

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2 \cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}} = \frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2 \cdot x^{-3}}$$

$$= \frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5 \cdot \left(x^4\right)^{\frac{3}{4}}}$$

$$= \frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{\frac{3}{4}}$$

$$= \frac{1}{x^5}\left(1 + \frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}dx = dt \Rightarrow \frac{1}{x^5}dx = -\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx = \int \frac{1}{x^5}\left(1 + \frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$= -\frac{1}{4}\int (1+t)^{\frac{3}{4}} dt$

$= -\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right] + C$

$= -\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}} + C$

$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}} + C$

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Q5 Integrate the function $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ $\left[\text{Hint: } \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)},\ \text{put } x = t^6\right]$

Answer.

$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$\therefore \int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} dx = \int \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)} dx$

$= \int \frac{6t^5}{t^2(1+t)} dt$

$= 6\int \frac{t^3}{(1+t)} dt$

On dividing, we obtain

$\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} dx = 6\int \left\{\left(t^2 - t + 1\right) - \frac{1}{1+t}\right\} dt$

$= 6\left[\left(\frac{t^3}{3}\right) - \left(\frac{t^2}{2}\right) + t - \log|1+t|\right]$

$= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(1+x^{\frac{1}{6}}\right) + C$

$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(1+x^{\frac{1}{6}}\right) + C$

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Q6 Integrate the function $\frac{5x}{(x+1)\left(x^2+9\right)}$

Answer.

Let $\frac{5x}{(x+1)\left(x^2+9\right)} = \frac{A}{(x+1)} + \frac{Bx+C}{\left(x^2+9\right)}$.... (1)

$\Rightarrow 5x = A\left(x^2+9\right) + (Bx+C)(x+1)$

$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}$, and $C = \frac{9}{2}$

From equation (1), we obtain

$$\frac{5x}{(x+1)\left(x^2+9\right)} = \frac{-1}{2(x+1)} + \frac{\frac{x}{2}+\frac{9}{2}}{\left(x^2+9\right)}$$

$$\int \frac{5x}{(x+1)\left(x^2+9\right)}dx = \int \left\{ \frac{-1}{2(x+1)} + \frac{(x+9)}{2\left(x^2+9\right)} \right\} dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{2}\int \frac{x}{x^2+9}dx + \frac{9}{2}\int \frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\int \frac{2x}{x^2+9}dx + \frac{9}{2}\int \frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log\left|x^2+9\right| + \frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log\left(x^2+9\right) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$$

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Q7 Integrate the function $\frac{\sin x}{\sin(x-a)}$

Answer.

$\frac{\sin x}{\sin(x-a)}$

Let $x - a = t \quad dx = dt$

$$\int \frac{\sin x}{\sin(x-a)}dx = \int \frac{\sin(t+a)}{\sin t}dt$$

$$= \int \frac{\sin t\cos a + \cos t\sin a}{\sin t}dt$$

$$= \int (\cos a + \cot t\sin a)dt$$

$$= t\cos a + \sin a\log|\sin t| + C_1$$

$$= (x-a)\cos a + \sin a\log|\sin(x-a)| + C_1$$

$$= x\cos a + \sin a\log|\sin(x-a)| - a\cos a + C_1$$

$$= \sin a\log|\sin(x-a)| + x\cos a + C$$

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Q8 Integrate the function : $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

Answer.

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}=\frac{e^{4\log x}\left(e^{\log x}-1\right)}{e^{2\log x}\left(e^{\log x}-1\right)}$$

$$=e^{2\log x}$$

$$=e^{\log x^2}$$

$$=x^2$$

$$\therefore \int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int x^2dx=\frac{x^3}{3}+C$$

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Q9 Integrate the function $\frac{\cos x}{\sqrt{4-\sin^2 x}}$

Answer.

$$\frac{\cos x}{\sqrt{4-\sin^2 x}}$$

Let $\sin x=t \quad \cos x dx=dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{4-\sin^2 x}x}dx=\int \frac{dt}{\sqrt{(2)^2-(t)^2}}$$

$$=\sin^{-1}\left(\frac{t}{2}\right)+\mathrm{C}$$

$$=\sin^{-1}\left(\frac{\sin x}{2}\right)+\mathrm{C}$$

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Q10 Integrate the function : $\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}$

Answer.

$$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^4 x-\cos^4 x\right)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{\left(\sin^2 x-\sin^2 x\cos^2 x\right)+\left(\cos^2 x-\sin^2 x\cos^2 x\right)}$$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x-\cos^2 x\right)}{\sin^2 x\left(1-\cos^2 x\right)+\cos^2 x\left(1-\sin^2 x\right)}$$

$$=\frac{-\left(\sin^4 x+\cos^4 x\right)\left(\cos^2 x-\sin^2 x\right)}{\left(\sin^4 x+\cos^4 x\right)}$$

$$=-\cos 2x$$

$$\therefore \int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=\int -\cos 2x dx=-\frac{\sin 2x}{2}+C$$

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Q11 Integrate the function $\frac{1}{\cos(x+a)\cos(x+b)}$

Answer.

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by $\sin(a-b)$, we obtain

$$\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cdot\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)}-\frac{\sin(x+b)}{\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}[\tan(x+a)-\tan(x+b)]$$

$$\int\frac{1}{\cos(x+a)\cos(x+b)}dx=\frac{1}{\sin(a-b)}\int[\tan(x+a)-\tan(x+b)]dx$$

$$=\frac{1}{\sin(a-b)}[-\log|\cos(x+a)|+\log|\cos(x+b)|]+C$$

$$=\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right|+C$$

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Q12 Integrate the function $\frac{x^3}{\sqrt{1-x^8}}$

Answer.

$$\frac{x^3}{\sqrt{1-x^8}}$$

Let $x^4=t \quad 4x^3dx=dt$

$$\Rightarrow\int\frac{x^3}{\sqrt{1-x^8}}dx=\frac{1}{4}\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\frac{1}{4}\sin^{-1}t+C$$

$$=\frac{1}{4}\sin^{-1}\left(x^4\right)+C$$

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Q13 Integrate the function $\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

Answer.

$$\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$$

Let $e^x=t \quad e^xdx=dt$

$$\Rightarrow\int\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}dx=\int\frac{dt}{(t+1)(t+2)}$$

$= \int \frac{1}{(t+1)} - \frac{1}{(t+2)}]dt$

$= \log|t+1| - \log|t+2| + C$

$= \log\left|\frac{t+1}{t+2}\right| + C$

$= \log\left|\frac{1+e^x}{2+e^x}\right| + C$

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Q14 Integrate the function $\frac{1}{\left(x^2+1\right)\left(x^2+4\right)}$

Answer.

$\therefore \frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{\left(x^2+4\right)}$

$\Rightarrow 1 = (Ax+B)\left(x^2+4\right) + (Cx+D)\left(x^2+1\right)$

$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$

Equating the coefficients of $x^3, x^2, x,$ and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$A = 0, B = \frac{1}{3}, C = 0,$ and $D = -\frac{1}{3}$

From equation (1), we obtain

$\frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{1}{3\left(x^2+1\right)} - \frac{1}{3\left(x^2+4\right)}$

$\int \frac{1}{\left(x^2+1\right)\left(x^2+4\right)}dx = \frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$

$= \frac{1}{3}\tan^{-1}x - \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$

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Q15 Integrate the function $\cos^3 x e^{\log \sin x}$

Answer.

$\cos^3 x e^{\log \sin x} = \cos^3 x \times \sin x$

Let $\cos x = t \quad -\sin x dx = dt$

$\Rightarrow \int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \sin x dx$

$= -\int t \cdot dt$

$= -\frac{t^4}{4} + C$

$= -\frac{\cos^4 x}{4} + C$

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Q16 Integrate the function $e^{3\log x}\left(x^4+1\right)^{-1}$

Answer.

$e^{3\log x}\left(x^4+1\right)^{-1} = e^{\log x^3}\left(x^4+1\right)^{-1} = \frac{x^3}{\left(x^4+1\right)}$

Let $x^4+1 = t \Rightarrow 4x^3dx = dt$

$\Rightarrow \int e^{3\log x}\left(x^4+1\right)^{-1}dx = \int \frac{x^3}{\left(x^4+1\right)}dx$

$= \frac{1}{4}\int \frac{dt}{t}$

$= \frac{1}{4}\log|t| + C$

$= \frac{1}{4}\log\left|x^4+1\right| + C$

$= \frac{1}{4}\log\left(x^4+1\right) + C$

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Q17 Integrate the function : $f'(ax+b)[f(ax+b)]^n$

Answer.

$f'(ax+b)[f(ax+b)]^n$

Let $f(ax+b) = t \Rightarrow af'(ax+b)dx = dt$

$\Rightarrow \int f'(ax+b)[f(ax+b)]^n dx = \frac{1}{a}\int t^n dt$

$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$

$= \frac{1}{a(n+1)}(f(ax+b))^{n+1} + C$

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Q18 Integrate the function $\frac{1}{\sqrt{\sin^3 x\sin\ (x+\alpha)}}$

Answer.

$\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}} = \frac{1}{\sqrt{\sin^3 x(\sin x\cos\alpha + \cos x\sin\alpha)}}$

$= \frac{1}{\sqrt{\sin^4 x\cos\alpha + \sin^3 x\cos x\sin\alpha}}$

$= \frac{1}{\sin^2 x\sqrt{\cos\alpha + \cot x\sin\alpha}}$

$= \frac{\csc^2 x}{\sqrt{\cos\alpha + \cot x\sin\alpha}}$

Let $\cos\alpha + \cot x\sin\alpha = t \Rightarrow -\csc^2 x\sin\alpha dx = dt$

$$\therefore \int \frac{1}{\sin^3 x\sin(x+\alpha)}dx = \int \frac{\csc^2 x}{\sqrt{\cos\alpha + \cot x\sin\alpha\alpha}}dx$$

$$= \frac{-1}{\sin\alpha}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{-1}{\sin\alpha}[2\sqrt{t}] + C$$

$$= \frac{-1}{\sin\alpha}[2\sqrt{\cos\alpha + \cot x\sin\alpha}] + C$$

$$= \frac{-2}{\sin\alpha}\sqrt{\cos\alpha + \frac{\cos x\sin\alpha}{\sin x}} + C$$

$$= \frac{-2}{\sin\alpha}\sqrt{\frac{\sin x\cos\alpha + \cos x\sin\alpha}{\sin x}} + C$$

$$= -\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$$

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Q19 Integrate the function $\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}, x \in [0, 1]$

Answer.

Let $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}dx$

It is known that, $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$

$$\Rightarrow I = \int \frac{\left(\frac{\pi}{2} - \cos^{-1}\sqrt{x}\right) - \cos^{-1}\sqrt{x}}{\frac{\pi}{2}}dx$$

$$= \frac{2\pi}{\pi}]\left(\frac{-2\cos^{-1}\sqrt{x}}{2}\sqrt{x}\right)dx$$

$$= \frac{2\pi}{\pi}\cdot\frac{}{2}\int 1\cdot dx - \frac{4}{\pi}\int \cos^{-1}\sqrt{x}dx$$

$$= x - \frac{4}{\pi}\int \cos^{-1}\sqrt{x}dx \ldots.(1)$$

Let $I_1 = \int \cos^{-1}\sqrt{x}dx$

Also, $\text{lct}\sqrt{x} = t \Rightarrow dx = 2tdt$

$$\Rightarrow I_1 = 2\int \cos^{-1}t\cdot tdt$$

$$= 2\left[\cos^{-1}t\cdot\frac{t^2}{2} - \int \frac{-1}{\sqrt{1-t^2}}\cdot\frac{t^2}{2}dt\right]$$

$$= t^2\cos^{-1}t + \int \frac{t^2}{\sqrt{1-t^2}}dt$$

$$= t^2\cos^{-1}t - \int \frac{1-t^2-1}{\sqrt{1-t^2}}dt$$

$$= t^2\cos^{-1}t - \int \sqrt{1-t^2}dt + \int \frac{1}{\sqrt{1-t^2}}dt$$

$= t^2\cos^{-1}t - \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}\sin^{-1}t + \sin^{-1}t$

$= t^2\cos^{-1}t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t$

From equation (1), we obtain

$I = x - \frac{4}{\pi}\left[t^2\cos t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right]$

$= x - \frac{4}{\pi}\left[x\cos^{-1}\sqrt{x} - \frac{\sqrt{x}}{2}\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{x}\right]$

$= x - \frac{4}{\pi}\left[x\left(\frac{-}{2} - \sin^{-1}\sqrt{x}\right) - \frac{\sqrt{x-\lambda^2}}{2} + \frac{-}{2}\sin^{-1}\sqrt{x}\right]$

$= x - 2x + \frac{4x}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - \frac{2}{\pi}\sin^{-1}\sqrt{x}$

$= -x + \frac{2}{\pi}\left[(2x-1)\sin^{-1}\sqrt{x}\right] + \frac{2}{\pi}\sqrt{x-x^2} + C$

$= \frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - x + C$

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Q20 Integrate the function $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

Answer.

$I = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$

Let $x = \cos^2\theta \Rightarrow dx = -2\sin\theta\cos\theta d\theta$

$I = \int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-2\sin\theta\cos\theta)d\theta$

$= -\int\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin 2\theta d\theta$

$= -\int\tan\frac{\theta}{2}\cdot 2\sin\theta\cos\theta d\theta$

$= -2\int\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta d\theta$

$= -4\int\sin^2\frac{\theta}{2}\cos\theta d\theta$

$= -4\int\sin^2\frac{\theta}{2}\cdot\left(2\cos^2\frac{\theta}{2} - 1\right)d\theta$

$= -4\int\left(2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)d\theta$

$= -8\int\sin^2\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}d\theta + 4\int\sin^2\frac{\theta}{2}d\theta$

$= -2\int\sin^2\theta d\theta + 4\int\sin^2\frac{\theta}{2}d\theta$

$= -2\int\frac{1-\cos 2\theta}{2})d\theta + 4\int\frac{1-\cos\theta}{2}d\theta$

$= -2\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right] + 4\left[\frac{\theta}{2} - \frac{\sin\theta}{2}\right] + C$

$= -\theta + \frac{\sin 2\theta}{2} + 2\theta - 2\sin\theta + C$

$= \theta + \frac{\sin 2\theta}{2} - 2\sin\theta + C$

$= \theta + \sqrt{1 - \cos^2\theta} \cdot \cos\theta - 2\sqrt{1 - \cos^2\theta} + C$

$= \cos^{-1}\sqrt{x} + \sqrt{1 - x} \cdot \sqrt{x} - 2\sqrt{1 - x} + C$

$= -2\sqrt{1 - x} + \cos^{-1}\sqrt{x} + \sqrt{x(1 - x)} + C$

$= -2\sqrt{1 - x} + \cos^{-1}\sqrt{x} + \sqrt{x - x^2} + C$

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Q21 Integrate the function $\frac{2 + \sin 2x}{1 + \cos 2x} e^x$

Answer.

$I = \int \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) e^x$

$= \int \left(\frac{2 + 2\sin x \cos x}{2\cos^2 x}\right) e^x$

$= \int \left(\frac{1 + \sin x \cos x}{\cos^2 x}\right) e^x$

$= \int \left(\sec^2 x + \tan x\right) e^x$

Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$\therefore I = \int \left(f(x) + f'(x)\right] e^x dx$

$= e^x f(x) + C$

$= e^x \tan x + C$

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Q22 Integrate the function $\frac{x^2 + x + 1}{(x + 1)^2 (x + 2)}$

Answer.

Let $\frac{x^2 + x + 1}{(x + 1)^2 (x + 2)} = \frac{A}{(x + 1)} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 2)} \ldots (1)$

$\Rightarrow x^2 + x + 1 = A(x + 1)(x + 2) + B(x + 2) + C\left(x^2 + 2x + 1\right)$

$\Rightarrow x^2 + x + 1 = A\left(x^2 + 3x + 2\right) + B(x + 2) + C\left(x^2 + 2x + 1\right)$

$\Rightarrow x^2 + x + 1 = (A + C)x^2 + (3A + B + 2C)x + (2A + 2B + C)$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 1$

$3A + B + 2C = 1$

$2A + 2B + C = 1$

On solving these equations, we obtain

$A = -2, B = 1,$ and $C = 3$

From equation (1), we obtain

$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{(x+1)} + \frac{3}{(x+2)} + \frac{1}{(x+1)^2}$$

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = -2\int \frac{1}{x+1}dx + 3\int \frac{1}{(x+2)}dx + \int \frac{1}{(x+1)^2}dx$$

$$= -2\log|x+1| + 3\log|x+2| - \frac{1}{(x+1)} + C$$

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Q23 Integrate the function $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Answer.

$$I = \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

Let $x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta d\theta)$$

$$= -\int \tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin\theta d\theta$$

$$= -\int \tan^{-1}\tan\frac{\theta}{2}\cdot\sin\theta d\theta$$

$$= -\frac{1}{2}\int \theta\cdot\sin\theta d\theta$$

$$= -\frac{1}{2}[\theta\cdot(-\cos\theta) - \int 1\cdot(-\cos\theta)d\theta]$$

$$= +\frac{1}{2}\theta\cos\theta - \frac{1}{2}\sin\theta$$

$$= \frac{1}{2}\cos^{-1}x\cdot x - \frac{1}{2}\sqrt{1-x^2} + C$$

$$= \frac{x}{2}\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C$$

$$= \frac{1}{2}\left(x\cos^{-1}x - \sqrt{1-x^2}\right) + C$$

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Q24 Integrate the function $\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right) - 2\log x\right]}{x^4}$

Answer.

$$\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right) - 2\log x\right]}{x^4} = \frac{\sqrt{x^2+1}}{x^4}\left[\log\left(x^2+1\right) - \log x^2\right]$$

$$= \frac{\sqrt{x^2+1}}{x^4}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$

$$= \frac{\sqrt{x^2+1}}{x^4}\log\left(1+\frac{1}{x^2}\right)$$

$$= \frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

$$= \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

Let $1+\frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3}dx = dt$

$$\therefore I = \int \frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)dx$$

$$= -\frac{1}{2}\int \sqrt{t}\log t dt$$

$$= -\frac{1}{2}\int t^{\frac{1}{2}}\cdot \log t dt$$

Integrating by parts, we obtain

$$I = -\frac{1}{2}\left[\log t\cdot\int t^{\frac{1}{2}}dt - \left\{\left(\frac{d}{dt}\log t\right)\int t^{\frac{1}{2}}dt\right\}dt\right]$$

$$= -\frac{1}{2}\left[\log t\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int_t^1\cdot\frac{t^{\frac{3}{2}}}{2}dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t - \frac{2}{3}\int t^{\frac{1}{2}}dt\right]$$

$$= -\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t - \frac{4}{9}t^{\frac{3}{2}}\right]$$

$$= -\frac{1}{3}t^{\frac{3}{2}}\log t + \frac{2}{9}t^{\frac{3}{2}}$$

$$= -\frac{1}{3}t^{\frac{3}{2}}\left[\log t - \frac{2}{3}\right]$$

$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$$

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Q25 Evaluate the definite integral $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

Answer.

$$I = \int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} \right) dx$$

$$= \int_{\frac{x}{2}}^{\pi} e^x \left(\frac{\csc^2\frac{x}{2}}{2} - \cot\frac{x}{2} \right) dx$$

Let $f(x) = -\cot\frac{x}{2}$

$\Rightarrow f'(x) = -\left(-\frac{1}{2}\csc^2\frac{x}{2} \right) = \frac{1}{2}\csc^2\frac{x}{2}$

$\therefore I = \int_{\frac{\pi}{2}}^{\pi} e^x \left(f(x) + f'(x) \right] dx$

$= \left[e^x \cdot f(x) dx \right]_{\frac{\pi}{2}}^{\pi}$

$= -\left[e^x \cdot \cot\frac{x}{2} \right]_{\frac{\pi}{2}}^{x}$

$= -\left[e^x \times \cot\frac{\pi}{2} - e^{\frac{\pi}{2}} \times \cot\frac{\pi}{4} \right]$

$= -\left[e^{\pi} \times 0 - e^{\frac{\pi}{2}} \times 1 \right]$

$= e^{\frac{\pi}{2}}$

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Q26 Evaluate the definite integral $\int_{0}^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Answer.

$I = \int_{0}^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

$\Rightarrow I = \int_{0}^{\frac{\pi}{4}} \frac{\frac{(\sin x \cos x)}{\cos^4 x}}{\frac{\left(\cos^4 x + \sin^4 x\right)}{\cos^4 x}} dx$

$\Rightarrow I = \int_{0}^{\frac{x}{4}} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx$

Let $\tan^2 x = t \Rightarrow 2\tan x \sec^2 x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{4}, t = 1$

$$\therefore I = \frac{1}{2}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\left[\tan^{-1}t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}1 - \tan^{-1}0\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{8}$$

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Q27 Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4\sin^2 x}$

Answer.

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\left(1 - \cos^2 x\right)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 - 4\cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3}\int_0^{\frac{x}{2}} \frac{4 - 3\cos^2 x - 4}{4 - 3\cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3}\int_0^{\frac{\pi}{2}} \frac{4 - 3\cos^2 x}{4 - 3\cos^2 x} dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4}{4 - 3\cos^2 x} dx$$

$$\Rightarrow I = -\frac{\pi}{6} + \frac{2}{3}\int_0^{\pi} \frac{2\sec^2 x}{1 + 4\tan^2 x} dx. \ldots . (1)$$

Consider, $\int_0^{\pi} \frac{2\sec^2 x}{1 + 4\tan^2 x} dx$

Let $2\tan x = t \Rightarrow 2\sec^2 x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \infty$

$$\Rightarrow \int_0^{\frac{x}{2}} \frac{2\sec^2 x}{1 + 4\tan^2 x} dx = \int_0^{\infty} \frac{dt}{1 + t^2}$$

$$= \left[\tan^{-1}t\right]_0^{\infty}$$

$$= \left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{\pi}{2}$$

Therefore, from (1), we obtain

$$I = -\frac{\pi}{6} + \frac{2}{3}\left[\frac{\pi}{2}\right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

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Q28 Evaluate the definite integral $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Answer.

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{-(-\sin 2x)}} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1+1-2\sin x \cos x)}} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)}{\sqrt{1-\left(\sin^2 x + \cos^2 x - 2\sin x \cos x\right)}} dx$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x + \cos x)\, dx}{\sqrt{1-(\sin x - \cos x)^2}}$

Let $(\sin x - \cos x) = t \Rightarrow (\sin x + \cos x)dx = dt$

When x $= \frac{\pi}{6}, t = \left(\frac{1-\sqrt{3}}{2}\right)$ $x = \frac{\pi}{3}, t = \left(\frac{\sqrt{3}-1}{2}\right)$

$I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

$\Rightarrow I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

$\frac{1}{\text{As}} \frac{1}{\sqrt{1-(-t)^2}} = \frac{1}{\sqrt{1-t^2}}$, therefore, $\frac{1}{\sqrt{1-t^2}}$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$\Rightarrow I = 2\int_0^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

$= \left[2\sin^{-1} t\right]_0^{\frac{\sqrt{3}-1}{2}}$

$= 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

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Q29 Evaluate the definite integral $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

Answer. Let I $= \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

$I = \int_0^1 \frac{1}{(\sqrt{1+x}-\sqrt{x})} \times \frac{(\sqrt{1+x}+\sqrt{x})}{(\sqrt{1+x}+\sqrt{x})} dx$

$= \int_0^d \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x} dx$

$= \int_0^4 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx$

$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$

$= \frac{2}{3}\left[(2)^{\frac{3}{2}} - 1\right] + \frac{2}{3}[1]$

$= \frac{2}{3}(2)^{\frac{3}{2}}$

$= \frac{2 \cdot 2\sqrt{2}}{3}$

$= \frac{4\sqrt{2}}{3}$

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Q30 Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

Answer.

Let $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

Also, let $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

When $x = 0, t = -1$ and when $x = \frac{\pi}{4}, t = 0$

$\Rightarrow (\sin x - \cos x)^2 = t^2$

$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x \cos x = t^2$

$\Rightarrow 1 - \sin 2x = t^2$

$\Rightarrow \sin 2x = 1 - t^2$

$\therefore I = \int_{-1}^{0} \frac{dt}{9 + 16\left(1 - t^2\right)}$

$= \int_{-1}^{0} \frac{dt}{9 + 16 - 16t^2}$

$= \int_{-1}^{0} \frac{dt}{25 - 16t^2} = \int_{-1}^{0} \frac{dt}{(5)^2 - (4t)^2}$

$= \frac{1}{4}\left[\frac{1}{2(5)} \log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$

$= \frac{1}{40}\left[\log(1) - \log\left|\frac{1}{9}\right|\right]_{-1}$

$= \frac{1}{40}\log 9$

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Q31 Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \sin 2x \tan^{-1}(\sin x) dx$

Answer.

Let $I = \int_0^{\frac{\pi}{4}} \sin 2x \tan^{-1}(\sin x) dx = \int_0^{\frac{\pi}{4}} 2\sin x \cos x \tan^{-1}(\sin x) dx$

Also, let $\sin x = t \Rightarrow \cos x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$\Rightarrow I = 2\int_0^1 t \tan^{-1}(t) dt. \ldots . (1)$

Consider $\int t \cdot \tan^{-1} t dt = \tan^{-1} t \cdot \int t dt - \int \left\{\frac{d}{dt}\left(\tan^{-1} t\right) \int t dt\right\} dt$

$= \tan^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{1 + t^2} \cdot \frac{t^2}{2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2}\int \frac{t^2 + 1 - 1}{1 + t^2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2}\int 1 dt + \frac{1}{2}\int \frac{1}{1 + t^2} dt$

$= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \cdot t + \frac{1}{2}\tan^{-1} t$

$$\Rightarrow \int_0^1 t \cdot \tan^{-1} t dt = \left[\frac{t^2 \cdot \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2}\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 1\right] = \frac{\pi}{4} - \frac{1}{2}$$

From equation (1), we obtain

$$I = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$$

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Q32 Evaluate the definite integral $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x} dx$

Answer. Let I = $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x} dx$....(1)

$$I = \int_0^{\pi} \left\{\frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)}\right\} dx \quad \left(\int_0^a f(x)dx = \int_0^e f(a - x)dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \left\{\frac{-(\pi - x)\tan x}{-(\sec x + \tan x)}\right\} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x} dx \ldots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi\tan x}{\sec x + \tan x} dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x + 1 - 1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi} 1.\, dx - \pi\int_0^{\pi} \frac{1}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi[x]_0^{\pi} - \pi\int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi^2 - \pi\int_0^{\pi} \left(\sec^2 x - \tan x \sec x\right) dx$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan x - \sec x]_0^x$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan\pi - \sec\pi - \tan 0 + \sec 0]$$

$$\Rightarrow 2I = \pi^2 - \pi[0 - (-1) - 0 + 1]$$

$$\Rightarrow 2I = \pi^2 - 2\pi$$

$$\Rightarrow 2I = \pi(\pi - 2)$$

$$\Rightarrow I = \frac{\pi}{2}(\pi - 2)$$

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Q33 Evaluate the definite integral $\int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$

Answer.

Let $I = \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$

$$\Rightarrow I = \int_1^4 |x - 1| dx + \int^4 |x - 2| dx + \int^4 |x - 3| dx$$

$$I = I_1 + I_2 + I_3 \ldots (3)$$

where, $I_1 = \int^4 |x-1|\,dx$, $I_2 = \int_1^4 |x-2|\,dx$, and $I_3 = \int_1^4 |x-3|\,dx$

$I_1 = \int_1^4 |x-1|\,dx$

$(x-1) \geq 0$ for $1 \leq x \leq 4$

$\therefore I_1 = \int_1^4 (x-1)dx$

$$\Rightarrow I_1 = \left[\frac{x^2}{x} - x\right]_1^4$$

$$\Rightarrow I_1 = \left[8 - 4 - \frac{1}{2} + 1\right] = \frac{9}{2} \ldots (2)$$

$I_2 = \int_1^4 |x-2|\,dx$

$x-2 \geq 0$ for $2 \leq x \leq 4$ and $x-2 \leq 0$ for $1 \leq x \leq 2$

$\therefore I_2 = \int_1^2 (2-x)dx + \int_2^1 (x-2)dx$

$\therefore I_2 = \int_1^2 (2-x)dx + \int_2^4 (x-2)dx$

$$\Rightarrow I_2 = \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$\Rightarrow I_2 = \left[4 - 2 - 2 + \frac{1}{2}\right] + [8 - 8 - 2 + 4]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \ldots\ldots (3)$$

$I_3 = \int_1^4 |x-3|\,dx$

$x-3 \geq 0$ for $3 \leq x \leq 4$ and $x-3 \leq 0$ for $1 \leq x \leq 3$

$\therefore I_3 = \int_1^3 (3-x)dx + \int_5^4 (x-3)dx$

$$\Rightarrow I_3 = \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

$$\Rightarrow I_3 = \left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right] + \left[8 - 12 - \frac{9}{2} + 9\right]$$

$$\Rightarrow I_3 = [6 - 4] + \left[\frac{1}{2}\right] = \frac{5}{2}$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

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Q34 Prove the following $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

Answer.

Let $I = \int_1^3 \frac{dx}{x^2(x+1)}$

Also, let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$$\Rightarrow 1 = Ax(x+1) + B(x+1) + C\left(x^2\right)$$

$$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$A + B = 0$

$B = 1$

On solving these equations, we obtain

$A = -1, C = 1,$ and $B = 1$

$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$

$\Rightarrow I = \int^3 \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$

$= \left[-\log x - \frac{1}{x} + \log(x+1) \right]_1^3$

$= \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x} \right]_1^3$

$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$

$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$

$= \log 2 - \log 3 + \frac{2}{3}$

$= \log\left(\frac{2}{3}\right) + \frac{2}{3}$

Hence, the given result is proved.

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Q35 Prove the following $\int_0^1 xe^x dx = 1$

Answer.

Let $I = \int_0^1 xe^x dx$

Integrating by parts, we obtain

$I = x\int_0^1 e^x dx - \int_0^1 \left\{ \left(\frac{d}{dx}(x) \right) \int e^x dx \right\} dx$

$= \left[xe^x \right]_0^1 - \int_0^1 e^x dx$

$= \left[xe^x \right]_0^1 - \left[e^x \right]_0^1$

$= e - e + 1$

$= 1$

Hence, the given result is proved.

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Q36 Prove the following $\int_{-1}^1 x^{17}\cos^4 x dx = 0$

Answer.

Let $I = \int_{-1}^1 x^{17}\cos^4 x dx$

Also, let $f(x) = x^{17}\cos^4 x$

$\Rightarrow f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

Therefore, $f(x)$ is an odd function.

It is known that if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$\therefore I = \int_{1}^{1} x^{17}\cos^4 x dx = 0$

Hence, the given result is proved.

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Q37 Prove the following $\int_{\theta}^{\pi} \sin^3 x dx = \frac{2}{3}$

Answer.

Let $I = \int_{\theta}^{\pi} \sin^3 x dx$

$I = \int_{\theta}^{\pi} \sin^2 x \cdot \sin x dx$

$= \int_{\theta}^{\pi} \left(1 - \cos^2 x\right) \sin x dx$

$= \int_{\theta}^{\pi} \sin x dx - \int_{\theta}^{\pi} \cos^2 x \cdot \sin x dx$

$= [-\cos x]_{\theta}^{\pi} + \left[\frac{\cos^3 x}{3}\right]_{0}^{\frac{\pi}{2}}$

$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$

Hence, the given result is proved.

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Q38 Prove the following $\int_{\theta}^{\pi} 2\tan^3 x dx = 1 - \log 2$

Answer.

Let $I = \int_{\theta}^{\pi} 2\tan^3 x dx$

$I = 2\int_{\theta}^{\pi} \tan^2 x \tan x dx = 2\int_{0}^{\pi} \left(\sec^2 x - 1\right) \tan x dx$

$= 2\int_{\theta}^{\pi} \sec^2 x \tan x dx - 2\int_{0}^{\pi} \tan x dx$

$= 2\left[\frac{\tan^2 x}{2}\right]_{0}^{\frac{\pi}{4}} + 2[\log\cos x]_{\theta}^{\pi}$

$= 1 + 2\left[\log\cos\frac{\pi}{4} - \log\cos 0\right]_{0}^{\frac{\pi}{4}}$

$= 1 + 2\left[\log\frac{1}{\sqrt{2}} - \log 1\right]$

$= 1 - \log 2 - \log 1 = 1 - \log 2$

Hence, the given result is proved.

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Q39 Prove the following $\int_{0}^{1} \sin^{-1} x dx = \frac{\pi}{2} - 1$

Answer.

Let $I = \int_0^1 \sin^{-1} x dx$

$\Rightarrow I = \int_0^1 \sin^{-1} x \cdot 1 \cdot dx$

Integrating by parts, we obtain

$$I = \left[\sin^{-1} x \cdot x\right]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x dx$$

$$= \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}\int_1^{\sqrt{1-x^2}} \frac{(-2x)}{\sqrt{1-x^2}} dx$$

Let $1 - x^2 = t - 2xdx = dt$
When $x = 0$, $t = 1$ and when $x = 1$, $t = 0$

$$I = \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}[2\sqrt{t}]_1^0$$

$$= \sin^{-1}(1) + [-\sqrt{1}]$$

$$= \frac{\pi}{2} - 1$$

Hence, the given result is proved.

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Q40 Evaluate $\int_0^1 e^{2-3x} dx$ as a limit of a sum.

Answer.

Let $I = \int_0^1 e^{2-3x} dx$

It is known that,

$\int_0^\infty f(x)dx = (b-a) \lim_{n \to \infty} \frac{1}{n}[f(a) + f(a+h) + \ldots + f(a+(n-1)h)]$

Where, $h = \frac{b-a}{n}$

Here, $a = 0$, $b = 1$, and $f(x) = e^{2-3x}$

$\Rightarrow h = \frac{1-0}{n} = \frac{1}{n}$

$\therefore \int_0^1 e^{2-3x} dx = (1-0) \lim_{n \to \infty} \frac{1}{n}[f(0) + f(0+h) + \ldots + f(0+(n-1)h)]$

$$= \lim_{n \to \infty} \frac{1}{n}\left[e^2 + e^{2-3h} + \ldots e^{2-3(n-1)h}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[e^2\left\{1 + e^{-3h} + e^{-6h} + e^{-9h} + \ldots e^{-3(n-1)k}\right\}\right]$$

$$= \lim_{h \to \infty} \frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-3h}\right)^n}{1-\left(e^{-3h}\right)}\right\}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n} \left[e^2 \left\{ \frac{1 - e^{-\frac{3}{n}}}{1 - e^{-\frac{3}{n}}} \right\} \right]$$

$$= \lim_{n \to \infty} \frac{1}{n} \left[\frac{e^2 \left(1 - e^{-3}\right)}{1 - e^{\frac{3}{n}}} \right]$$

$$= e^2 \left(e^{-3} - 1\right) \lim_{n \to \infty} \frac{1}{n} \left[\frac{1}{e^{-\frac{3}{3}} - 1} \right]$$

$$= e^2 \left(e^{-3} - 1\right) \lim_{n \to \infty} \left(-\frac{1}{3} \right) \left[\frac{-\frac{3}{n}}{e^{\frac{3}{n}} - 1} \right]$$

$$= \frac{-e^2 \left(e^{-3} - 1\right)}{3} \lim_{n \to \infty} \left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}} \right]$$

$$= \frac{-e^2 \left(e^{-3} - 1\right)}{3} (1) \qquad \left[\lim_{n \to \infty} \frac{x}{e^x - 1} \right]$$

$$= \frac{-e^{-1} + e^2}{3}$$

$$= \frac{1}{3} \left(e^2 - \frac{1}{e} \right)$$

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Q41 Choose the correct answer $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(A) $\tan^{-1}\left(e^x\right) + C$ (B) $\tan^{-1}\left(e^x\right) + C$

(C)$\log\left(e^x - e^{-x}\right) + C$ (D) $\log\left(e^x + e^{-x}\right) + C$

Answer.

Let $I = \int \frac{dx}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx$

Also, let $e^x = t \Rightarrow e^x dx = dt$

$$\therefore I = \int \frac{dt}{1 + t^2}$$

$$= \tan^{-1} t + C$$

$= \tan^{-1}\left(e^x\right) + C$

Hence, the correct Answer is A.

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Q42 Choose the correct answer $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

(A) $\frac{-1}{\sin x+\cos x}$ + C (B) $\log|\sin x+\cos x|+C$

(C)$\log|\sin x-\cos x|$ + C (D)$\frac{1}{(\sin x+\cos x)^2}$

Answer.

Let $I=\frac{\cos 2x}{(\cos x+\sin x)^2}$

$I=\int\frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}dx$

$=\int\frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$

$=\int\frac{\cos x-\sin x}{\cos+\sin x}dx$

Let $\cos x+\sin x=t \Rightarrow (\cos x-\sin x)dx=dt$

$\therefore I=\int\frac{dt}{t}$

$=\log|t|+C$

$=\log|\cos x+\sin x|+C$

Hence, the correct Answer is *B*

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Q43 Choose the correct answer

If$f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(C)$\frac{b-a}{2}\int_a^b f(x)dx$ (D) $\frac{a+b}{2}\int_a^b f(x)dx$

Answer. Let $I=\int_s^b xf(x)dx$ ....(1)

$I=\int_a^b(a+b-x)f(a+b-x)dx$

$\Rightarrow I=\int_a^b(a+b-x)f(x)dx \quad \left(\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right)$

$\Rightarrow I=(a+b)\int_a^b f(x)dx \quad -I \quad [Using(1)]$

$\Rightarrow I+I=(a+b)\int_c^b f(x)dx$

$\Rightarrow 2I=(a+b)\int_a^b f(x)dx$

$\Rightarrow I=\left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

Hence, the correct Answer is D.

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Q44 Choose the correct answer

The value of $\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(A) 1 (B) 0 (C) −1 (D) $\frac{\pi}{4}$

Answer.

Let $I=\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$\Rightarrow I=\int_0^1\tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$

$\Rightarrow I = \int_0^1 \left[\tan^{-1}x - \tan^{-1}(1-x)\right]dx \ldots (1)$

$\Rightarrow I = \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}(1-1+x)\right]dx$

$\Rightarrow I = \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}(x)\right]dx$

$\Rightarrow I = \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}(x)\right]dx \ldots (2)$

Adding (1) and (2), we obtain

$2I = \int_0^1 \left(\tan^{-1}x + \tan^{-1}(1-x) - \tan^{-1}(1-x) - \tan^{-1}x\right)dx$

$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

Hence, the correct Answer is B

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