

1. Simplify each of the following:

(i) $\sqrt[3]{4} \times \sqrt[3]{16}$

(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Sol:

(i) $\frac{\sqrt[3]{1250}}{\sqrt[3]{2}}$

(Note: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$)

$$= \sqrt[3]{4 \times 16}$$

$$= \sqrt[3]{64}$$

$$= \sqrt[3]{4^3}$$

$$= (4^3)^{\frac{1}{3}}$$

$$= 4^{(3 \times \frac{1}{3})}$$

$$= 4^1$$

$$= 4$$

(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

(Note: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$)

$$= \sqrt[4]{\frac{1250}{2}}$$

$$= \sqrt[4]{\frac{2 \times 625}{2}}$$

$$= \sqrt[4]{625}$$

$$= \sqrt[4]{15^4}$$

$$= 15^{(4 \times \frac{1}{4})}$$

$$= 15$$

2. Simplify the following expressions:

(i) $(4 + \sqrt{7})(3 + \sqrt{2})$

(ii) $(3 + \sqrt{3})(5 - \sqrt{2})$

(iii) $(\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$

Solution:

$$\begin{aligned}
 & \text{(i)} (4 + \sqrt{7})(3 + \sqrt{2}) \\
 &= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{7 \times 2} \\
 &= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(ii)} (3 + \sqrt{3})(5 - \sqrt{2}) \\
 &= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3 \times 2} \\
 &= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(iii)} (\sqrt{5} - 2)(\sqrt{3} - \sqrt{5}) \\
 &= \sqrt{15} - \sqrt{25} - 2\sqrt{3} + 2\sqrt{5} \\
 &= \sqrt{15} - \sqrt{5 \times 5} - 2\sqrt{3} + 2\sqrt{5} \\
 &= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}
 \end{aligned}$$

3. Simplify the following expressions:

$$\text{(i)} (11 + \sqrt{11})(11 - \sqrt{11})$$

$$\text{(ii)} (5 + \sqrt{7})(5 - \sqrt{7})$$

$$\text{(iii)} (\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$$

$$\text{(iv)} (3 + \sqrt{3})(3 - \sqrt{3})$$

$$\text{(v)} (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

Solution:

$$\text{(i)} (11 + \sqrt{11})(11 - \sqrt{11})$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$\text{So, } 11^2 - 11$$

$$121 - 11 = 110$$

$$\text{(ii)} (5 + \sqrt{7})(5 - \sqrt{7})$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$\text{So, } 5^2 - 7$$

$$25 - 7 = 18$$

$$\text{(iii)} (\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$\sqrt{8 \times 8} - \sqrt{2 \times 2} = 8 - 2$$

$$= 6$$

$$\text{(iv)} (3 + \sqrt{3})(3 - \sqrt{3})$$

$$\text{As we know, } (a + b)(a - b) = (a^2 - b^2)$$

$$= 9 - \sqrt{3 \times 3}$$

$$= 6$$

$$(v) (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

$$\text{As we know, } (a + b)(a - b) = (a^2 - b^2)$$

$$= \sqrt{5 \times 5} - \sqrt{2 \times 2}$$

$$= 5 - 2$$

$$= 3$$

4. Simplify the following expressions:

$$(i) (\sqrt{3} + \sqrt{7})^2$$

$$(ii) (\sqrt{5} - \sqrt{3})^2$$

$$(iii) (2\sqrt{5} + 3\sqrt{2})^2$$

Solution:

$$(i) (\sqrt{3} + \sqrt{7})^2$$

$$\text{As we know, } (a + b)^2 = (a^2 + 2 \times a \times b + b^2)$$

$$= \sqrt{3}^2 + 2 \times \sqrt{3} \times \sqrt{7} + \sqrt{7}^2$$

$$= 3 + 2 \times \sqrt{3 \times 7} + 7$$

$$= 10 + 2 \times \sqrt{21}$$

$$(ii) (\sqrt{5} - \sqrt{3})^2$$

$$\text{As we know, } (a - b)^2 = (a^2 - 2 \times a \times b + b^2)$$

$$(iii) (2\sqrt{5} + 3\sqrt{2})^2$$

$$\text{As we know, } (a + b)^2 = (a^2 + 2 \times a \times b + b^2)$$

$$= 4\sqrt{5 \times 5} + 2 \times 2\sqrt{5} \times 3\sqrt{2} + 9\sqrt{2 \times 2}$$

$$= 20 + 12\sqrt{10} + 18$$

$$= 28 + 12\sqrt{10}$$