

1. Simplify the following:

(i) $3(a^4 b^3)^{10} \times 5 (a^2 b^2)^3$

Solution:

$$= 3(a^{40} b^{30}) \times 5 (a^6 b^6)$$

$$= 15 (a^{46} b^{36})$$

(ii) $(2x^{-2} y^3)^3$

Solution:

$$= (2^3 x^{-2 \times 3} y^{3 \times 3})$$

$$= 8x^{-6} y^9$$

(iii) $\frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^4}$

Solution:

$$\frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^4}$$

$$= \frac{(24 \times 10^7 \times 10^{-5})}{8 \times 10^4}$$

$$= \frac{(24 \times 10^{7-5})}{8 \times 10^4}$$

$$= \frac{(24 \times 10^2)}{8 \times 10^4}$$

$$= \frac{(3 \times 10^2)}{10^4}$$

$$= \frac{3}{100}$$

(iv) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$

Solution:

$$= \frac{-20a^2b^5}{10a^2b^2}$$

$$= -2b^3$$

(v) $\left(\frac{x^2y^2}{a^2b^3}\right)^n$

Solution:

$$= \frac{x^{2n}y^{2n}}{a^{2n}b^{3n}}$$

(vi) $\frac{(a^{3n-9})^6}{a^{2n-4}}$

Solution:

$$\begin{aligned}
&= \frac{a^{18n-54}}{a^{2n-4}} \\
&= a^{18n-2n-54+4} \\
&= a^{16n-50}
\end{aligned}$$

2. If $a = 3$ and $b = -2$, find the values of:

(i) $a^a + b^b$

(ii) $a^b + b^a$

(iii) $a^b + b^a$

Solution:

(i) We have,

$$\begin{aligned}
&a^a + b^b \\
&= 3^3 + (-2)^{-2} \\
&= 3^3 + \left(-\frac{1}{2}\right)^2 \\
&= 27 + \frac{1}{4} \\
&= \frac{109}{4}
\end{aligned}$$

(ii) $a^b + b^a$

$$\begin{aligned}
&= 3^{-2} + (-2)^3 \\
&= \left(\frac{1}{3}\right)^2 + (-2)^3 \\
&= \frac{1}{9} - 8 \\
&= -\frac{71}{9}
\end{aligned}$$

(iii) We have,

$$\begin{aligned}
&a^b + b^a \\
&= (3 + (-2))^{3(-2)} \\
&= (3 - 2)^{-6} \\
&= 1^{-6} \\
&= 1
\end{aligned}$$

3. Prove that:

(i) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$

$$(ii) \left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2} = x^{2(a^3+b^3+c^3)}$$

$$(iii) \left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$$

Solution:

(i) To prove

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\begin{aligned} & \frac{x^{a^3+a^2b+ab^2}}{x^{a^2b+ab^2+b^3}} \times \frac{x^{b^3+b^2c+bc^2}}{x^{b^2c+bc^2+c^3}} \times \frac{x^{c^3+c^2a+ca^2}}{x^{c^2a+ca^2+a^3}} \\ & x^{a^3+a^2b+ab^2-(b^3+a^2b+ab^2)} \times x^{b^3+b^2c+bc^2-(c^3+b^2c+bc^2)} \times x^{c^3+c^2a+ca^2-(a^3+c^2a+ca^2)} \\ & x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3} \\ & x^{a^3-b^3+b^3-c^3+c^3-a^3} \\ & x^0 \\ & 1 \end{aligned}$$

Or,

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2} = x^{2(a^3+b^3+c^3)}$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\begin{aligned} & x^{(a+b)(a^2-ab+b^2)} \times x^{(b+c)(b^2-bc+c^2)} \times x^{(c+a)(c^2-ca+a^2)} \\ & x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3} \\ & x^{a^3+b^3+b^3+c^3+c^3+a^3} \\ & x^{2(a^3+b^3+c^3)} \end{aligned}$$

Therefore, LHS = RHS

Hence proved

(iii) To prove,

$$\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\left(\frac{x^{ac}}{x^{bc}}\right) \times \left(\frac{x^{ba}}{x^{ca}}\right) \times \left(\frac{x^{cb}}{x^{ab}}\right)$$

$$\begin{aligned}
 & x^{ac-bc} \times x^{ba-ca} \times x^{bc-ab} \\
 & x^{ac-bc+ba-ca+bc-ab} \\
 & x^0 \\
 & 1
 \end{aligned}$$

Therefore, LHS = RHS

Hence proved

4. Prove that:

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

$$(ii) \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$$

Solution:

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\begin{aligned}
 & \frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}} \\
 & \frac{\frac{x^b}{x^b}}{\frac{x^b+x^a}{x^b+x^a}} + \frac{\frac{x^a}{x^a}}{\frac{x^a+x^b}{x^a+x^b}} \\
 & \frac{x^b+x^a}{x^a+x^b} \\
 & 1
 \end{aligned}$$

Therefore, LHS = RHS

Hence proved

$$(ii) \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\begin{aligned}
 & \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} \\
 & \frac{\frac{x^a}{x^a}}{\frac{x^a+x^b+x^c}{x^a+x^b+x^c}} + \frac{\frac{x^b}{x^b}}{\frac{x^b+x^a+x^c}{x^b+x^a+x^c}} + \frac{\frac{x^c}{x^c}}{\frac{x^c+x^b+x^a}{x^c+x^b+x^a}} \\
 & \frac{x^a+x^b+x^c}{x^a+x^b+x^c} \\
 & 1
 \end{aligned}$$

Therefore, LHS = RHS

Hence proved

5. Prove that:

$$(i) \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = abc$$

$$(ii) (a^{-1} + b^{-1})^{-1}$$

Solution:

(i) To prove,

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = abc$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{\frac{a+b+c}{\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}}}{\frac{a+b+c}{\frac{a+b+c}{abc}}} = abc$$

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$$(a^{-1} + b^{-1})^{-1} = \frac{ab}{a+b}$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{\frac{1}{(a^{-1}+b^{-1})}}{\frac{\frac{1}{(\frac{1}{a}+\frac{1}{b})}}{\frac{1}{(\frac{a+b}{ab})}}} = \frac{ab}{a+b}$$

Therefore, LHS = RHS

Hence proved

$$6. \text{ If } abc = 1, \text{ show that } \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$$

Solution:

To prove,

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}} \\ \frac{b}{b+ab+1} + \frac{c}{c+bc+1} + \frac{a}{a+ac+1} \dots (1)$$

We know $abc = 1$

$$c = \frac{1}{ab}$$

By substituting the value c in equation (1), we get

$$\frac{b}{b+ab+1} + \frac{\frac{1}{ab}}{\frac{1}{ab}+b(\frac{1}{ab})+1} + \frac{a}{a+a(\frac{1}{ab})+1} \\ \frac{b}{b+ab+1} + \frac{\frac{1}{ab} \times ab}{1+b+ab} + \frac{ab}{1+ab+b} \\ \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{ab}{1+ab+b} \\ \frac{1+ab+b}{b+ab+1} \\ 1$$

Therefore, LHS = RHS

Hence proved

7. Simplify:

$$(i) \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

Solution:

$$= \frac{3^n \times 9^n \times 9}{\frac{3^n}{3} \times \frac{9^n}{9}}$$

$$= 9 \times 3 \times 9$$

$$= 243$$

$$(ii) \frac{(5 \times 25^{n+1})(25 \times 5^{2n})}{(5 \times 5^{2n+3}) - (25)^{n+1}}$$

Solution:

$$= \frac{(5 \times 25^n \times 25) - (25 \times 25^n)}{(5 \times 25^n \times 125) - (25^n \times 25)}$$

$$= \frac{25^n \times 25(5-1)}{25^n \times 25(25-1)}$$

$$= \frac{4}{24}$$

$$= \frac{1}{6}$$

$$(iii) \frac{(5^{n+3}) - (6 \times 5^{n+1})}{(9 \times 5^n) - (2^2 \times 5^n)}$$

Solution:

$$\begin{aligned}
&= \frac{(5^{n+3}) - (6 \times 5^{n+1})}{(9 \times 5^n) - (2^2 \times 5^n)} \\
&= \frac{(5^n \times 5^3) - (6 \times 5^n \times 5)}{(9 \times 5^n) - (2^2 \times 5^n)} \\
&= \frac{5^n(125 - 30)}{5^n(9 - 4)} \\
&= \frac{95}{5} \\
&= 19
\end{aligned}$$

$$(iv) \frac{(6 \times 8^{n+1}) + (16 \times 2^{3n-2})}{(10 \times 2^{3n+1}) - 7 \times (8)^n}$$

Solution:

$$\begin{aligned}
&= \frac{(6 \times 8^n \times 8) + (16 \times 8^n \times \frac{1}{4})}{(10 \times 8^n \times 2) - (7 \times (8)^n)} \\
&= \frac{8^n(48 + 4)}{8^n(20 - 7)} \\
&= \frac{52}{13} \\
&= 4
\end{aligned}$$

Level 2

8. Solve the following equations for x:

$$(i) 7^{2x+3} = 1$$

$$(ii) 2^{x+1} = 4^{x-3}$$

$$(iii) 2^{5x+3} = 8^{x+3}$$

$$(iv) 4^{2x} = \frac{1}{32}$$

$$(v) 4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$(vi) 2^{3x-7} = 256$$

Solution:

(i) We have,

$$\Rightarrow 7^{2x+3} = 1$$

$$\Rightarrow 7^{2x+3} = 7^0$$

$$\Rightarrow 2x + 3 = 0$$

$$\Rightarrow 2x = -3$$

$$\Rightarrow x = -\frac{3}{2}$$

(ii) We have,

$$2^{x+1} = 4^{x-3}$$

$$2^{x+1} = 2^{2x-6}$$

$$x + 1 = 2x - 6$$

$$x = 7$$

(iii) We have,

$$2^{5x+3} = 8^{x+3}$$

$$2^{5x+3} = 2^{3x+9}$$

$$5x + 3 = 3x + 9$$

$$2x = 6$$

$$x = 3$$

(iv) We have,

$$4^{2x} = \frac{1}{32}$$

$$2^{4x} = \frac{1}{2^5}$$

$$2^{4x} = 2^{-5}$$

$$4x = -5$$

$$x = \frac{-5}{4}$$

(v) We have,

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$2^{2x-2} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2}\right)^{3x}$$

$$2^{2x-2} \times 2^{2x-3} = \left(\frac{1}{2}\right)^{3x}$$

$$2^{2x-2+2x-3} = \left(\frac{1}{2}\right)^{3x}$$

$$2^{4x-5} = 2^{-3x}$$

$$4x-5 = -3x$$

$$7x = 5$$

$$x = \frac{5}{7}$$

(vi) $2^{3x-7} = 256$

$$2^{3x-7} = 2^8$$

$$3x - 7 = 8$$

$$3x = 15$$

$$x = 5$$

9. Solve the following equations for x:

(i) $2^{2x} - 2^{x+3} + 2^4 = 0$

(ii) $3^{2x+4} + 1 = 2 \times 3^{x+2}$

Solution:

(i) We have,

$$\Rightarrow 2^{2x} - 2^{x+3} + 2^4 = 0$$

$$\Rightarrow 2^{2x} + 2^4 = 2^x \cdot 2^3$$

$$\Rightarrow \text{Let } 2^x = y$$

$$\Rightarrow y^2 + 2^4 = y \times 2^3$$

$$\Rightarrow y^2 - 8y + 16 = 0$$

$$\Rightarrow y^2 - 4y - 4y + 16 = 0$$

$$\Rightarrow y(y-4) - 4(y-4) = 0$$

$$\Rightarrow y = 4$$

$$\Rightarrow x^2 = 2^2$$

$$\Rightarrow x = 2$$

(ii) We have,

$$3^{2x+4} + 1 = 2 \times 3^{x+2}$$

$$(3^{x+2})^2 + 1 = 2 \times 3^{x+2}$$

$$\text{Let } 3^{x+2} = y$$

$$y^2 + 1 = 2y$$

$$y^2 - 2y + 1 = 0$$

$$y^2 - y - y + 1 = 0$$

$$y(y-1) - 1(y-1) = 0$$

$$(y-1)(y-1) = 0$$

$$y = 1$$

10. If $49392 = a^4 b^2 c^3$, find the values of a, b and c, where a, b and c, where a, b, and c are different positive primes.

Solution:

Taking out the LCM, the factors are 2^4 , 3^2 and 7^3

$$a^4 b^2 c^3 = 2^4, 3^2 \text{ and } 7^3$$

$$a = 2, b = 3 \text{ and } c = 7 \text{ [Since, a, b and c are primes]}$$

11. If $1176 = 2^a \times 3^b \times 7^c$, Find a, b, and c.

Solution:

Given that 2, 3 and 7 are factors of 1176.

Taking out the LCM of 1176, we get

$$2^3 \times 3^1 \times 7^2 = 2^a \times 3^b \times 7^c$$

By comparing, we get

$$a = 3, b = 1 \text{ and } c = 2.$$

12. Given $4725 = 3^a \times 5^b \times 7^c$, find

(i) The integral values of a, b and c

Solution:

Taking out the LCM of 4725, we get

$$3^3 \times 5^2 \times 7^1 = 3^a \times 5^b \times 7^c$$

By comparing, we get

$$a = 3, b = 2 \text{ and } c = 1.$$

(ii) The value of $2^{-a} \times 3^b \times 7^c$

Solution:

$$\begin{aligned} (2^{-a} \times 3^b \times 7^c) &= (2^{-3} \times 3^2 \times 7^1) \\ (2^{-3} \times 3^2 \times 7^1) &= \frac{1}{8} \times 9 \times 7 \\ &= \frac{63}{8} \end{aligned}$$

13. If $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$, prove that $a^{q-r}b^{r-p}c^{p-q} = 1$

Solution:

Given,

$$a = xy^{p-1}, b = xy^{q-1} \text{ and } c = xy^{r-1}$$

$$\text{To prove, } a^{q-r}b^{r-p}c^{p-q} = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$= a^{q-r}b^{r-p}c^{p-q} \quad \dots\dots(i)$$

By substituting the value of a, b and c in equation (i), we get

$$\begin{aligned} &= (xy^{p-1})^{q-r}(xy^{q-1})^{r-p}(xy^{r-1})^{p-q} \\ &= xy^{pq-pr-q+r}xy^{qr-pq-r+p}xy^{rp-rq-p+q} \\ &= xy^{pq-pr-q+r+qr-pq-r+p+rp-rq-p+q} \\ &= xy^0 \\ &= 1 \end{aligned}$$