

Q1. Define an irrational number.

Solution:

An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers. It cannot be expressed as terminating or repeating decimal.

Q2. Explain how an irrational number is differing from rational numbers?

Solution: An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers. It cannot be expressed as terminating or repeating decimal.

For example, 0.10110100 is an irrational number

A rational number is a real number which can be written as a fraction and as a decimal i.e. it can be expressed as a ratio of integers. . It can be expressed as terminating or repeating decimal.

For examples,

0.10 and $0.\overline{4}$ both are rational numbers

Q3. Find, whether the following numbers are rational and irrational

(i) $\sqrt{7}$

(ii) $\sqrt{4}$

(iii) $2 + \sqrt{3}$

(iv) $\sqrt{3} + \sqrt{2}$

(v) $\sqrt{3} + \sqrt{5}$

(vi) $(\sqrt{2}-2)^2$

(vii) $(2-\sqrt{2})(2+\sqrt{2})$

(viii) $(\sqrt{2} + \sqrt{3})^2$

(ix) $\sqrt{5} - 2$

(x) $\sqrt{23}$

(xi) $\sqrt{225}$

(xii) 0.3796

(xiii) 7.478478.....

(xiv) 1.101001000100001.....

Solution:

(i) $\sqrt{7}$ is not a perfect square root so it is an Irrational number.

(ii) $\sqrt{4}$ is a perfect square root so it is an rational number.

We have,

$\sqrt{4}$ can be expressed in the form of

$\frac{a}{b}$, so it is a rational number. The decimal

representation of $\sqrt{9}$ is 3.0. 3 is a rational number.

(iii) $2 + \sqrt{3}$

Here, 2 is a rational number and $\sqrt{3}$ is an irrational number

So, the sum of a rational and an irrational number is an irrational number.

(iv) $\sqrt{3} + \sqrt{2}$

$\sqrt{3}$ is not a perfect square and it is an irrational number and $\sqrt{2}$ is not a perfect square and is an irrational number. The sum of an irrational number and an irrational number is an irrational number, so $\sqrt{3} + \sqrt{2}$ is an irrational number.

(v) $\sqrt{3} + \sqrt{5}$

$\sqrt{3}$ is not a perfect square and it is an irrational number and $\sqrt{5}$ is not a perfect square and is an irrational number. The sum of an irrational number and an irrational number is an irrational number, so $\sqrt{3} + \sqrt{5}$ is an irrational number.

(vi) $(\sqrt{2}-2)^2$

We have, $(\sqrt{2}-2)^2$

$$= 2 + 4 - 4\sqrt{2}$$

$$= 6 + 4\sqrt{2}$$

6 is a rational number but $4\sqrt{2}$ is an irrational number.

The sum of a rational number and an irrational number is an irrational number, so $(\sqrt{2} + \sqrt{4})^2$ is an irrational number.

(vii) $(2-\sqrt{2})(2 + \sqrt{2})$

We have,

$$(2-\sqrt{2})(2 + \sqrt{2}) = (2)^2 - (\sqrt{2})^2 \quad [\text{Since, } (a + b)(a - b) = a^2 - b^2]$$

$$4 - 2 = \frac{2}{1}$$

Since, 2 is a rational number.

$(2-\sqrt{2})(2 + \sqrt{2})$ is a rational number.

(viii) $(\sqrt{2} + \sqrt{3})^2$

We have,

$$(\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + \sqrt{6} \quad [\text{Since, } (a+b)^2 = a^2 + 2ab + b^2]$$

The sum of a rational number and an irrational number is an irrational number, so $(\sqrt{2} + \sqrt{3})^2$ is an irrational number.

(ix) $\sqrt{5} - 2$

The difference of an irrational number and a rational number is an irrational number.

$(\sqrt{5} - 2)$ is an irrational number.

(x) $\sqrt{23}$

$$\sqrt{23} = 4.795831352331...$$

As decimal expansion of this number is non-terminating, non-recurring so it is an irrational number.

(xi) $\sqrt{225}$

$$\sqrt{225} = 15 = \frac{15}{1}$$

$\sqrt{225}$ is rational number as it can be represented in $\frac{p}{q}$ form.

(xii) 0.3796

0.3796, as decimal expansion of this number is terminating, so it is a rational number.

(xiii) 7.478478.....

7.478478 = 7.478, as decimal expansion of this number is non-terminating recurring so it is a rational number.

(xiv) 1.101001000100001.....

1.101001000100001....., as decimal expansion of this number is non-terminating, non-recurring so it is an irrational number

Q4. Identify the following as irrational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$

(ii) $3 \times \sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{64}$

(vi) $\sqrt{100}$

Solution:

(i) We have,

$\sqrt{4}$ can be written in the form of

$\frac{p}{q}$. So, it is a rational number. Its decimal

representation is 2.0

(ii). We have,

$$\begin{aligned} & 3 \times \sqrt{18} \\ &= 3 \times \sqrt{2 \times 3 \times 3} \\ &= 9 \times \sqrt{2} \end{aligned}$$

Since, the product of a ratios and an irrational is an irrational number.

$9 \times \sqrt{2}$ is an irrational.

$3 \times \sqrt{18}$ is an irrational number.

(iii) We have,

$$\begin{aligned} & \sqrt{1.44} \\ &= \sqrt{\frac{144}{100}} \\ &= \frac{12}{10} \\ &= 1.2 \end{aligned}$$

Every terminating decimal is a rational number, so 1.2 is a rational number.

Its decimal representation is 1.2.

(iv) $\sqrt{\frac{9}{27}}$

We have,

$$\begin{aligned} & \sqrt{\frac{9}{27}} \\ &= \frac{3}{\sqrt{27}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Quotient of a rational and an irrational number is irrational numbers so

$\frac{1}{\sqrt{3}}$ is an irrational number.

$\sqrt{\frac{9}{27}}$ is an irrational number.

(v) We have,

$$\begin{aligned} & -\sqrt{64} \\ &= -8 \\ &= -\frac{8}{1} \\ &= -\frac{8}{1} \text{ can be expressed in the form of } \frac{a}{b}, \end{aligned}$$

so $-\sqrt{64}$ is a rational number.

Its decimal representation is -8.0 .

(vi) We have,

$$\sqrt{100}$$

$= 10$ can be expressed in the form of $\frac{a}{b}$,

so $\sqrt{100}$ is a rational number

Its decimal representation is 10.0 .

Q5. In the following equations, find which variables x , y and z etc. represent rational or irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = \frac{17}{4}$

(v) $v^2 = 3$

(vi) $w^2 = 27$

(vii) $t^2 = 0.4$

Solution:

(i) We have,

$$x^2 = 5$$

Taking square root on both the sides, we get

$$x = \sqrt{5}$$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have,

$$y^2 = 9$$

$$y = 3$$

$= \frac{3}{1}$ can be expressed in the form of $\frac{a}{b}$, so it is a rational number.

(iii) We have,

$$z^2 = 0.04$$

Taking square root on both sides, we get

$$z = 0.2$$

$\frac{2}{10}$ can be expressed in the form of $\frac{a}{b}$, so it is a rational number.

(iv) We have,

$$u^2 = \frac{17}{4}$$

Taking square root on both sides, we get,

$$u = \sqrt{\frac{17}{4}}$$

$$u = \frac{\sqrt{17}}{2}$$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v) We have,

$$v^2 = 3$$

Taking square root on both sides, we get,

$$v = \sqrt{3}$$

$\sqrt{3}$ is not a perfect square root, so v is irrational number.

(vi) We have,

$$w^2 = 27$$

Taking square root on both the sides, we get,

$$w = 3\sqrt{3}$$

Product of a irrational and an irrational is an irrational number. So w is an irrational number.

(vii) We have,

$$t^2 = 0.4$$

Taking square root on both sides, we get,

$$t = \sqrt{\frac{4}{10}}$$

$$t = \frac{2}{\sqrt{10}}$$

Since, quotient of a rational and an irrational number is irrational number. $t^2 = 0.4$ is an irrational number.

Q6. Give an example of each, of two irrational numbers whose:

(i) Difference is a rational number.

(ii) Difference is an irrational number.

(iii) Sum is a rational number.

(iv) Sum is an irrational number.

(v) Product is a rational number.

(vi) Product is an irrational number.

(vii) Quotient is a rational number.

(viii) Quotient is an irrational number.

Solution:

(i) $\sqrt{2}$ is an irrational number.

Now, $\sqrt{2} - \sqrt{2} = 0$.

0 is the rational number.

(ii) Let two irrational numbers are $3\sqrt{2}$ and $\sqrt{2}$.

$$3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$$

$5\sqrt{6}$ is the rational number.

(iii) $\sqrt{11}$ is an irrational number.

Now, $\sqrt{11} + (-\sqrt{11}) = 0$.

0 is the rational number.

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

$$4\sqrt{6} + \sqrt{6}$$

$5\sqrt{6}$ is the rational number.

(iv) Let two Irrational numbers are $7\sqrt{5}$ and

$$\sqrt{5}$$

Now, $7\sqrt{5} \times \sqrt{5}$

$$= 7 \times 5$$

= 35 is the rational number.

(v) Let two irrational numbers are $\sqrt{8}$ and $\sqrt{8}$.

Now, $\sqrt{8} \times \sqrt{8}$

8 is the rational number.

(vi) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

$$\text{Now, } \frac{4\sqrt{6}}{\sqrt{6}}$$

= 4 is the rational number

(vii) Let two irrational numbers are $3\sqrt{7}$ and $\sqrt{7}$

Now, 3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{8}$ and $\sqrt{2}$

Now $\sqrt{2}$ is an rational number.

Q7. Give two rational numbers lying between 0.232332333233332 and 0.212112111211112.

Solution: Let a = 0.212112111211112

And, $b = 0.232332333233332...$

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3. If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b .

Let, $x = 0.22$

$y = 0.22112211...$ Then $a < x < y < b$

Hence, x , and y are required rational numbers.

Q8. Give two rational numbers lying between 0.515115111511115 and 0.5353353335

Solution: Let, $a = 0.515115111511115...$

And, $b = 0.5353353335..$

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, $a < b$.

So If we consider rational numbers

$x = 0.52$

$y = 0.52062062...$

We find that,

$a < x < y < b$

Hence x and y are required rational numbers.

Q9. Find one irrational number between 0.2101 and $0.2222 \dots = 0.\overline{2}$

Solution:

Let, $a = 0.2101$ and,

$b = 0.2222...$

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore $a < b$. In the third decimal place a has digit 0.

So, if we consider irrational numbers

$x = 0.211011001100011....$

We find that $a < x < b$

Hence x is required irrational number.

Q10. Find a rational number and also an irrational number lying between the numbers $0.3030030003...$ and $0.3010010001...$

Solution: Let,

$a = 0.3010010001$ and,

$b = 0.3030030003...$

We observe that in the third decimal place a has digit 1 and b has digit

3, therefore $a < b$. In the third decimal place a has digit 1. So, if we

consider rational and irrational numbers

$$x=0.302$$

$$y = 0.302002000200002.....$$

We find that $a < x < b$ and, $a < y < b$.

Hence, x and y are required rational and irrational numbers respectively.

Q11. Find two irrational numbers between 0.5 and 0.55.

Solution: Let $a = 0.5 = 0.50$ and $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b$ so, if we consider irrational numbers

$$x = 0.51051005100051...$$

$$y = 0.530535305353530...$$

We find that $a < x < y < b$

Hence x and y are required irrational numbers.

Q12. Find two irrational numbers lying between 0.1 and 0.12.

Solution:

$$\text{Let } a = 0.1 = 0.10$$

$$\text{And } b = 0.12$$

We observe that In the second decimal place a has digit 0 and b has digit 2.

Therefore, $a < b$.

So, if we consider irrational numbers

$$x = 0.1101101100011... \quad y = 0.111011110111110... \quad \text{We find that } a < x < y < b$$

Hence, x and y are required irrational numbers.

Q13. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

If possible, let $\sqrt{3} + \sqrt{5}$ be a rational number equal to x .

Then,

$$x = \sqrt{3} + \sqrt{5}$$

$$x^2 = (\sqrt{3} + \sqrt{5})^2$$

$$x^2 = 8 + 2\sqrt{15}$$

$$\frac{x^2 - 8}{2} = \sqrt{15}$$

Now, $\sqrt{\frac{x^2 - 8}{2}}$ is rational

$\sqrt{15}$ is rational

Thus, we arrive at a contradiction.

Hence, $\sqrt{3} + \sqrt{5}$ is an irrational number.