Std. X C Time: 3 Hrs.
Date: 06-02-04 Maths Max Marks: 100

Note:(1) There are five questions in this paper. All of them are compulsory
(2) Figure to the right indicates total marks of the question.

I (A) If for a correspondence between two pairs of corresponding sides are proportional and the included angles are congruent, then prove that the correspondence is a similarity. (4)

(B) Solve any two of the following: (6)

1) Factorise: $2ab(a+2b)+2bc(b+2c)+2ca(c+2a)+9abc$

2) Factorize: $a^3+b^3+c^3-3abc$

3) Simplify: $\frac{x^3-x^2y+xy^2}{x^4+xy^3}+\frac{y^3+y^2x+yx^2}{x^3y-y^4}+\frac{2x^3y+2xy^3}{x^5-xy^4}$

(C) Factorize any two of the following: (4)

1) $x^3+9x^2+23x+15$

2) x^4-34x^2+225

3) $x(x-2)(x^2-2x+1)-12$

(D) Solve any one of the following: (2)

1) Simplify: $\left(\frac{x^2}{2}+\frac{4}{x}\right)\left(\frac{1}{x^2-2x+4}\right)-\frac{x}{2x-4}$

2) If $2a:3b = 3:2$, evaluate $(a-b):(a+b)$

(E) Fill in the blanks by selecting the proper alternative from those given in the brackets. (4)

1) $A=\{2^1, 2^2, 2^3, 2^4\}$ and $B = Z$. The function f is defined as $f: A \to B$, $f(x): A \to B$, $f(x) = \log_2 x$. Then the range of the function f is _____
($\{2\}, \{4\}, \{1,2,3,4\}$)

2) $f: N-\{1\} \to N$, $f(n)$ = the smallest prime factor of n. Then, $f(210)$ = _____
(1, 2, 3)

3) In a computer, huge quantity of data is stored in _____ unit.
(memory, control, arithmetical and logical)

4) In a flow chart, the flow of computation can go in two possible directions only from _____ box. ()

II A) Prove that in the same circle congruent chords are equidistant from the centre. (4)

B) Solve any two: (6)

1) If $\frac{a^2+b^2+c^2}{a^2+ac+c^2}=\frac{b^2}{ac}$, prove that b is the geometric mean between a and c. $(a, b, c \neq 0)$

2) If $\frac{x}{a+2b+2c}=\frac{y}{b+2c+2a}=\frac{z}{c+2a+2b}$ prove that each ratio $=\frac{x-y+z}{a+3b+c}$

3) x-y varies directly as $\frac{x}{y}$ and x+y varies directly as $\frac{y}{x}$. If $x, y \neq 0$, prove that x^2-y^2 is a non-zero constant.

(C) Solve any two. (4)

1) Weekly (Monday to Sunday) mean earning of a dining hall is Rs.225. If the mean of its earning from Monday to Saturday is Rs.200. Find its earning for Sunday.

2) Find the mean of $\frac{2}{5}, \frac{3}{5}, \frac{3}{7}, \frac{4}{7}, \frac{5}{9}, \frac{4}{9}$

3) Information regarding the number of children in twelve families is given below. 3,2,2,2,2,3,2,1,2,2,2,2,2. Find the mean, median and mode of the data, which of the three measures of central tendency is more appropriate.

(D) Solve any one: (2)

1) For the function $f: R \to R$, $f(x) = (2^x - x^2)$, x; Find f(0).

2) Find the range of the function $f: N \to Z$, f(x) = the remainder obtained on dividing x by 5.

(E) Fill in the blanks by selecting the proper alternative from those given in the brackets. (4)

1) $\frac{x^2-1}{x^3-1} \times \frac{x^2+x+1}{x+1} =$ _____ (1, -1, 0)

2) If $\frac{a+3}{b}=\frac{b-4}{c}=\frac{c+1}{a}$ and $a+b+c \neq 0$, then each ratio = _____ (10, 1, 0)

3) If $x^2 - 6xy + 9y^2 = 0$ then x ∝ _____ (y, $\sqrt{y}$, y2)

4) If $\cos^2\theta = \frac{1}{4}$, then $\theta =$ _____ $(30°, 45°, 60°)$

III (A) Prove that an angle inscribed in a semicircle is a right angle. (4)

(B) Solve any two. (6)

1) From the top of 400 metre high hill, the angles of depression of two houses situated in opposite directions from the hill are found to be 30° and 60°. Find the distance between these houses. ($\cot 30° = 1.7321$, $\cot 60° = 0.5774$)

2) The diameter of the base of a cone is 20 cm and its slant height is 25 cm. Find the total surface area.

3) Find the volume of the largest sphere that can be possibly carved out from a wooden cube of length 7 cm.

(C) Solve any two. (4)

1) Evaluate: $\frac{\operatorname{cosec} 30° \cdot \tan 30° \sec 45° \sin 45° \cos 60°}{\cot 60° \cdot \sin 30°}$

2) Prove that: $\frac{(\sec\theta - \tan\theta)^2 + 1}{\sec\theta - \tan\theta} = 2\sec\theta$

3) Prove that: $\operatorname{cosec}^6\theta - \cot^6\theta = 1 + 3\cot^2\theta + 3\cot^4\theta$

(D) Solve any one. (2)

1) x varies inversely as y and z jointly. x=6 when y=2 and z=3. Find x, when y.

2) Evaluate : $4\tan^2 30° - 5\operatorname{cosec}^2 45° - \frac{1}{3}\sin^4 45°$.

(E) Answer the following. (4)

1) If the equagtion $4x^2+mx+1=0$ has equal roots in R. Find m.

2) If $x^2-5x=1$, find the value of $x-\frac{1}{x}$.

3) Define: Median of a triangle.

4) Define: Similarity of triangles.

IV (A) In ΔABC, $\overline{AD}$ is a median. Prove that $AB^2+AC^2=2(BD^2+AD^2)$ (4)

(B) Solve any two. (6)

1) Simplify: $\frac{x^2-4(x+3)}{x^2-9(x-2)} \times \frac{x^2(x-2)^2-16}{x^3+8} \times \frac{x^3-27}{(x+3)^2-3x}$

2) Find the mean and median of the following observations.
47, 50, 53, 56, 59, 62, 65, 68, 49, 52, 55, 58

3) Frequency distribution of daily wages of 50 workers is given below. Find the median

Wages in Rs.	20-29	30-39	40-49	50-59
No. of workers	5	27	15	3

(C) Solve any two: (4)

1) Chords $\overline{AD}$ and $\overline{BC}$ of a circle intersect each other at right angles at M. If $m\angle ABM = 40$, find $m\angle BCD$.

2) In $\triangle ABC$, $\angle B$ is right angle. AB=5 and BC=12. Find the radius of the incircle of $\triangle ABC$.

3) A line passing through a point P lying in the exterior of $\odot(0,4,5)$ touches a circle at a point A. AP=6. If $\overline{OP}$ intersects the circle in the point B, find PB.

(D) Solve any one. (2)

1) For what values of 'm' are the roots of the equation $(m+1)x^2 + 2(m+3)x + (2m+3) = 0$ equal ?

2. $\left(\frac{-3}{2}\right)$ and $\frac{7}{3}$ are the zeros of the quadratic polynomial and the coefficient of x^2 is 6, then obtain the factors of the polynomial.

(E) Answer the following. (4)

1) Define: Chord of a circle

2) Define: Cyclic quadrilateral

3) The surface area of a sphere A = _____

4) Measures of radius and height of a cylinder and a cone are equal. Volume of the cylinder = ____ x volume of the cone.

V (a) $\overline{PQ}$ is given. Constant $\triangle ABC$ such that BC = 4PQ, $m\angle A = 45$ and the length of median $\overline{AD}$ = 3PQ. Write the steps of construction. (4)

(b) Solve anytwo. (6)

1. The area of a triangle varies jointly as the length of its base and the altitude. The area of a triangle with base 9 cm and altitude 8 cm is 36 sq.cm. If the base and the altitude of triangle are 17 cm and 14 cm respectively, find its area.

2. Solve in R: $\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) - 10 = 0$

3. A housewife spends Rs.180 per month from her total household expenses for buying of groundnut oil. With the increase in price of Rs.5 per kilogram in oil, she could purchase 1/2 kg of oil less for Rs.180. Find the original price of oil per kg.

(C) Solve any two (4)

1. $\overline{AD}$ is the bisector of $\angle A$ in ΔABC intersects $\overline{BC}$ in point D. If AB=6, AC = 7.5 and BC=9, then find BD.

2. A-P-B, A-Q-C and $\overline{PQ} \parallel \overline{BC}$ in ΔABC. If AP = 3x, AB =10, AQ =3 and QC = x, find x.

3. In ΔABC, AB = 29, BC = 40, AC = 29, $\overline{AD}$ is a median and G is a centroid, find AG.

(D) Attempt any one:

1) In ΔABC, $\angle B$ is a right angle. $\overline{BM}$ is the altitude of the triangle. If AC=13, BM=6, then find AB.

2) In a circle with radius 21 cm, a minor arc subtends an angle of measure 150 at the centre. Find the length of the corresponding major arc.

(E) $\angle B$ is a right angle in ΔABC, prove that B is on the circle with $\overline{AC}$ as a diameter. (4)

OR

Answer the following (4)

1) "Two circle having same centre but radii of different measures are called concentric circle". State whether the given statement is true or false.

2) $\overline{AB}$ is a diameter of $\odot(P,10)$ and $\angle ACB$ is inscribed in a semicircle. If BC=16, find AC.

3) P is a point in the exterior of $\odot(0,1.5)$, OP = 2.5. A line passing through P touches the circle at the point A. Find AP.

4) $\square ABCD$ is a cyclic quadrilateral and $\overline{AB}$ is a diameter of the circumcircle of $\square ABCD$. If m $\angle ADC = 130$, find m $\angle BAC$.

(C) Solve any two (4)

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