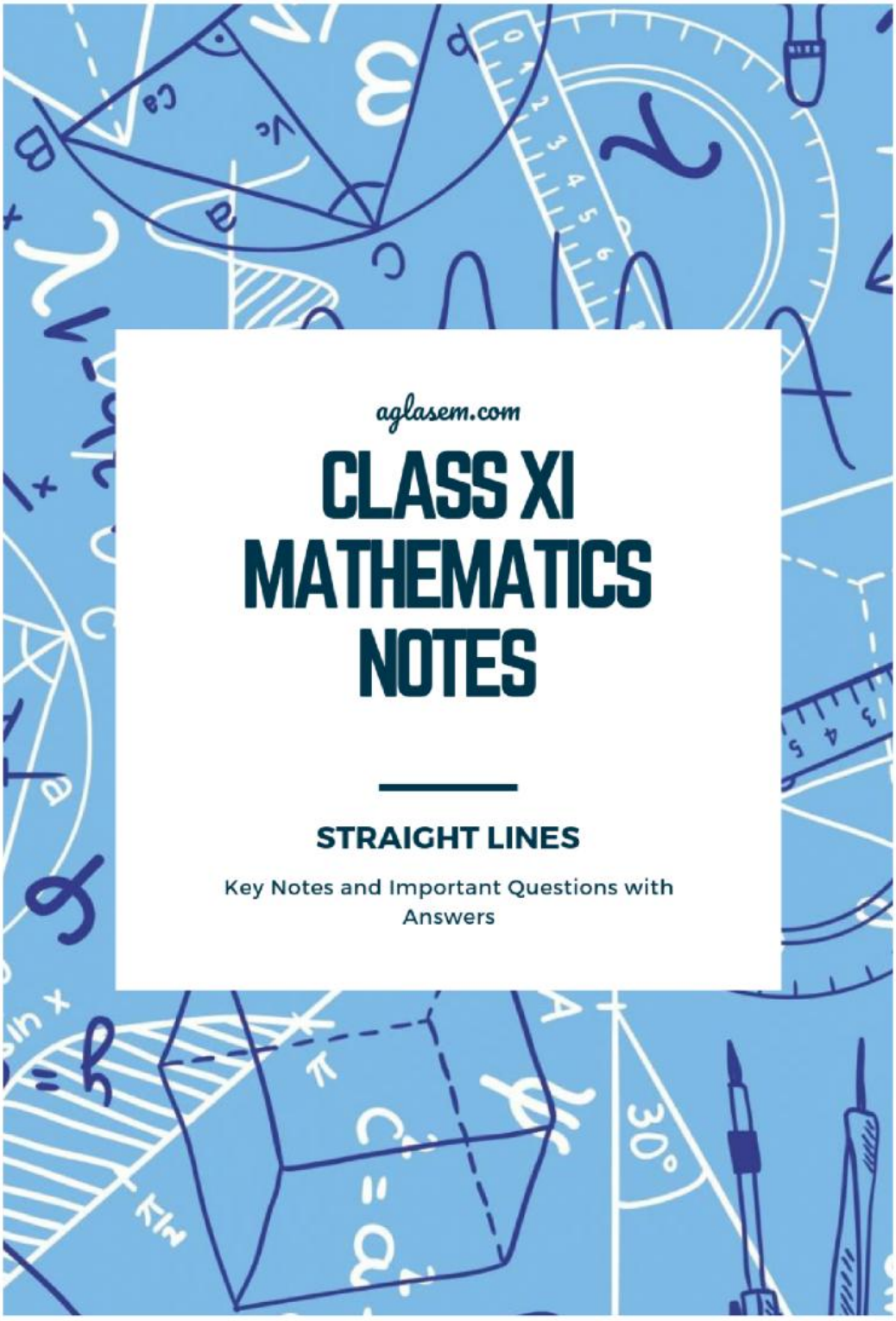


aglasem.com

CLASS XI MATHEMATICS NOTES

STRAIGHT LINES

Key Notes and Important Questions with Answers

CHAPTER - 10

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

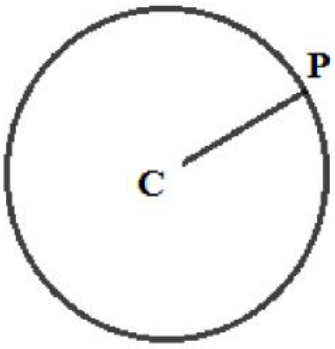
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ B (x_2, y_2) and $C(x_3, y_3)$.Then area of triangle

$$ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

- NOTE: Area of a Δ is always positive. If the above expression is zero, then a Δ is not possible. Thus the points are collinear.

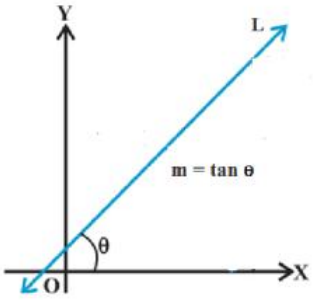
- LOCUS: When a variable point P(x,y) moves under certain condition then the path traced out by the point P is called the locus of the point.

  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.



CP = constant

- Locus of an equation: In the coordinate plane, locus of an equation is the pictorial representation of the set of all those points which satisfy the given equation.
- Equation of a locus: is the equation in x and y that is satisfied by the coordinates of every point on the locus.
- A line is also defined as the locus of a point satisfying the condition ax + by + c = 0 where a, b, c are constants.
- Slope of a straight line: If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



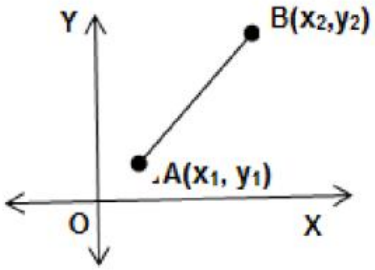
$m = \tan\theta,\ \theta \neq 90°$

If $0° < \theta < 90°$ *then* $m > 0$ *and*

$90° < \theta < 180°$ *then* $m < 0$

- NOTE 1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined
- NOTE 2: Slope of any horizontal line i.e. || to x-axis is zero.Slope of a vertical line i.e. || to y-axis is not zero.

- Three points *A, B* and *C* lying in a plane are collinear, if slope of *AB* = Slope of *BC*.
- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.



- Two lines are parallel to each other if and only if their slopes are equal.

  $i.e\ l_1 \| l_2 \Leftrightarrow m_1 = m_2$

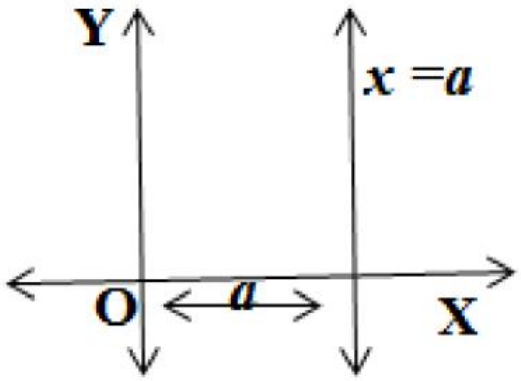
- NOTE: If slopes of lines l_1 and l_2 are not defined then they must be $\perp$ to x-axis, so they are ||. Thus $l_1 \| l_2 \Leftrightarrow$ they have same slope or both of them have no slope.
- Two non- vertical lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

  $i.e\ l_1 \perp l_2 \Leftrightarrow m_1\, m_2 = -1$

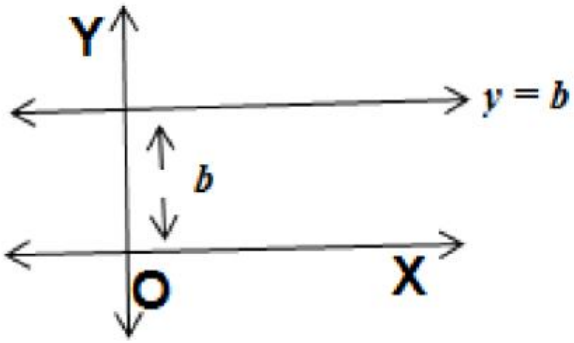
- NOTE: The above condition holds when the lines have non-zero slopes i.e none of them $\perp$ to any axis.
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, 1 + m_1 m_2 \neq 0$

  & obtuse angle is $\phi = 180 - \alpha$

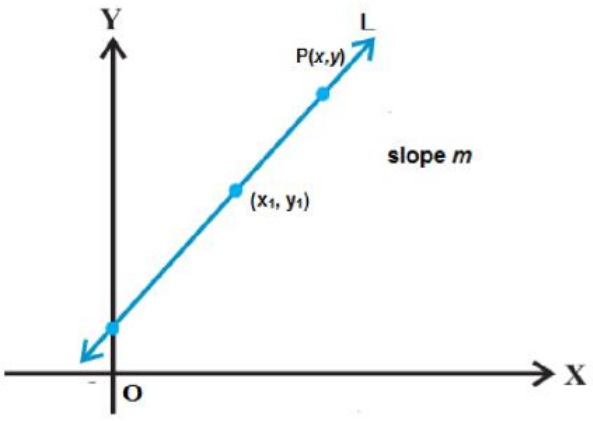
- x = a is a line parallel to y-axis at a distance of a units from y-axis. x = a lies on right or left of y-axis according as a is positive or negative.



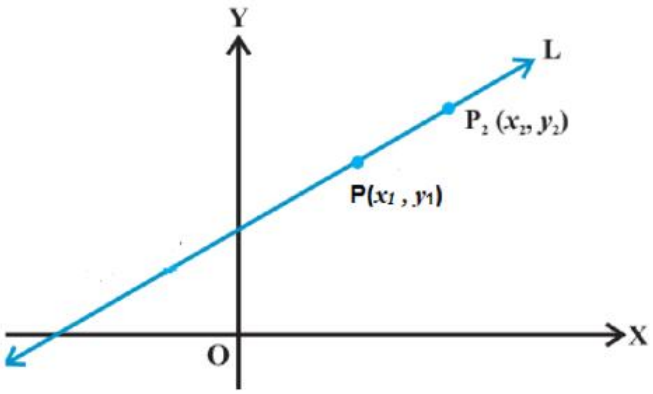
- y = b is a line parallel to x-axis at a distance of 'b' units fromx-axis. y=b lies above or below x-axis, according as b is positive or negative.



- Point slope form
- Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$

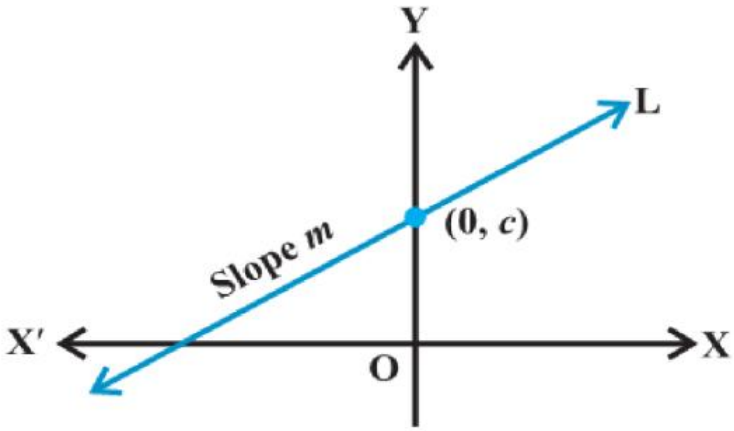


- Two point form



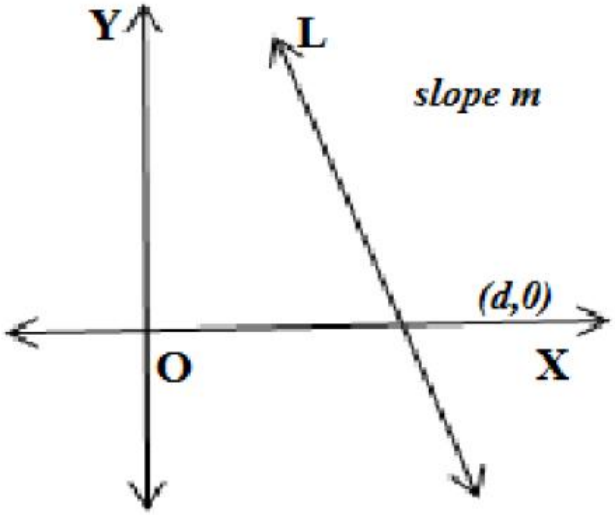
- Equation of a line passing through given points (x_1 , y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Slope intercept form(y - intercept)
- Equation of a line having slope m and y-intercept ‘c’ is given by

  y = mx + c



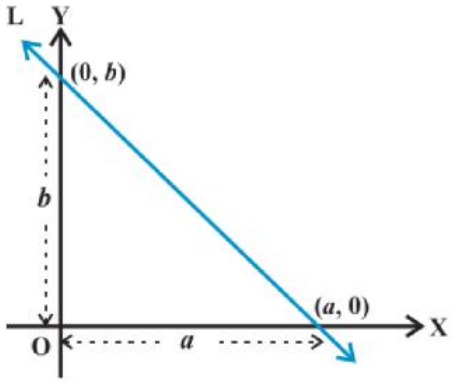
- (x –intercept)
- Equation of a line having slope m and y-intercept c is given by

  y = m (x–d)



- Intercept form.
- Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

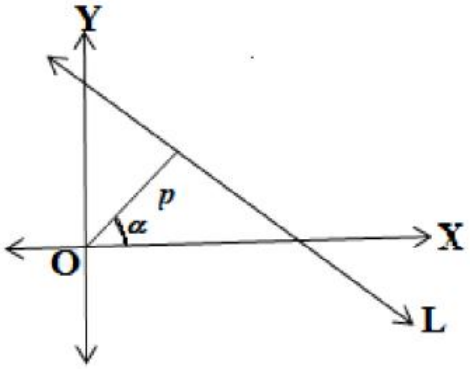
  $\frac{x}{a} + \frac{y}{b} = 1$



- Normal Form
- Equation of line in normal form is given by x cosα + y sinα = p,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis



- General Equation of a line
- Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non zero.

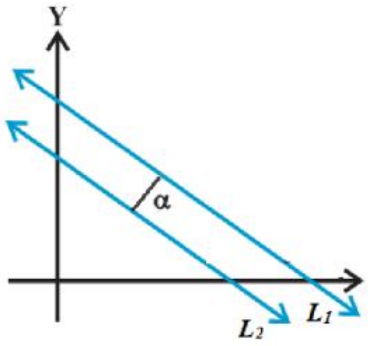
  Slope $= \frac{-A}{B}$ and y-intercept $= \frac{-C}{B}$ x-intercept $= \frac{-C}{A}$

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

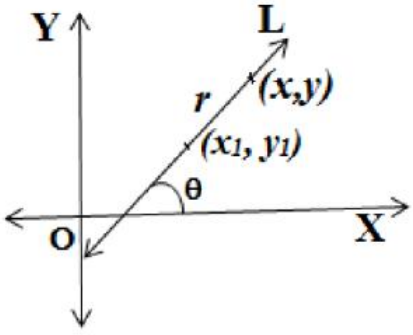
$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$



- Symmetrical (or distance) Form

  A straight line passing through the point (x_1, y_1) and inclination θ with x-axis is given by

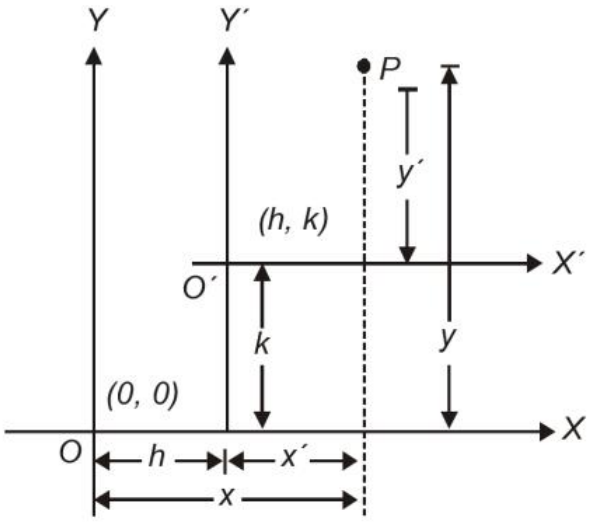
$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$



  Where r is the directed distance of any point (x, y) from the point (x_1, y_1)

- Shifting of Origin
- Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



If the origin is shifted to the point (h,k), then new coordinates (x^{1}, y^{1}) and the original coordinates (x,y) of a point are related to each other by the relation

x' = x – h, y' = y – k

- Equation of family of lines parallel to Ax + By + C = 0 is given by Ax + By + k = 0, for different real values of k
- Equation of family of lines perpendicular to Ax + By + C = 0 is given by Bx – Ay + k = 0, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

Important Questions with Solutions

Section – A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.

2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?

3. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.

4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?

5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.

6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.

7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

8. Find k so that the line 2x + ky – 9 = 0 may be perpendicular to2x + 3y – 1 = 0

9. Find the acute angle between lines x + y = 0 and y = 0

10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

11. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

Section – A

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

12. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

13. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

14. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

15. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

16. If the medians through A and B of the triangle with vertices A (0, b) , B (0, 0) and C (a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. If the image of the point (3, 8) in the line px + 3y – 7 = 0 is the point (–1, –4), then find the value of p.

18. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines x + 2y = 5 and x – 3y + 5 = 0

19. The line 2x – 3y = 4 is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

प्रसार हेतु निःशुल्क

20. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

21. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

22. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

23. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

24. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)?

25. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?

26. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and –6 respectively.

27. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

28. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

29. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

30. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

31. The vertices of a Δ are (6,0), (0,6) and (6,6). Find the distance between its circumcentre and centroid.

Section-D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

32. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

33. One side of a rectangle lies along the line 4x + 7y + 5 = 0. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

34. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

35. Find the equations of the straight lines which cut off intercepts on x–axis twice that on y–axis and are at a unit distance from origin.

36. Two adjacent sides of a parallelogram are 4x + 5y = 0 and 7x + 2y = 0. If the equation of one of the diagonals is 11x +7y = 4, find the equation of the other diagonal.

37. A line is such that its segment between the lines $5x - y + 4 = 0$ and$3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

38. If one diagonal of a square is along the line 8x – 15y = 0 and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

39. If the slope of a line passing through te point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

40. Find the equation of straight line which passes through the intersection of the straight line 3x + 2y + 4 = 0 and x – y – 2 = 0 and forms a triangle with the axis whose area is 8 sq. unit.

41. Find points on the line x + y + 3 = 0 that are at a distance of 5 unitsfrom the line x + 2y + 2 = 0

42. Show that the locus of the midpoint of the distance between the axes of the variable line $x\cos\alpha + y\sin\alpha = p$ is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$, where p is a constant.

43. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ where c is constant. Show that the locus of the foot of perpendicular from the origin to the given line is $x^2+y^2=c^2$.

44. A point p is such that the sum of squares of its distance from the axes of coordinates is equal to the square of its distance from the line x –y =1. Find the locus of P.

45. A straight line L is perpendicular to the line 5x – y =1. The area of the Δ formed by the line L and the coordinate axes is 5. Find the equation of the line L.

46. The vertices of a Δ are $\left[at_1t_2,\ a(t_1+t_3)\right],\left[at_2t_3,\ a(t_2+t_3)\right]$. Find the orthocentre of the Δ.

47. Two equal sides of an isosceles Δ are given by the equation 7x – y + 3 =0 & x + y –3 = 0 & its third side passes through the point (1,–10). Determine the equation of the third side.

48. Let A (2,–3) & B (–2,1) be the vertices of a Δ ABC. If the centroid of this triangle moves on the line 2x + 3y = 1. Then find the locus of the vertex C.

49. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1,1) & (2,–1) respectively. Then find the coordinates of A.

50. Find the area enclosed within the curve $|x|+|y|=1$.

51. If the area of the triangle formed by a line with coordinates axes is $54\sqrt{3}$ square units and the perpendicular drawn from the origin to the line makes an angle 60° with the x-axis, find the equation of the line.

52. Find the coordinators of the circumcentre of the triangle whose vertices are (5,7), (6,6),& (2, -2).

53. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

54. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2}+b^{-2}=p^{-2}+q^{-2}$.

ANSWERS

1. (1, 2)
2. k = 3
3. p = 2, q = –5
4. 3 : 2 (internally)
6. y = *x* – 1 and *y* = – *x* – 1.
7. *x* + *y* = 7
8. $\frac{4}{3}$
9. $\frac{\pi}{4}$
10. $\frac{2\pi}{3}$
11. (–3, –1)
12. p = –3, q = –3
13. *y* + 3 = 0
14. No
15. $\left(1,\frac{7}{3}\right)$

17. 1

18. $\frac{10}{\sqrt{26}}$

19. (1, –5)

20. c = –4, (2,0), (4, 4)

21. 49 square units

22. $x + y + 5\sqrt{2} = 0,\ x + y - 5\sqrt{2} = 0$

23. 4 square units

24. $y = 9$

25. 1 : 2

26. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$

27. 4 square units

28. (–4, –3)

29. x + 2y = 1

30. x + y – 5 = 0

31. $3\sqrt{2}$

32. 3x – 4y + 15 = 0

33. 4x + 7y – 11 = 0, 7x – 4y + 25 = 0

    7x – 4y – 3 = 0

34. x – 2y + 3 = 0, 2x + y – 14 = 0,

$x - 2y + 13 = 0,\ 2x + y - 4 = 0$

$3x - y - 1 = 0,\ x + 3y - 17 = 0$

35. $x + 2y + \sqrt{5} = 0,\ x + 2y - \sqrt{5} = 0$

36. $x = y$

37. $107x - 3y - 92 = 0$

38. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

39. $(-1, -1)$ or $(7, 5)$

40. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$

41. $(1,-4),\ (-9,6)$

44. $x^2 + y^2 + 2xy + 2x - 2y - 1 = 0$

45. $x + 5y = \pm 5\sqrt{2}$

46. $\left(-a, a(t_1 + t_2 + t_3 + t_1 t_2 t_3\right)$

47. $x - 3y - 31 = 0$
$3x + y + 7 = 0$

48. $\frac{x}{-2} + \frac{y}{1} = 1$

49. $\left(1, \frac{-3}{2}\right)$ *or* $\left(3, \frac{-1}{2}\right)$

50. $\sqrt{3}$

51. $x + \sqrt{3}y = 18$

52. $(2,3)$

53. $3x - 4y - 3a = 0$ *and* $x - a = 0$