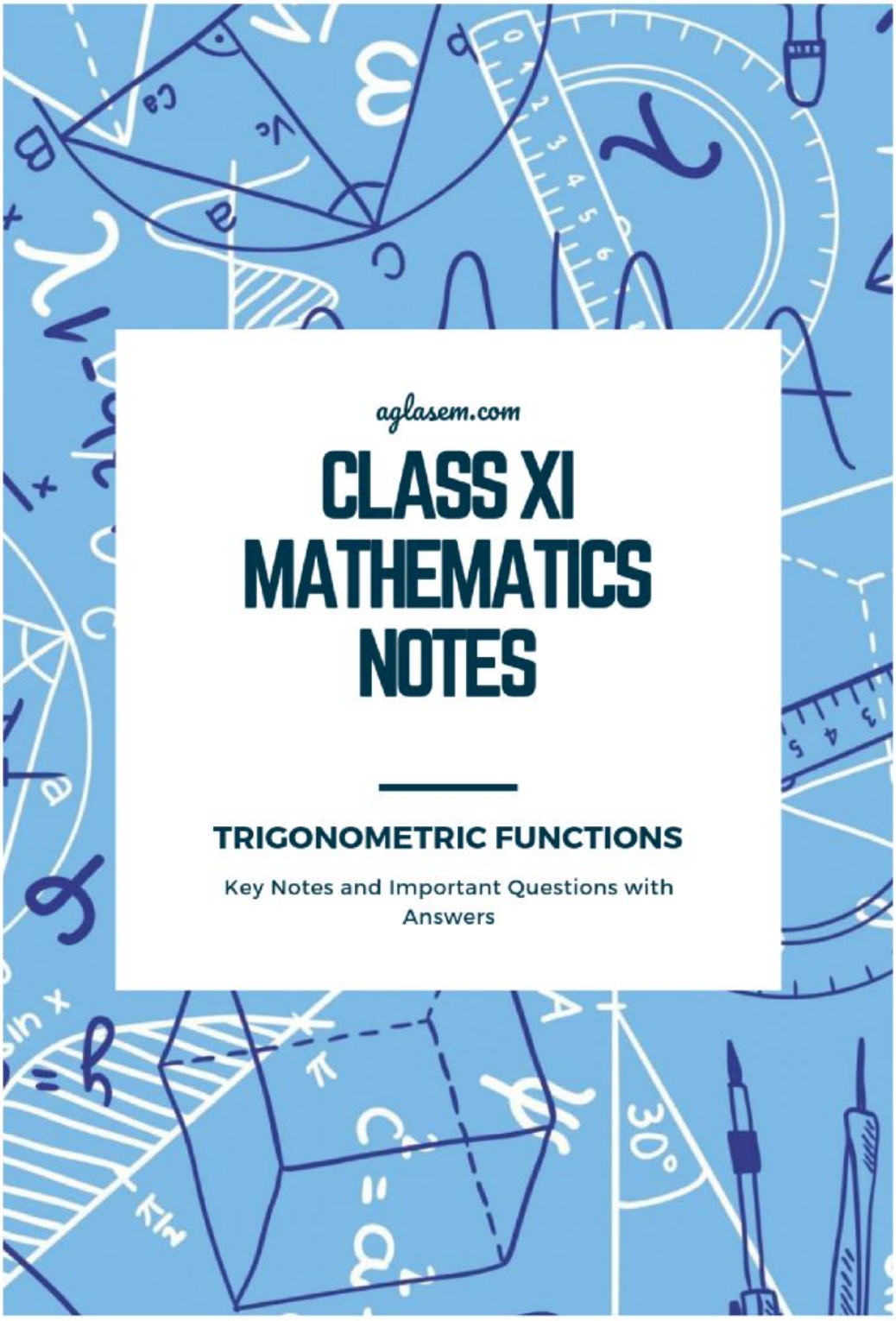

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CLASS XI
MATHEMATICS
NOTES
TRIGONOMETRIC FUNCTIONS
Key Notes and Important Questions with Answers

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by 1^c.
- π radian = 180 degree, 1° = 60'

  1 radian = $\frac{180}{\pi}$ degree and 1' = 60''

  1 degree = $\frac{\pi}{180}$ radian
- If an arc of length l makes an angle qradian at the centre of a circle of radius r, we have

  $\theta = \frac{l}{r}$

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Quadrant →	I	II	III	IV
t- functions which are positive	All	sin x cosec x	tan x cot x	cos x sec x

Function	–x	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$	$\pi - x$	$\pi + x$	$2\pi - x$	$2\pi + x$
sin	–sin x	cos x	cos x	sin x	–sin x	–sin x	sin x
cos	cos x	sin x	–sin x	–cos x	–cos x	cos x	cos x
tan	–tan x	cot x	–cot x	–tan x	tan x	–tan x	tan x
cosec	–cosec x	sec x	sec x	cosec x	–cosec x	–cosec x	cosec x
sec	sec x	cosec x	–cosec x	–sec x	–sec x	sec x	sec x
cot	–cot x	tan x	–tan x	–cot x	cot x	–cot x	cot x

Function	Domain	Range
sin x	R	[–1,1]
cos x	R	[–1,1]
tan x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R
Cosec x	$R - \{n\pi; n \in z\}$	R – (–1,1)
Sec x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R – (–1,1)
cot x	$R - \{n\pi, n \in z\}$	R

Some Standard Results

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$

  $\cos(x + y) = \cos x \cos y \quad \sin x \sin y$

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

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$$\cot(x+y) = \frac{\cot x.\cot y-1}{\cot y+\cot x}$$

- $\sin(x \quad y) = \sin x \cos y \quad \cos x \sin y$

  $\cos(x \quad y) = \cos x \cos y + \sin x \sin y$

$$\tan(x \ y) = \frac{\tan x - \tan y}{1+\tan x.\tan y}$$

$$\cot(x \ y) = \frac{\cot x.\cot y+1}{\cot y-\cot x}$$

- $$\tan(x+y+z) = \frac{\tan x+\tan y+\tan z-\tan x\tan y\tan z}{1-\tan x\tan y-\tan y.\tan z-\tan z\tan x}$$

- $2\sin x \cos y = \sin(x+y) + \sin(x \quad y)$

  $2\cos x \sin y = \sin(x+y) \quad \sin(x \quad y)$

  $2\cos x \cos y = \cos(x+y) + \cos(x \quad y)$

  $2\sin x \sin y = \cos(x \quad y) \quad \cos(x+y)$

- $$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

  $$\sin x \quad \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

  $$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

  $$\cos x \quad \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)$$

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- $\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$
- $\cos 2x = \cos^2 x \quad \sin^2 x = 2 \cos^2 x \quad 1 = 1 \quad 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
- $\sin 3x = 3 \sin x \quad 4 \sin^3 x$
- $\cos 3x = 4 \cos^3 x \quad 3 \cos x$
- $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$
- $\sin(x + y) \sin(x \quad y) = \sin^2 x \quad \sin^2 y = \cos^2 y \quad \cos^2 x$
- $\cos(x + y) \cos(x \quad y) = \cos^2 x \quad \sin^2 y = \cos^2 y \quad \sin^2 x$
- General solution A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.
- Principal solutions The solutions of a trigonometric equation which lie in $[0, 2\pi)$ are called its principal solutions.
- $\sin \theta = 0 \Rightarrow \theta = n\pi$, where $n \in z$
- $\cos \theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$, where $n \in z$
- $\tan \theta = 0 \Rightarrow \theta = n\pi$, where $n \in z$
- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (\ 1)^n \alpha$, where $n \in z$
- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$, where $n \in z$
- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$, where $n \in z$

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$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}; \qquad \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}; \qquad \cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

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Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the radian measure of 5° 37' 30".

2. Write the value of $\left(\frac{11}{16}\right)^c$.

3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$

4. What is the value of sin (1125°)?

5. What is the radian measure of 37° 30'?

6. Write the general salection of $\sin\left(x+\frac{\pi}{12}\right)=0$

7. Write the value of 2sin 75° sin 15°?

8. What is the maximum value of 3 7 cos 5*x?*

9. Express sin 12θ + sin 4θ as the product of sines and cosines.

10. Express 2 cos 4*x* sin 2*x* as an algebric sum of sines and cosines

11. Write the maximum value of cos (cos *x*).

12. Write the minimum value of cos (cos *x*).

13. Write the radian measure of 22° 30'

14. Write the value of $\tan\frac{\pi}{12}$

15. Write the value of $\tan\frac{\pi}{8}$

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SHORT ANSWER TYPE QUESTIONS (2 MARKS)

16. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°.

17. If $\sin A \quad \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find $\cos A$, $\sin 2A$

18. What is the sign of cos x/2 sin x/2 when

    (i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$

19. Prove that $\cos 510° \cos 330° + \sin 390° \cos 120° = 1$

20. If $A + B = \pi/4$ from that $(1 + \tan A)(1 + \tan B) = 2$

21. Find the maximum and minimum value of $7 \cos x + 24 \sin x$

22. Evaluate $\sin(\pi + x) \sin(\pi \quad x) \operatorname{cosec}^2 x$

23. Find the angle in radians between the hands sof a clock at 7 : 20 pm.

24. If $\cot\alpha = \frac{1}{2}$ $\sec\beta = \frac{-5}{3}$ where $\pi < \alpha < 3\pi/2$ and $\frac{}{2}$. Find the value of $\tan(\alpha + \beta)$

25. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of $\cos x/2$, $\tan x/2$

26. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

27. Find angle in radians between the hands of a clock at 7:20 pm.

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28. A hourse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

29. Find the minimum and maximum value of

$\text{Sin}^4 x + \cos^2 x; x \in R.$

30. Solve $\sec x.\ \cos 5x + 1 = 0$

31. Solve $2\tan^2 x + \sec^2 x = 2$ for $0 < x^2\pi$.

32. Solve $\sqrt{3}\cos x \quad \sin x \quad 1.$

33. Solve $\sqrt{2}\sec \quad \tan \quad \sqrt{3}.$

34. Solve $3\tan x + \cot x = 5\ \text{cosec}\ x.$

35. Find x if $3\tan(x \quad 15°) = \tan(x + 15°)$

36. Solve $\tan x \quad \tan 2x \quad \sqrt{3}\tan x.\tan 2x \quad \sqrt{3}.$

37. Solve $\tan x \quad \sec x \quad \sqrt{3}.$

38. If $\sec x \quad \sqrt{2}$ and $\frac{3}{2} \quad x \quad 2$, find the value of $\frac{1 \quad \tan x \quad \text{cosec}\ x}{1 \quad \cot x \quad \text{cosec}\ x}$.

39. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$.

40. If $f(x) = \frac{\cot x}{1 \quad \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha).f(\beta)$.

41. Prove that $\tan 70° = \tan 20° + 2\tan 50°$

42. Prove that $\tan 13x = \tan 4x + \tan 9x + \tan 4x \tan 9x \tan 13x.$

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Prove the following Identities

43. $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta.\cos 4\theta$.

44. $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$.

45. $\dfrac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x.\sin x + \cos 6x.\cos x} = \tan 2x$.

46. $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$.

47. $\tan\alpha.\ \tan(60° - \alpha).\tan(60° + \alpha) = \tan 3\alpha.$

48. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$.

49. $\dfrac{\cos x}{1 - \sin x} = \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$.

50. $\cos 10° + \cos 110° + \cos 130° = 0.$

51. $\dfrac{\sin(x+y) - 2\sin x + \sin(x-y)}{\cos(x+y) - 2\cos x + \cos(x-y)} = \tan x$.

52. $\sin x + \sin 2x + \sin 4x + \sin 5x = 4\cos\dfrac{x}{2}.\cos\dfrac{3x}{2}.\sin 3x$.

53. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

54. Find the value of $\sqrt{3}$ cosec 20° sec 20°

55. $\cos\dfrac{\pi}{5}.\cos\dfrac{2\pi}{5}.\cos\dfrac{4\pi}{5}.\cos\dfrac{8\pi}{5} = \dfrac{1}{16}$.

56. $\cos 20°.\cos 40°.\cos 80° = \dfrac{1}{8}$.

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Find the general solution of the following equations
(Q.No. 57 to Q. No. 59)

57. $\sin 7x = \sin 3x$.

58. $\cos 3x \quad \sin 2x = 0$.

59. $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

60. Draw the graph of cosx, sin x and tan x in [0, 2π].

61. Draw sin x, sin 2x and sin 3x on same graph and with same scale.

62. Evaluate:

(ii) Cos 36° (ii) $\tan\left(\frac{13\pi}{12}\right)$

63. Evaluate:

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right).$$

64. If $\tan A \quad \tan B = x$, $\cot B \quad \cot A = y$ prove that $\cot(A \quad B) = \frac{1}{x}+\frac{1}{y}$

65. If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$ then prove that $\frac{\tan x}{\tan y}=\frac{a}{b}$.

66. If $\cos x = \cos\alpha.\cos\beta$ then prove that

$$\tan\left(\frac{x+\alpha}{2}\right).\tan\left(\frac{x-\alpha}{2}\right)=\tan^2\frac{\beta}{2}$$

67. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then prove that

$$\cos\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}.$$

68. If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$ then prove that

$$\cos 2(\alpha-\beta)-4ab\cos(\alpha-\beta)=1-2a^2-2b^2.$$

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69. Find the range of 5 sin x 12 cos x + 7.

70. If α and β are the solution of the equation, $a\tan\theta + b\sec\theta = c$ then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2 - c^2}$.

71. Prove that

$$\cos^2 x + \cos^2 y - 2\cos x.\cos y.\cos(x+y) = \sin^2(x+y)$$

72. Prove that

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

73. Solve

$$81^{\sin^2 x} + 81^{\cos^2 x} = 30 \quad 0 < x < \pi$$

74. Find the minimum value of p for which $\cos(p\sin x) = \sin(p\cos x)$ has a solution in $[0, 2\pi]$.

75. Prove that

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16.\sin A}.$$

76. Solve:

$4\sin x.\ \sin 2x.\ \sin 4x = \sin 3x$

77. Solve:

$\cos\theta\ \cos 2\theta\ \cos 3\theta = \frac{1}{4}$

78. Evaluate:

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

79. Prove that

$$4\sin\alpha.\sin\left(\alpha+\frac{\pi}{3}\right).\sin\left(\alpha+\frac{2\pi}{3}\right) = \sin 3\alpha.$$

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Answers

1. $\left(\frac{\pi}{32}\right)^C$

2. 39° 22' 30"

3. $\sqrt{3}$

4. $\frac{-1}{\sqrt{2}}$

5. $-\left(\frac{5\pi}{24}\right)^C$

6. $n \quad \overline{12}$

7. $\frac{1}{2}$

8. 10

9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x \quad \sin 2x$

11. 1

12. $\cos 1$

13. $\frac{\pi}{8}$

14. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

15. $\sqrt{2}-1$

16. 70m

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17. $\frac{-4}{5}, \frac{-24}{25}$

18. (i) + ve (ii) ve

21. Max value 25; Min value 25

22. 1

23. $\frac{5\pi}{9}$

24. $\frac{2}{11}$

25. $-1/\sqrt{3}, -2$

26. $\pi/4$

27. $\frac{5\pi}{9}$

28. 70 m

29. $\min = \frac{3}{4}, \quad \max = 1$

30. $x = (2n+1)\frac{\pi}{6}, \quad or \quad x = (2n+1)\frac{\pi}{4}, n \in z$

31. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

32. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, \quad n \in z$

33. $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}, \quad n \in z$

34. $2n\pi \pm \frac{\pi}{3}, \quad n \in z$

35. $x = \frac{n\pi}{2} + (-1)^4 \frac{\pi}{4}, \quad n \in z$

36. $\frac{n\pi}{3} + \frac{\pi}{9}$

37, $2n\pi + \frac{\pi}{6}$

38. 1

40. $\frac{1}{2}$

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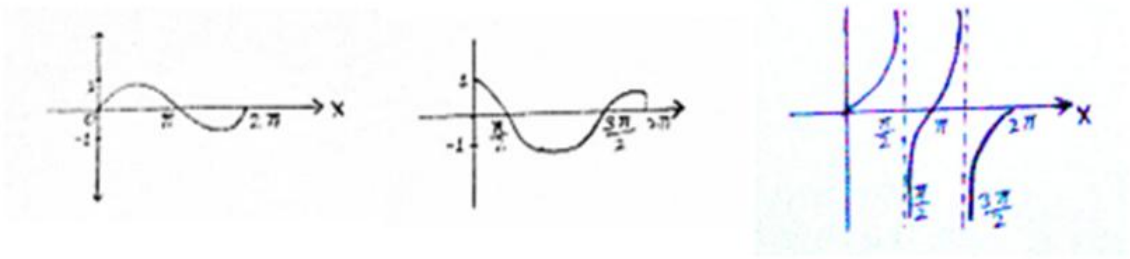
54. 4

57. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, \quad n \in z$

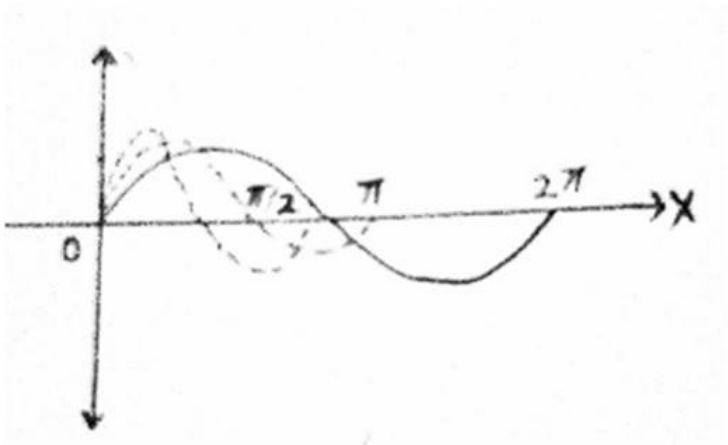
58. $\frac{1}{5}(2n\pi + \frac{\pi}{2}), 2n\pi - \frac{\pi}{2}, \quad n \in z$

59. $x = \frac{n\pi}{2}, \quad n \in z$

60.



61.



62. (i) $\frac{1+\sqrt{5}}{4}$

(ii) $2-\sqrt{3}$

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63. $\frac{3}{2}$

69. $[\ 6,20]$

73. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

74. $\frac{\pi}{\sqrt{8}}, \frac{5\sqrt{2}\pi}{4}$

76. $n\pi,\ n\pi \pm \frac{\pi}{3},\ n \in z$

77. $(2n+1)\frac{\pi}{8},\ n\pi \pm \frac{\pi}{3},\ n \in z$

78. $\frac{1}{8}$

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