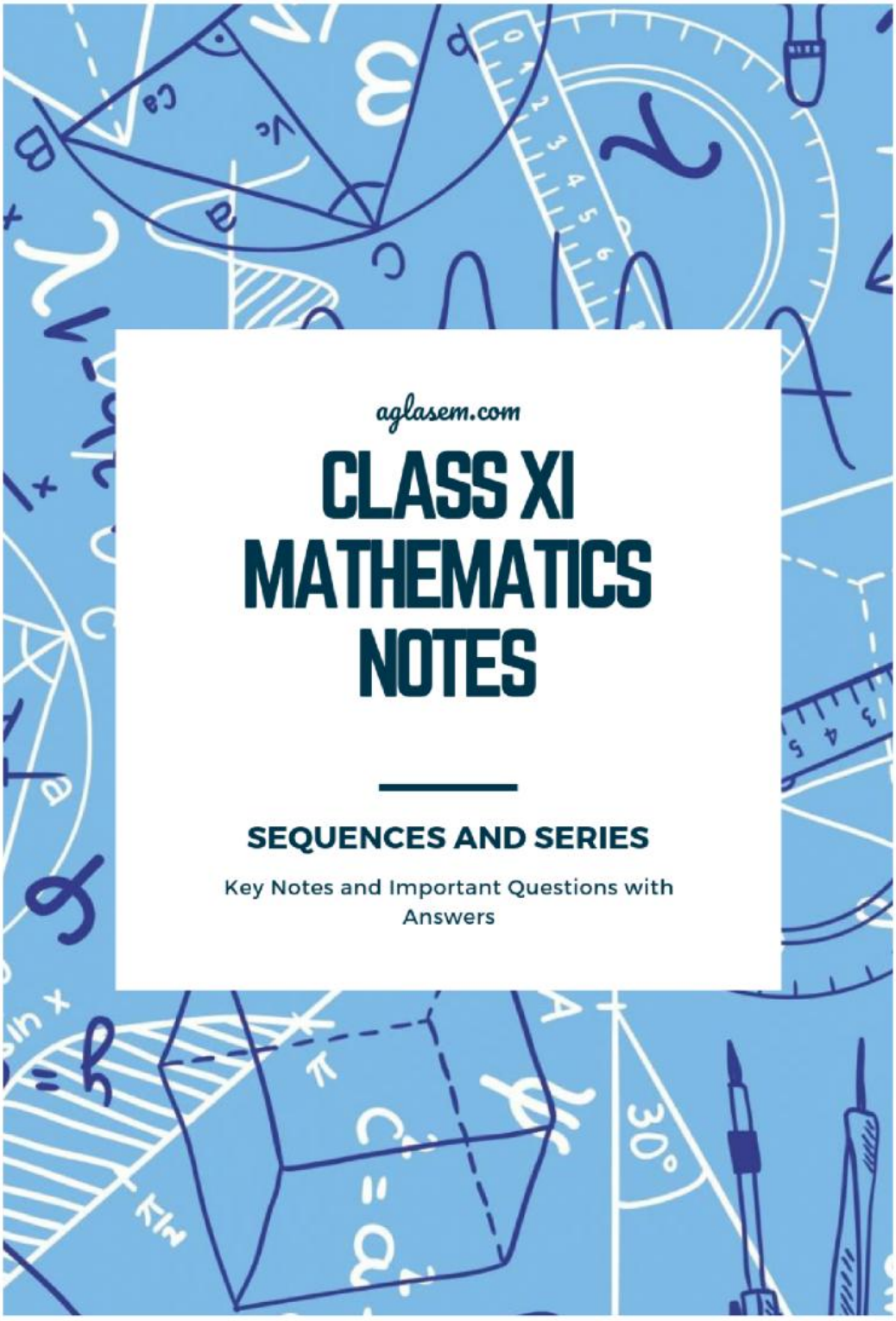


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# CLASS XI MATHEMATICS NOTES

## SEQUENCES AND SERIES

Key Notes and Important Questions with Answers

# CHAPTER - 9

# SEQUENCES AND SERIES

## KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- A sequence whose range is a subset of R is called a real sequence.
- A sequence is said to be a progression if the term of the sequence can be expressed by some formula
- A sequence is called an arithmetic progression if the difference of a term and previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=$d$) for all $n \in N$.
- The term 'series' is associated with the sequence in following way

  Let $a_1, a_2, a_3$ ..... be a sequence. Then, the expression $a_1 + a_2 + a_3$ + ..... is called series associated with given sequence.
- A series is finite or infinite according as the given sequence is finite or infinite.
- General A.P. is,

  a, a + d, a + 2d, .....
- $a_n = a + (n - 1)d = n^{th}$ term of A.P. = $l$
- $S_n$ = Sum of first n terms of A.P.

$= \frac{n}{2}[a + l]$ where $l$ = last term. N

$= \frac{n}{2}[2a + (n-1)d]$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P.

  ak, bk, ck are also in A.P., k ≠ 0, $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where k ≠ 0.

- Three numbers in A.P.

  a – d, a, a + d

- Arithmetic mean between a and b is $\frac{a + b}{2}$.

- If $A_1$, $A_2$, $A_3$, .....$A_n$ are n numbers inserted between a and b, such that the resulting sequence is A.P. then,

  $A_n = a + n.\frac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$

- $a_m = n$, $a_n = m \Rightarrow a_r = m + n - r$

- $S_m = S_n \Rightarrow S_{m+n} = 0$

- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term

- If three terms of A.P. are to be taken then we choose then as a – d, a, a + d.

- If four terms of A.P. are to be taken then we choose then as a – 3d, a – d, a + d, a + 3d.
- If five terms of A.P are to be taken, then we choose then as:

  $a-2d,\ a-d,\ a,\ a+d\ \ a+2d$ .
- G.P. (Geometrical Progression)

  a, ar, $ar^2$, .....(General G.P.)

  $a = ar^{n-1}$

  $$S_n = \frac{a\left(r^n - 1\right)}{r-1}, \qquad r \neq 1$$
- Geometric mean between a and b is $\sqrt{ab}$
- Reciprocals of terms in GP always form a G.P.
- If $G_1, G_2, G_3, \ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}} \quad 1 \le k \le n$$
- If three terms of G.P. are to be taken, then we those then as

  $\frac{a}{r}$, a, ar.
- If four terms of G.P. are to be taken, then we choose then as

  $\frac{a}{r^3}, \frac{a}{r}$, ar, $ar^3$.
- If a, b, c are in G.P. then ak, bk, ck are also in G.P. where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. wherek ≠ o..

- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.
- Sum of infinite G.P. is possible if | r | < 1 and sum is given by $\frac{a}{1-r}$
- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
- $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $S_\infty = a + ar + ar^2 + ar^3 + .....\infty$ term if –1 < r <1 $\Rightarrow S_\infty = \frac{a}{1-r}$.

  Such that –1 < r <1 or $|r| < |$
- If a, b, c are in A.P. then 2b = a+c.
- If a, b, c are in G.P. then $b^2 = ac$
- If A and G be A.M and G.M of two given positive real number 'a' and 'b' respectively then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $A \geq G$.

# Important Questions with Solutions

## Section - A

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If $n^{th}$ term of an A.P. is $6n - 7$ then write its $50^{th}$ term.

2. If $S_n = 3n^2 + 2n$, then write $a_2$

3. Which term of the sequence,

   3, 10, 17, ..... is 136?

4. If in an A.P. $7^{th}$ term is 9 and $9^{th}$ term is 7, then find $16^{th}$ term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its $n^{th}$ term.

6. Which term of the G.P.

   $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots\ldots\ldots\ldots$ is $\frac{1}{1024}$ ?

7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, \ldots\ldots, 128\sqrt{2}$, find the $4^{th}$ term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, \ldots\ldots$

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ ..... upto infinity.

12. Write the value of $0.\overline{3}$

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

## Section - B

### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

14. Write the $n^{th}$ term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + .....$

15. Find the number of terms in the A.P. 7, 10, 13, ....., 31.

16. In an A.P.,

    8, 11, 14, ..... find $S_n - S_{n-1}$

17. Find the number of squares that can be formed on chess board?

18. Find the sum of given terms:-

    (a) 81 + 82 + 83 ..... + 89 + 90

    (b) 251 + 252 + 253 + ..... + 259 + 260

19. (a) If a, b, c are in A.P. then show that 2b = a + c.

    (b) If a, b, c are in G.P. then show that $b^2 = a.c.$

20. If a, b, c are in G.P. then show that $a^2+b^2, ab+bc, b^2+c^2$ are also in G.P.

## Section - C

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

21. Find the least value of n for which

    $1+ 3 + 3^2 + ... + 3^{n-1} > 1000$

25. Find the sum of the series

    $(1+ x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + ...$

23. Write the first negative term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots$

24. Determine the number of terms in A.P. 3, 7, 11, ..... 407. Also, find its $11^{th}$ term from the end.

25. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

26. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

27. Find the sum of the sequence,

    72 + 70 + 68 + …. + 40

28. If in an A.P

    $\frac{a_7}{a_{10}} = \frac{5}{7}$ find $\frac{a_4}{a_7}$.

29. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

30. Solve: 1 + 6 + 11 + 16 + ..... + x = 148

31. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their $10^{th}$ terms.

32. If the $I^{st}$, $2^{nd}$ and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

33. If $\frac{b+c-2a}{a}\frac{c+a-2a}{b}\frac{a+b-2c}{c}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term] abc

34. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

35. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

36. Find the sum to infinity of the series:

$$1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+.....\infty$$

37. If $A = 1 + r^a + r^{2a} + .....$ up to infinity, then express r in terms of 'a' & 'A'.

38. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + ...

39. If $x=a+\frac{a}{r}+\frac{a}{r^2}+.....+\infty;\ y=b-\frac{b}{r}+\frac{b}{r^2}-.....\infty$ and

$z=c+\frac{c}{r^2}+\frac{c}{r^4}+.....\infty$ prove that $\frac{xy}{z}=\frac{ab}{c}$.

40. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

41. Prove that $0.00\bar{3}1=\frac{7}{225}$

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

42. If log2, log($2^n$–1) and log ($2^n$+3) are in A.P. Show that $n=\frac{\log 5}{\log 2}$

43. If a, b, c are in G.P. that the following are also in G.P.

(i) $a^2, b^2, c^2$

(ii) $a^3, b^3, c^3$

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

44. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b+c, c+a, a+b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

45. If the numbers $a^2, b^2, c^2$ are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

46. Show that: $0.3\overline{56} = \frac{353}{990}$

## Section-D

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

47. Prove that the sum of n numbers between a and b such that the resultingseries becomes A.P. is $\frac{n(a+b)}{2}$

48. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of

the first square is 15 cm, then find the sum of the areas of all the squares so formed.

49. If a, b, c are in G.P., then prove that

$$\frac{1}{a^2-b^2}-\frac{1}{b^2-c^2}=-\frac{1}{b^2}$$

[**Hint :** Put b = ar, c = $ar^2$]

50. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

51. If a is A.M. of b and c and c, $G_1$, $G_2$, b are in G.P. then prove that

$$G_1^3+G_2^3=2abc$$

52. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + ..... upto n terms.

53. Evaluate

$$\sum_{r=1}^{10}(2r-1)^2$$

.

54. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

55. If $10^9+2(11)^1(10)^8+3(11)^2(10)^7+.....+10(11)^9=k.(10)^9.$ then find the value of k such that $k\in N$.

56. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms.

57. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

58. Show that if the positive number a, b, c are in A.P. so are the numbers $\frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

59. Find the sum of the series:- $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\frac{9}{16}\ldots\ldots\ldots\infty$ .

60. If $a_1, a_2, a_3, \ldots\ldots a_n$ are in A.P. where $a_i > 0 \forall i \in N$. then show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\frac{1}{\sqrt{a_3}+\sqrt{a_4}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

61. If the sum of first 'n' terms of an A.P. is $c.n^2$ then prove that the sum of squares of these 'n' terms is $\frac{nc^2\left(4n^2-1\right)}{3}$.

62. Let 'p' and 'q' be the roots of the equation $x^2-2x+A=0$ and let 'r' and 's' be the roots of the equation $x^2-18x+B=0$ if p<q<r<s are in A.P. then prove that A= –3 and B= 77.

63. If $S_1, S_2, S_3 \ldots S_n$ are the sums of infinite geometric series whose first terms are 1,2,3,…n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{n+1}$ respectively, then show that the value of

$$S_1^2+S_2^2+S_3^2+\ldots S_{2n-1}^2=\frac{1}{6}(2n)(2n+1)(4n+1)-1$$

64. Let $A_n=\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^4+\ldots+(-1)^{n-1}.\left(\frac{3}{4}\right)^n$ and $B_n=1-A_n$.

Find at least odd natural number $n_o$, so that $B_n > A_n \forall n \geq n_o$.

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65. If $p^{th}, q^{th}$ and $r^{th}$ terms of an A.P. and G.P. are equal and are x, y and z respectively, then prove that $x^{y-z}.y^{z-x}.z^{x-y}=1$

# ANSWERS

1. 293
2. 11
3. $20^{th}$
4. 0
5. 4n + 5
6. 12th
7. $\frac{9}{2}$
8. 64
9. 3
10. $20\left(1-\frac{1}{2^8}\right)$
11. 5
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. $\frac{2n+1}{(n+6)(n+10)^2}$
15. 9
16. 3n + 5
17. 204
18. (a) 855 (b) 2555
21. n = 7
22. $\frac{n}{1-x}-\frac{x^2(1-x^n)}{(1-x)^2}$
23. $-\frac{1}{4}$
24. 102, 367
25. 33
26. 7999

27. 952

28. $\frac{3}{5}$

29. 11

30. 36

31. 33:17

32. $\frac{(b+c-2a)(a+c)}{2(b-a)}$

34. 5, 10, 20, .....; or 20, 10, 5,

35. 18, 6, 2; or 2, 6, 18

36. 6

37. $\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$

38. $\frac{7}{81}\left[9n-1+10^{-n}\right]$

40. $\frac{15}{7}\left(2^{n}-1\right)$

48. 450 cm²

50. 16, 4

52. $\frac{n(n+1)}{3}(48n^{2}-16n-14)$

53. 1330

54. $19,\frac{38}{3},\frac{76}{9},.....$

55. k = 100

56. $n+2^{-n}-1$

57. $r=2+\sqrt{3}$

59. $\frac{2}{9}$

64. $n_0=7$