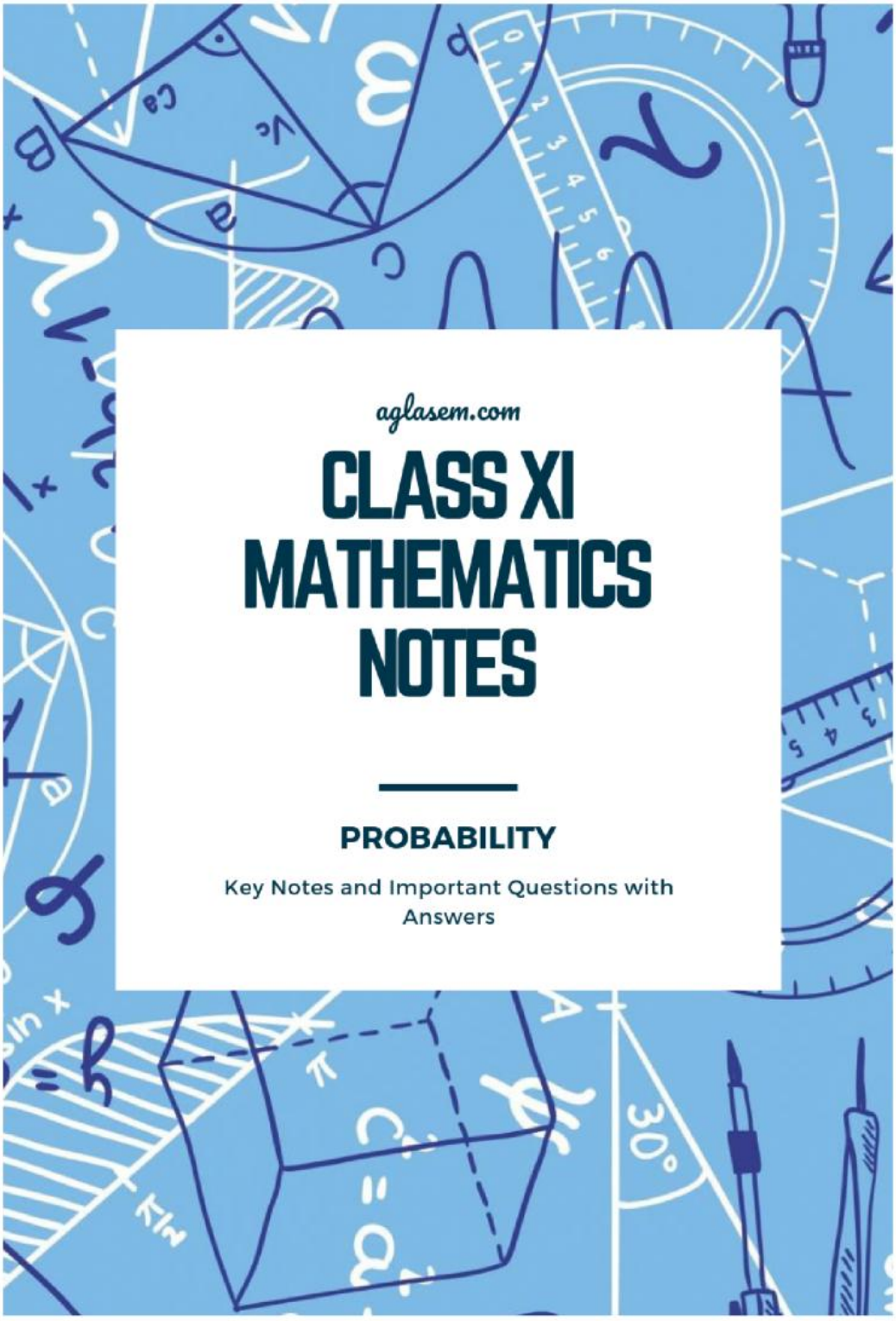


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CLASS XI MATHEMATICS NOTES

PROBABILITY

Key Notes and Important Questions with Answers

CHAPTER - 16

PROBABILITY

KEY POINTS

- **Random Experiment:** If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.

- **Sample Space:** The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.

- **Some examples of random experiments and their sample spaces**

  (i) A coin is tossed

  S = {H, T}, n(S) = 2

  Where n(S) is the number of elements in the sample space S.

  (ii) A die is thrown

  S = { 1, 2, 3, 4, 5, 6], n(S) = 6

  (iii) A card is drawn from a pack of 52 cards n (S) = 52.

  (iv) Two coins are tossed

  S = {HH, HT, TH, TT}, n(S) = 4.

  (v) Two dice are thrown

$$S=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),.....,(2,6)\\\vdots\\\vdots\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix} \qquad n(s)=36.$$

(vi) Two cards are drawn from a well shuffled pack of 52cards

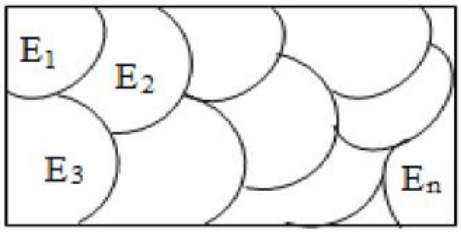
(a) with replacement n(S) = 52 × 52

(b) without replacement $n(S)={}^{52}C_2$

- **Event:** A subset of the sample space associated with a random experiment is called an event.
- **Elementary or Simple Event:** An event which has only one Sample point is called a simple event.
- **Compound Event:**An event which has more than one Sample point is called a Compound event.
- **Sure Event:** If event is same as the sample space of the experiment, then event is called sure event.
- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, $\phi$is called impossible event.
- **Exhaustive and Mutually Exclusive Events:** Events E_1, E_2, E_3……..E_n are such that

(i) $E_1 \cup E_2 \cup E_3 \cup \ldots\ldots\ldots \cup E_n = S$ then Events E_1, E_2, E_3……..E_n are called exhaustive events.

(ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events E_1, E_2, E_3……..E_n are called mutually exclusive.



Then we say that $E_1, E_2, \ldots\ldots E_n$ partitions the sample space S.

Probability of an Event: For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A

and n(S) is number of elements in set S and $0 \leq P(A) \leq 1$.

- (a) If A and B are any two events then

  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  $= P(A) + P(B) - P(A \text{ and } B)$

  (b) If A and B are mutually exclusive events, then

  $P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B)=0$ for mutually exclusive events)

  (c) $P(A) + P(\overline{A}) = 1$ or P(A) + P(not A) = 1

  (d) P (Sure event) = P(S) = 1

  (e) P (impossible event) = $P(\phi) = 0$

  (f) $P(A - B) = P(A) - P(A \cap B) = P(A \cap \overline{B})$

  (g) $P(B-A) = P(B) - P(A \cap B) = P(B \cap \overline{A})$

  (h) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

  (i) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

- Addition theorem for three events

  Let A, Band Cbe any three events associated with a random experiment, then

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C).$$

- Axiomatic Approach to Probability:

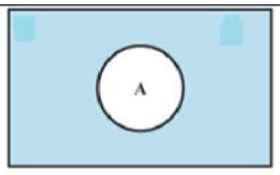
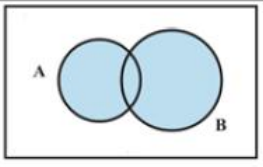
  Let S be a sample space containing elementary outcomes $w_1, w_2, \ldots\ldots\ldots w_n$

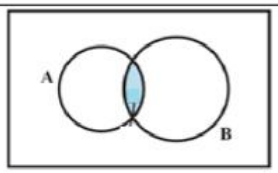
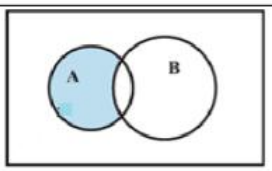
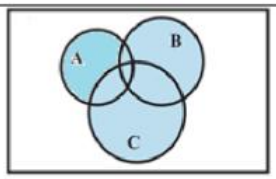
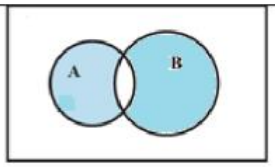
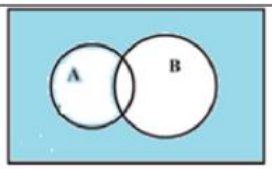
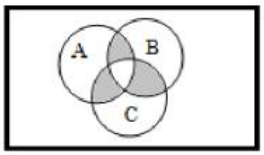
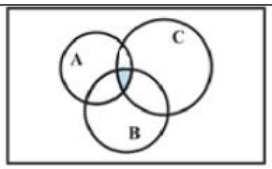
  i.e. $S=\{w_1, w_2, \ldots\ldots\ldots w_n\}$

  (i) $0\le P(w_i)\le 1$ for each $w_i \in S$

  (ii) $P(w_1)+P(w_2)+\ldots\ldots\ldots+P(w_n)=1$

  (iii) $P(A)=\sum P(w_i)$ for any event A containing elementary events w_i.

Verbal description of the event	Equivalent Set Theoretic Notation	Venn Diagram
Not A	$\overline{A}$ or A'	
A or B	$A\cup B$	

A and B	$A \cap B$	
A but not B (only A)	$A \cap \overline{B}$	
At least one of A, B or C	$A \cup B \cup C$	
Exactly one of A and B	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$	
Neither A nor B	$(A \cap \overline{B})$	
Exactly two of A, B and C	$(A \cap B \cap \overline{C}) \cup$ $(A \cap \overline{B} \cap C)$ $\cup (\overline{A} \cap B \cap C)$	
All there of A, B and C	$A \cap B \cap C$	

The cards J, Q and K are called face cards. There are 12 face cards in a deck of 52 cards.

There are 64 squares in a chess board i.e.32 white and 32 Black.

Total n digit numbers = $9 \times 10^{n-1}$, $n \geq 2$

e.g. there are $9 \times 10^2 = 900$. Three digit numbers.

Important Questions with Solutions

Section-A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Describe the Sample Space for the following experiments

(Q. No. 1 to 5)

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up.
4. A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.
6. A coin is tossed n times. Find the number of element in its sample space.
7. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?
8. What is the probability that a given two-digit number is divisible by 15?
9. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?
10. If $P(A \cup B) = P(A \cap B)$, then find relation between P(A) and P(B).
11. Is $P(A \cap B) = 0$? If A and B are exhaustive events.

Section-B

SHORT ANSWER TYPE QUESTIONS (2 MARK)

12. Let A and B be two events such that P(A) = 0.3 and P(A$\cup$B) = 0.8. find P(B) if P(A$\cap$B) = P(A) P(B).

13. Three identical dice are rolled. Find the probability that the same number appears on each of them.

14. In an experiment of rolling of a fair die.

    Let A,B and C be three events defined as under:

    A : a number which is a perfect square

    B : a prime number

    C : a number which is greater than 5.

    Is A, B, and C exhaustive events?

15. Punching time of an employee is given below:

Day	Mon	Tue	Wed	Thurs	Fri	Sat
Time(a.m)	10:35	10:20	10:22	10:27	10:25	10:40

    If the reporting time is 10:30 a.m, then find the probability of his coming late.

16. A game has 18 triangular block out of which 8 are blue and rest are red, and 19 square blocks out of which 7 are blue and rest are yellow. On piece is lost. Find the probability that it was a square of blue colour.

17. A card is drawn from a pack of 52 cards. Find the probability of getting:

(i) a jack or a queen

(ii) a king or a diamond

(iii) a heart or a club

(iv) either a red or a face card.

(v) neither a heart nor a king

(vi) neither an ace nor a jack

(vii) a face card

18. In a leap year find the probability of

(i) 53 Mondays and 53 Tuesdays

(ii) 53 Mondays and 53 Wednesday

(iii) 53 Mondays or 53 Tuesdays.

19. In a non-leap year, find the probability of

(i) 53 Mondays and 53 Tuesdays.

(ii) 53 Mondays or 53 Tuesdays.

20. Two card are drawn at random from a deck of 52 playing cards. Find the probability of drawing two kings.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

21. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I's come together.

22. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

23. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

24. Find the probability of at most two tails or at least two heads in a toss of three coins.

25. A, B and C are events associated with a random experiment such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A $\cap$ B) = 0.08 P(A $\cap$ C) =0.28and P(A$\cap$B$\cap$C) = 0.09. If P(A$\cup$B$\cup$C) $\geq$ 0.75

then prove that P(B $\cap$ C)lies in the interval [0.23, 0.48]

[***Hint:*** P (A $\cup$ B $\cup$ C) = P(A) + P(B) + P(C) – P(A $\cap$ B) – P(B $\cap$ C) – P(A $\cap$ C) + P (A $\cap$ B $\cap$ C)]

26. If $\frac{1+3p}{3}, \frac{1-p}{4}$, and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Then find the set of all values of p.

27. An urn A contains 6 red and 4 black balls and urn B contain 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. Now if one ball is drawn at random from urn A then find the probability that it is found to be red.

28. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.

29. S = {1, 2, 3, -----, 30}, A = {x : x is multiple of 7}, B = { x : x is multiple of 5}, C = {x : x is a multiple of 3}. If x is a member of S chosen at random find the probability that

    (i) $x \in A \cup B$

    (ii) $x \in B \cap C$

    (iii) $x \in A \cap C'$

30. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

31. The number lock of a suitcase has 4 wheels with 10 digits, i.e. fro 0 to 9. The lock open with a sequence of 4 digits with repeats allowed. What is the probability of a person getting the right sequence to open the suit case?

32. If A and B are any two events having P(A $\cup$ B) = $\frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$ then find the $P(\overline{A} \cap B)$.

33. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

34. A typical PIN (Personal identification number) is a sequence of any four symbol chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

35. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

36. One mapping /function is selected at random from all the mappings of the set A = {1,2,3,4,5}. Into itself. Find the probability that the mapping selected is one to one.

37. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?

38. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?

39. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

40. If A,B,C are three mutually exclusive and exhaustive events of an experiment such that 3P(A) = 2P(B) = P(C), then find the value of P(A).

ANSWERS

1. {0, 1, 2}
2. {Red, Black}
3. {T, HT, HHT, HHHT.....}
4. {HR1, HR2, HB1, HB2, HB3, TH, TT}
5. {RW, WR, WW}
6. 2^n
7. $\frac{8}{21}$
8. $\frac{1}{15}$
9. A and B are Mutually exclusive events.
10. P(A) = P(B)
11. No.
12. $\frac{5}{7}$
13. $\frac{1}{36}$
14. Yes as $A \cup B \cup C = S$
15. $\frac{1}{3}$
16. $\frac{1}{4}$

17. (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{8}{13}$

(v) $\frac{9}{13}$ (vi) $\frac{11}{13}$ (vii) $\frac{3}{13}$

18. (i) $\frac{1}{7}$ (ii) 0 (iii) $\frac{3}{7}$

19. (i) 0 (ii) $\frac{2}{7}$

20. $\frac{1}{221}$

21. $\frac{1}{5}$

22. 3

23. $\frac{23}{28}$

24. $\frac{7}{8}$

25. $0.23 \le P(B) \le 0.48$

26. $\frac{-1}{3} \le p \le \frac{1}{2}$

27. $\frac{32}{55}$

28. $\frac{4}{1155}$

29. (i) $\frac{1}{3}$ (ii) $\frac{1}{15}$ (iii) $\frac{1}{10}$

30. $\frac{3}{10}$

31. $\frac{1}{10000}$

32. $\frac{1}{6}$

33. $\frac{1}{10}$

34. $\frac{265896}{1679616}$

35. $\frac{63}{190}$

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36. $\frac{5!}{5^5} = \frac{24}{125}$

37. 120

38. $\frac{5}{12}$

39. $\frac{3}{5}$

40. $\frac{2}{11}$