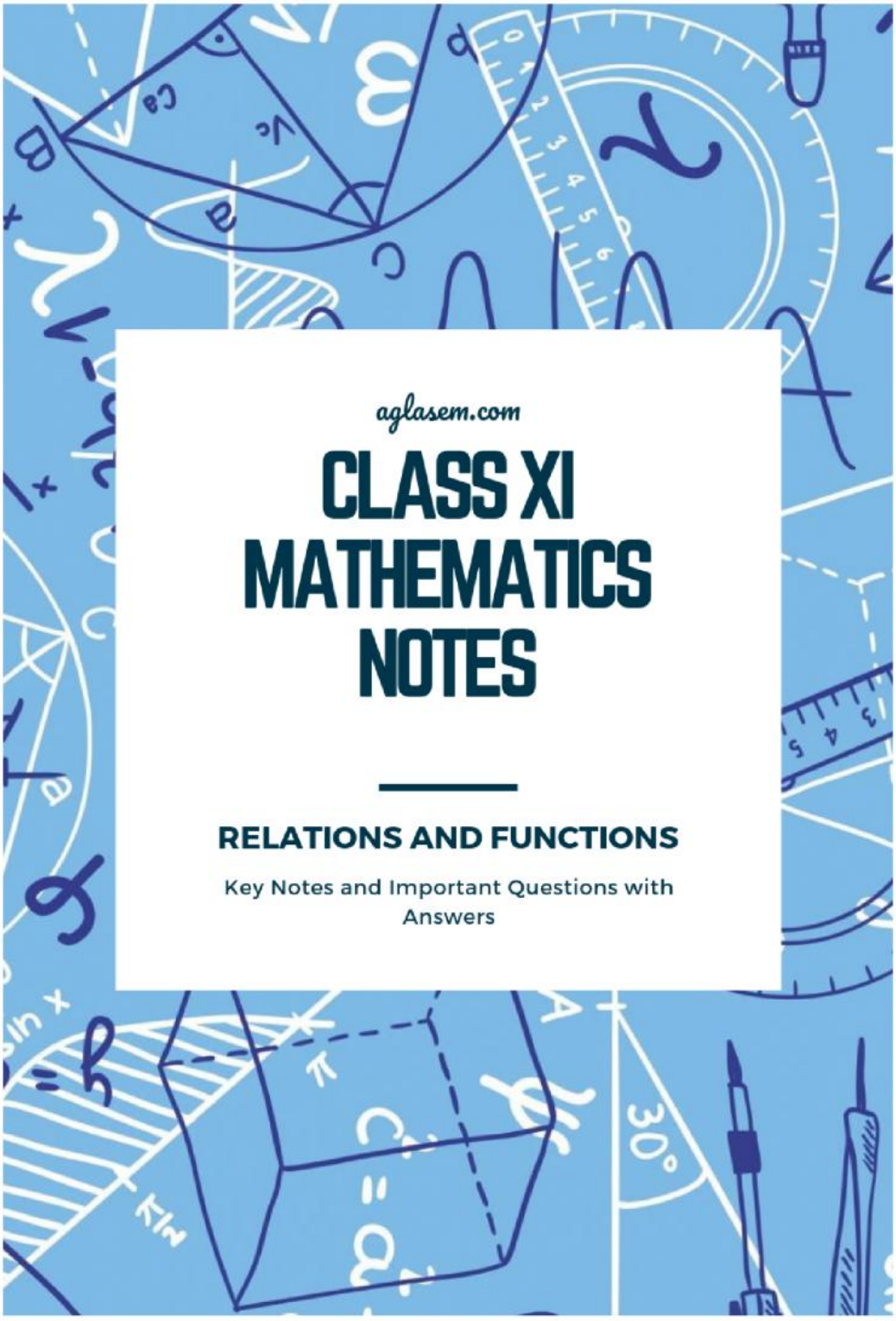


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CLASS XI MATHEMATICS NOTES

RELATIONS AND FUNCTIONS

Key Notes and Important Questions with Answers

CHAPTER – 2

RELATIONS AND FUNCTIONS

KEY POINTS

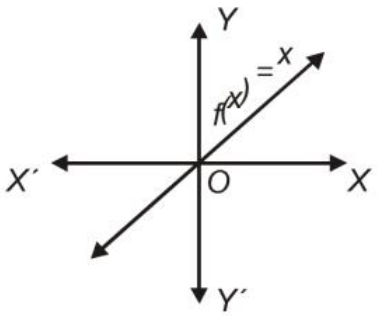
- Cartesian Product of two non-empty sets A and B is given by, $A \times B = \{ (a, b) : a \in A, b \in B\}$
- If $(a, b) = (x, y)$, then $a = x$ and $b = y$
- Relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.
- Domain of $R = \{a : (a, b) \in R\}$
- Range of $R = \{ b : (a, b) \in R\}$
- Co-domain of R = Set B
- Range $\subseteq$ Co-domain
- If $n(A) = p$, $n(B) = q$ then $n(A \times B) = pq$ and number of relations = 2^{pq}
- Image : If the element x of A corresponds to $y \in B$ under the function f, then we say that y is image of x under 'f'

  $\Rightarrow \quad f(x) = y$
- If $f(x) = y$, then x is preimage of y.
- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

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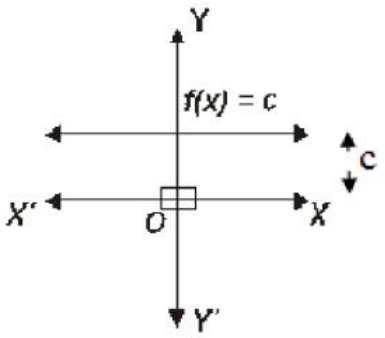
- $D_f = \{x : f(x) \text{ is defined}\}$ $R_f = \{f(x) : x \in D_f\}$
- Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.
- Identity function, $f : R \to R$; $f(x) = x \ \forall \ x \in R$, where R is the set of realnumbers.

  $D_f = R$ $R_f = R$



- Constant function, $f : R \to R$; $f(x) = c \ \forall \ x \in R$ where c is a constant

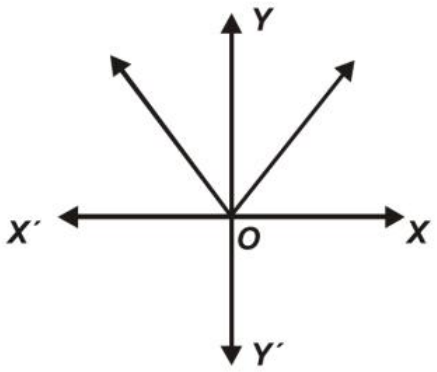
  $D_f = R$ $R_f = \{c\}$



- Modulus function, $f : R \to R$; $f(x) = |x| \ \forall \ x \in R$

  $D_f = R$

  $R_f = R^+ \cup \{0\} = \{ x : x \in R : x \geq 0\}$



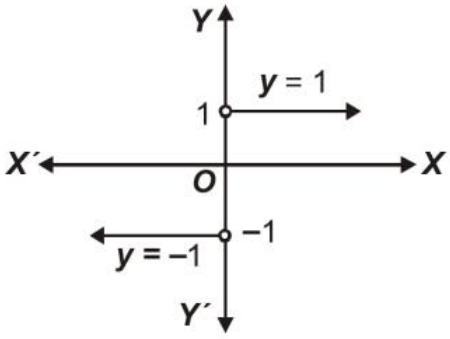
- Signum function

$$f: R \to R;\ f(x) = \begin{cases} 1, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ 1, \text{if } x < 0 \end{cases} \text{ and } f(x) = \begin{cases} \dfrac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$$

Then

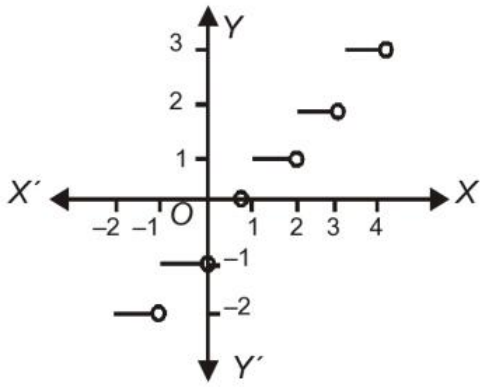
$D_f = R$

and $R_f = \{-1,0,1\}$



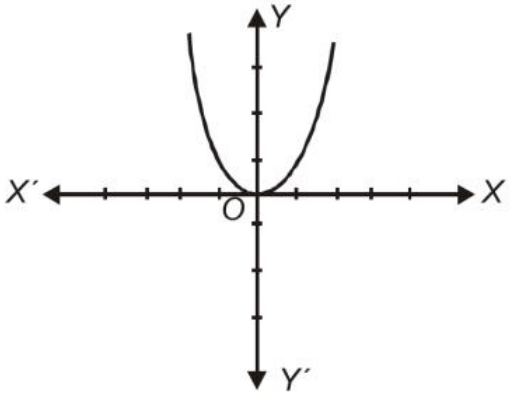
- Greatest Integer functionf : R $\to$ R; f(x) = [x], x $\in$ R assumes the value of the greatest integer, less than or equal to x

$D_f = R$ $R_f = Z$



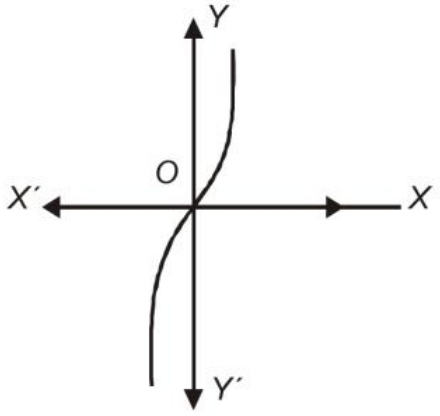
- $f : R \to R$, $f(x) = x^2$

  $D_f = R$ $R_f = [0, \infty)$



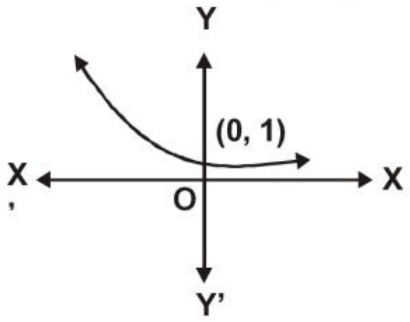
- $f : R \to R$, $f(x) = x^3$

  $D_f = R$ $R_f = R$

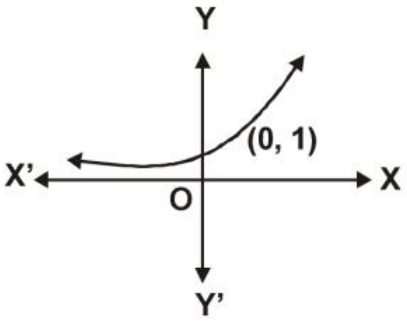


- Exponential function, $f : R \to R$; $f(x) = a^x$, $a > 0$, $a \neq 1$

$D_f = R$ $R_f = (0, \infty)$



$0 < a < 1$

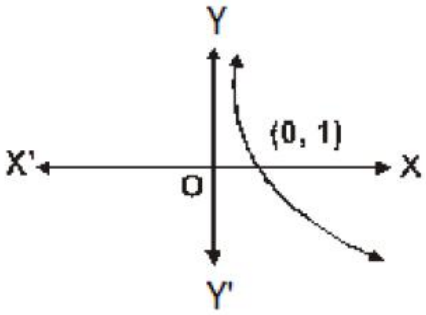


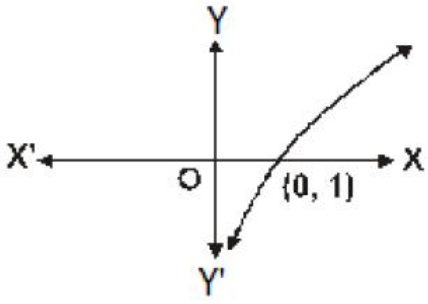
$a > 1$

- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty, 2 < e < 3$$

- Logarithmic functions, $f : (0, \infty) \to R$; $f(x) \log_a x$, $a > 0$, $a \neq 1$





$D_f = (0, \infty)$ $R_f = R$

- Natural logarithrnic function $f(x) = \log_e x$ or $\log x$.

- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

$(f \pm g)(x) = f(x) \pm g(x) \; \forall \; x \in X$

$(fg)(x) = f(x)\, g(x) \; \forall \; x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall \; x \in X \;\; provided \;\; g(x) \neq 0$$

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if $(a - 1, b + 5) = (2, 3)$

   If A = {1,3,5}, B = {2,3}, find : (Question-2, 3)

2. $A \times B$

3. $B \times A$

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. $A \times (B \cap C)$

5. $A \times (B \cup C)$

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from P to Q

7. If $R = \{(x,y): x,y \in Z, x^2 + y^2 = 64\}$, then,

   Write R in roster form

   Which of the following relations are functions? Give reason. (Questions 8 to 10)

8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

11. If A and B are finite sets such that n(A) = 5 and n(B) = 7, then find the number of functions from A to B.

12. If $f(x) = x^2 - 3x + 1$ find $x \in R$ such that $f(2x) = f(x)$

Let f and g be two real valued functions, defined by, $f(x) = x$, $g(x) = |x|$.

Find: (Question 13 to 16)

13. $f + g$

14. $f - g$

15. fg

16. $\frac{f}{g}$

17. If $f(x) = x^3$, find the value of,

$$\frac{f(5) - f(1)}{5 - 1}$$

18. Find the domain of the real function,

$$f(x) = \sqrt{x^2 - 4}$$

19. Find the domain of the function, $f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$

Find the range of the following functions. (Question- 20,21)

20. $f(x) = \frac{1}{4 - x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,

$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R = \left\{\left(x, \frac{1}{x}\right) : x \in z, 0 < x < 6\right\}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y): x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

26. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): a∈ A, b∈ B, a is a factor of b and a < b}, then find in Roster form. Also find its domain and range.

27. Let $f(x) = \begin{cases} x^2, when\ 0 \le x \le 2. \\ 2x, when\ 2 \le x \le 5. \end{cases}$

$g(x) = \begin{cases} x^2, when\ 0 < x \le 3. \\ 2x, when\ 3 \le x \le 5. \end{cases}$

Show that f is a function while g is not a function.

28. Find the domain and range of,

f(x) = |2x – 3| – 3

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant functionf : R $\rightarrow$ R; f(x) = 2 $\forall$ x $\in$ R. Also find its domain and range.

31. Draw the graph of the function $|x - 2|$

Find the domain and range of the following real functions (Question 32 to 37)

32. $f(x) = \sqrt{x^2 + 4}$

33. $f(x) = \frac{x+1}{x-2}$

34. $f(x) = \frac{|x+1|}{x-1}$

35. $f(x) = \frac{x^2 - 9}{x-3}$

36. $f(x) = \frac{4-x}{x-4}$

37. $f(x) = 1 - |x-3|$

38. Determine a quadratic function (f) is defined by f(x) = ax² + bx + c. If f(0) = 6; f(2) = 11, f(–3) = 6

39. Draw the graph of the function $f(x) = \begin{cases} 1+2x & x < 0. \\ 3+5x & x \geq 0. \end{cases}$ also find its range.

40. Draw the graph of following function

$$f(x)=\begin{cases}\dfrac{|x|}{x} & x\neq 0.\\ 0 & x=0.\end{cases}$$

Also find its range.

Find the domain of the following function.

41. $f(x)=\dfrac{1}{\sqrt{x+|x|}}$

42. $f(x)=\dfrac{1}{\sqrt{x-|x|}}$

43. $f(x)\quad\dfrac{1}{\sqrt{[x]^2\quad[x]\quad 6}}$

44. $f(x)=\dfrac{1}{\sqrt{9-x^2}}$

45. $f(x)=\sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$

46. Find the domain for which the followings:

$f(x)=2x^2-1$ and $g(x)=1-3x$ are equal.

47. If $f(x)=x-\dfrac{1}{x}$ prove that $\left[f(x)\right]^3=f(x^3)-3f\left(\dfrac{1}{x}\right)$.

48. If [x] denotes the greatest integer function. Find the solution set of equation.

$[x]^2-5[x]+6=0$

49. If $f(x)=\dfrac{ax-b}{bx-a}=y$

Find the value of f(y)

50. Draw the graph of following function and find range (R_f) of

$$f(x)=|x-2|+|2-x| \quad \forall \quad -3\le x\le 3$$

ANSWERS

1. a = 3, b = –2
2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}
3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}
4. {(1,4), (2,4)}
5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}
6. $2^6 = 64$
7. R = { (0,8), (0,–8), (8,0), (–8,0)}
8. Not a function because 4 has two images.
9. Not a function because 2 does not have a unique image.
10. Function because every element in the domain has its unique image.
11. 7^5
12. 0,1
13. $f+g=\begin{cases}2x & x\ge 0\\ 0 & x<0\end{cases}$
14. $f-g=\begin{cases}0 & x\ge 0\\ 2x & x<0\end{cases}$
15. $fg=\begin{cases}x^2 & x\ge 0\\ -x^2 & x<0\end{cases}$
16. $\frac{f}{g}=\begin{cases}1 & x>0\\ -1 & x<0\end{cases}$ and Note -: $\frac{f}{g}$ is not defined at x = 0
17. 31
18. $(-\infty, -2] \cup [2, \infty)$

19. $R - \{2,3\}$

20. $(-\infty, 0) \cup [1/4, \infty)$

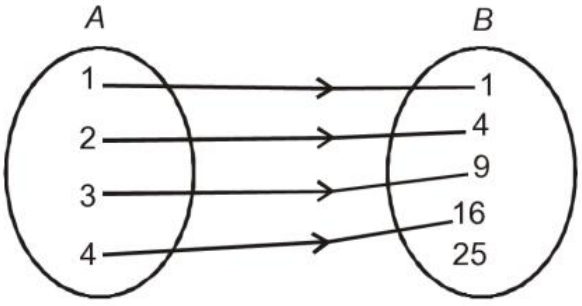
21. $[2,\infty)$

22. $\{-4, -2, -1,1,2,4\}$

23. $\{2,4,6,8\}$

24. $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\right\}$

25. (a)



(b) $\{1,2,3,4\}$

(c) $\{1,4,9,16\}$

(d) $\{1,4,9,16,25\}$

26. $R = \{ (2,4) (2,6) (2,18) (2,54) (6,18) (6,54) (9,18) (9,27) (9,54) \}$

Domain is $R = \{2,6,9\}$

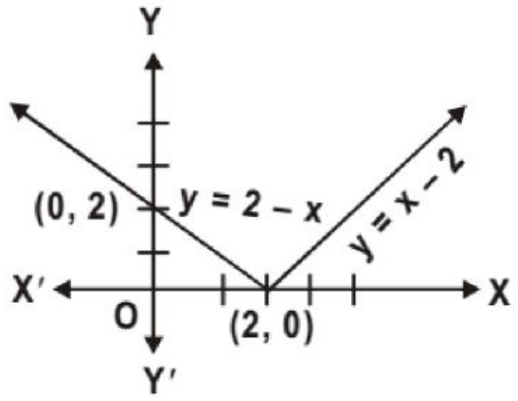
Range of $R = \{ 4, 6, 18, 27, 54\}$

28. Domain is R

Range is $[-3, \infty)$

30. Domain = R, Range = {2}

31.



32. .Domain = R

Range =[2, ∞)

33. Domain = R – {2}

Range = R – {1}

34. Domain = R

Range = {1, –1}

35. Domain = R – {3}

Range = R – {6}

36. Domain = R – {4}

Range = {–1}

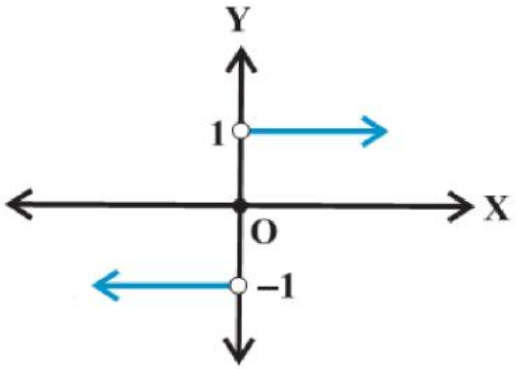
37. Domain = R

Range =(–∞, 1]

38. $\frac{1}{2}x^2 + \frac{3}{2}x + 6$

39. $(-\infty, 1) \cup [3, \infty)$

40. Range of f = {–1,0,1}



41. $(0,\infty)$

42. ϕ (given function is not defined)

43. $(-\infty,-2)\cup[4,\infty)$

44. (–3,3)

45. $(-\infty,-1)\cup(1,4]$

46. $\left\{-2,\frac{1}{2}\right\}$

48. [2,4)

49. x

50. $R_f = [4,6]$ and graph is

