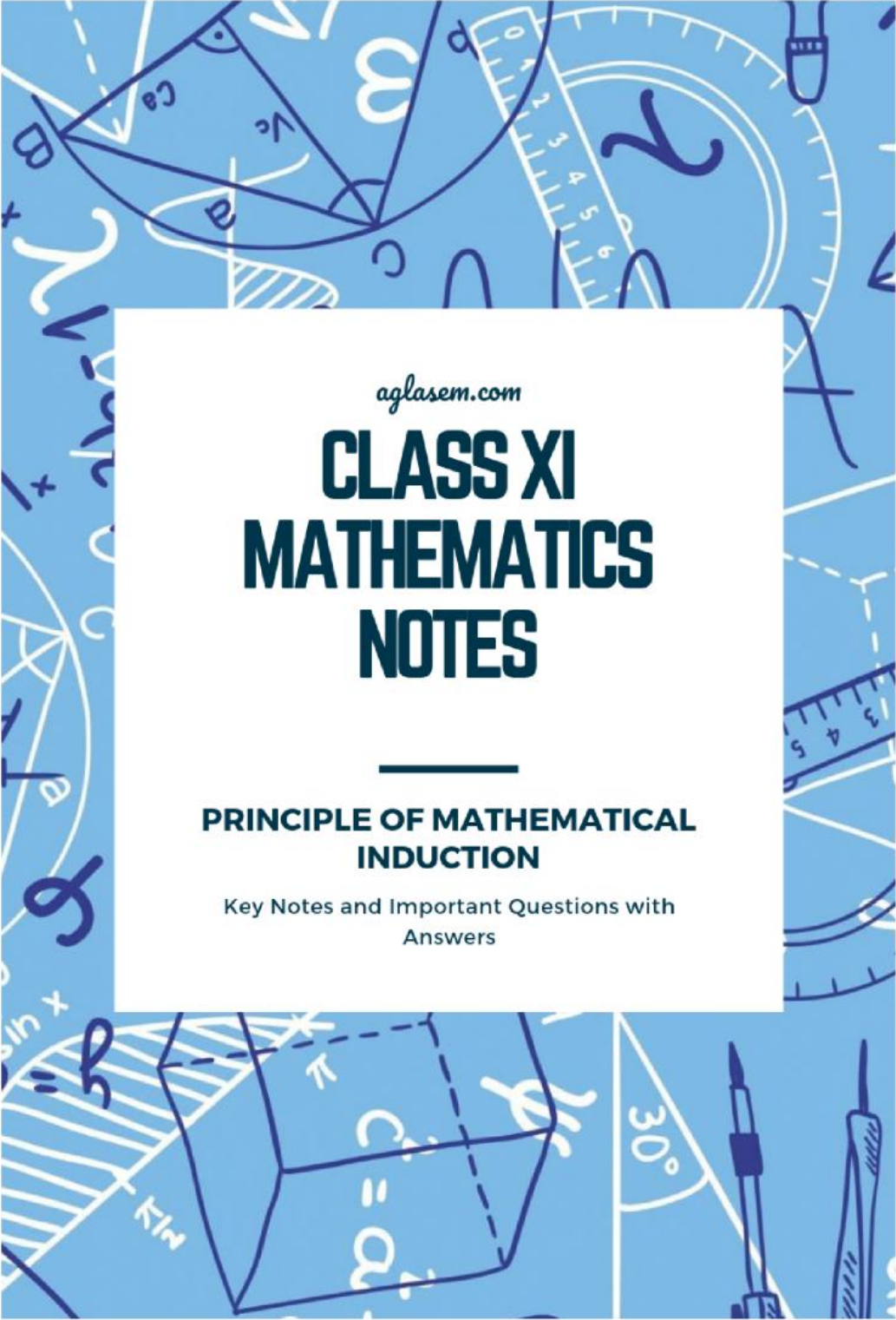


The background is a vibrant blue with a collage of white mathematical sketches. These include a circle with inscribed angles and arcs, a ruler with numerical markings, a 3D cube with dashed lines, a 30-degree angle, a compass, and various mathematical symbols like π , ϕ , ψ , and χ .

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CLASS XI MATHEMATICS NOTES

PRINCIPLE OF MATHEMATICAL INDUCTION

Key Notes and Important Questions with
Answers

CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- A meaningful sentence which can be judged to be either true or false is called a statement.
- A statement involving mathematical relations is called as mathematical statement.
- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Induction being with observations. From observations we arrive at tentative conclusions called conjectures. The process of induction help in proving the conjectures which may be true.
- Statements like(i) $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \forall n \in \mathbb{N}$.
(ii) $2^n \geq 2 \forall n \in \mathbb{N}$.
(iii) If $n(A)=n$ then number of all subsets of $A = 2^n \forall n \in \mathbb{N}$.

(iv) $S_n = \frac{2(r^n - 1)}{r - 1}$ where S_n is sum of n terms of G.P, $a = 1^{\text{st}}$

term and $r =$ common ratio. Are all concerned with $n \in \mathbb{N}$ which takes values $1, 2, 3, \dots$. Such statements are denoted by $P(n)$. By giving particular values to n , we get particular statement as $P(1)$, $P(2), \dots, P(k)$ for some $k \in \mathbb{N}$.

Principle of mathematical Induction:

Let $P(n)$ be any statement involving natural number n such that

- (i) $P(1)$ is true, and
- (ii) If $P(k)$ is true $\Rightarrow P(k+1)$ is true for some $k \in \mathbb{N}$. that is $P(k)$ is true whenever $P(k)$ is true for some $k \in \mathbb{N}$ then $P(n)$ is true $\forall n \in \mathbb{N}$.

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARKS)

1. Let $P(n)$: $n^2 + n$ is even. Is $P(1)$ true?
2. Let $P(n)$: $n(n+1)(n+2)$ is divisible by 3. What is $P(3)$?
3. Let $P(n)$: $n^2 > 9$. Is $P(2)$ true?

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

4. Give an example of a statement such that $P(3)$ is true but $P(4)$ is not true.
5. If $P(n) : 1 + 4 + 7 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$. Verify $P(n)$ for $n = 1, 2$,
6. If $P(n)$ is the statement " $n^2 - n + 41$ is Prime" Prove that $P(1)$ and $P(2)$ are true but $P(41)$ is not true.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Prove the following by using the principle of mathematical induction $\forall n \in \mathbb{N}$.

Type-1

1. $3.6 + 6.9 + 9.12 + \dots + 3n(3n + 3) = 3n(n + 1)(n + 2)$
2. $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$
3. $a + (a + d) + (a + 2d) + \dots + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

HOTS

4. $7 + 77 + 777 + \dots + \text{to } n \text{ terms} = \frac{7}{81}(10^{n+1} - 9n - 10)$
5. $\sin x + \sin 2x + \sin 3x + \dots + \sin nx = \frac{\sin\left(\frac{n+1}{2}x\right)\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$
6. $\sin x + \sin 3x + \dots + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x}.$
7. $\cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \dots \cos(2^{n-1}\alpha) = \frac{\sin 2^n \alpha}{2^n \sin \alpha}.$
8. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$

Type II

9. $2^{3n-1} - 1$ is divisible by 7.
10. 3^{2n} when divided by 8 leaves the remainder 1.

11. $4^n + 15n - 1$ is divisible by 9.

HOTS

12. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9

13. $11^{n+2} + 12^{2n+1}$ is divisible by 133

14. $x^n - y^n$ is divisible by $(x-y)$ if x and y are any two distinct integers.

15. Given that $5^n - 5$ is divisible by 4 $\forall n \in \mathbb{N}$. Prove that $2 \cdot 7^n + 3 \cdot 5^n - 5$ is a multiple of 24.

16. $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25.

Type III

17. $2^{n+1} > 2n + 1$

18. $3^n > 2^n$

19. $n < 2^n$

HOTS

20. $1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}$.

21. $(1+x)^n \geq 1+nx$ where $x > -1$.

22. $2^{n+3} \leq (n+3)!$

ANSWER

1. True

2. P(3): $3(3+1)(3+2)$ is divisible by 3

3. NO.
4. $P(n)$: $3n^2+n$ is divisible by 3 and soon
5. $P(1)$ and $P(2)$ are true.