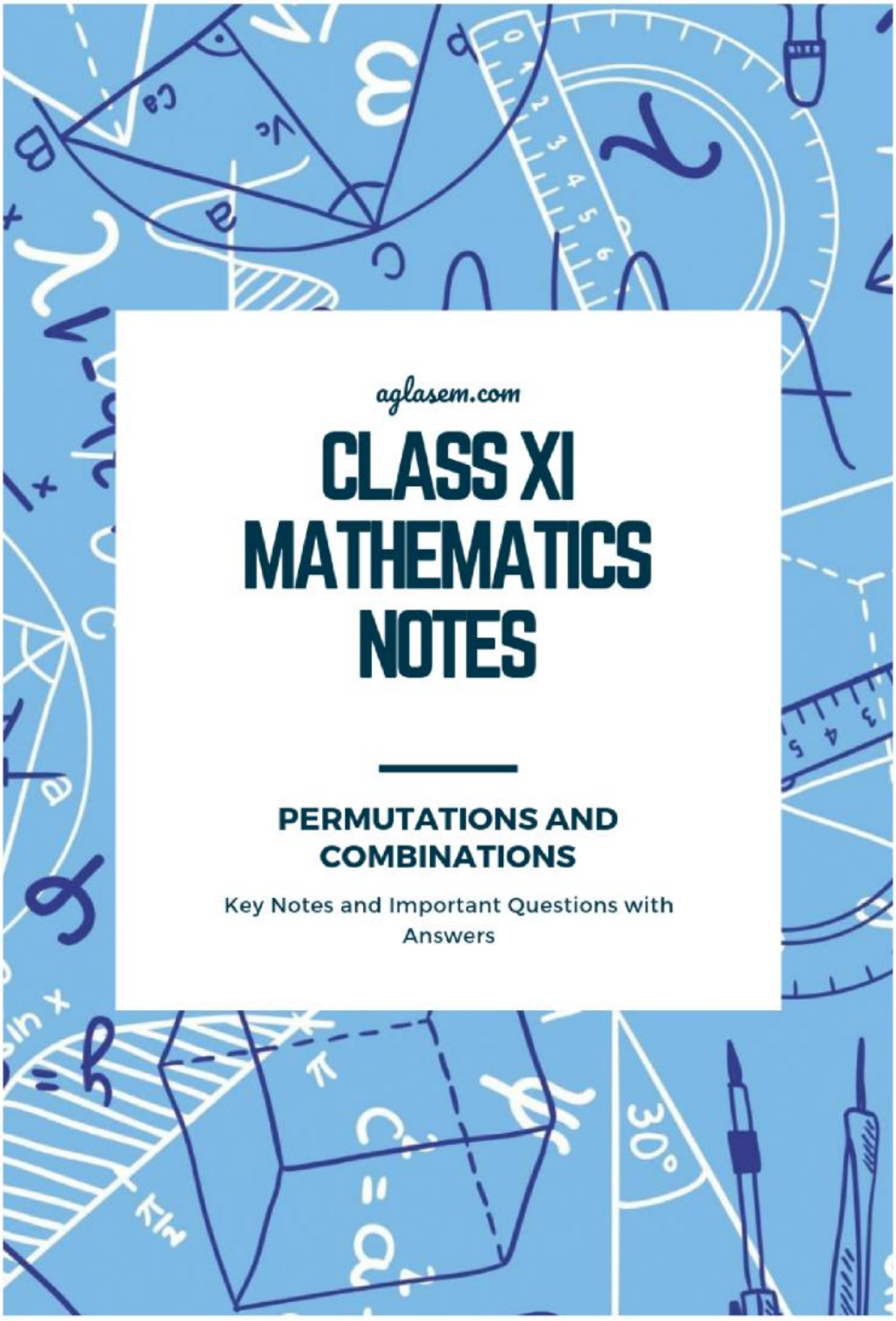

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CLASS XI
MATHEMATICS
NOTES
PERMUTATIONS AND
COMBINATIONS
Key Notes and Important Questions with
Answers

CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- **Multiplication Principle (Fundamental Principle of Counting:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is m × n.
- **Fundamental Principle of Addition:**If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.
- **Factorial:**Factorial of a natural number n, denoted by n! or $\lfloor n$ is the continued product of first n natural numbers.

  $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

  $= n \times ((n-1)!)$

  $= n \times (n-1) \times ((n-2)!)$
- **Permutation:**A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.
- The number of permutation of n different objects taken r at a time where$0 \leq r \leq n$ and the objects do not repeat is denoted by ${}^{n}P_{r}$or P(n, r) where,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\frac{n!}{p_1!p_2!.....p_k!}$
- **Combination:**Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n objects taken r at a time where

$0 \le r \le n$, is denoted by ${}^{n}C_r$ or C(n, r) or $\binom{n}{r}$ where ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

Some important result :

$0! = 1$

${}^{n}C_0 = {}^{n}C_n = 1$

${}^{n}C_r = {}^{n}C_{n-r}$ where $0 \le r \le n$, and r are positive integers

${}^{n}P_r = \underline{|n}\ {}^{n}C_r$ where $0 \le r \le n$, r and n are positive integers.

${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

If ${}^{n}C_a = {}^{n}C_b$ if either a = b or a + b = n

Important Questions with Solutions

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^{n}P_4 : {}^{n}P_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word "SCHOOL"?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

Section B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

13. From a class of 40 students, in how many ways can five students be chosen

    (i) For an excursion party.
    (ii) As subject monitor (one from each subject)

14. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together.

15. If $n_{C_{12}} \quad n_{C_{13}}$ then find the value of the 25_{C_n}.

16. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

Section - C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

18. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

19. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

20. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

21. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

22. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

23. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

24. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

25. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

26. A polygon has 35 diagonal. Find the number of its sides.

27. How many different products can be obtained by multiplying two or more of the numbers 2, 5, 6, 7, 9?

28. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

29. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

30. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

31. Prove that 33! in divisible by 2^{15} what is the largest integer n such that 33! is divisible by 2^n?

32. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has

(i) no girl (ii) at least 3 girls

(iii) at least one girl and one boy?

33. Find n if

$$16^{n+2}C_8 = 57^{n-2}P_4$$

34. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

35. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

36. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

37. There are 15 points in a plane out of which only 6 are in a straight line, then

(a) How many different straight lines can be made?

(b) How many triangles can be made?

38. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that (i) No two girls sit together?

(iii) All the girls never sit together?

39. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

40. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

(a) 3 are red and 1 is black.

(b) All 4 cards are from different suits.

(c) Atleast 3 are face cards.

(d) All 4 cards are of the same colour.

41. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

42. How many different four letter words can be formed (with or without meaning) using the letters of the word “MEDITERRANEAN” such that the first letter is E and the last letter is R.

43. If all letters of word ‘MOTHER’ are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word ‘MOTHER’?

44. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats?

45. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

46. The set S= {1,2,3,.....12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S,\ A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

47. Find the value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$.

48. There are two parallel lines l_1 and l_2 in a plane l_1 contains m different points $A_1, A_2 \ldots\ldots\ldots\ldots A_m$ and l_2 contains n different points $B_1, B_2 \ldots\ldots\ldots\ldots B_n$. How many triangles are possible with these vertices?

ANSWERS

1. $\frac{6!}{2} = 360$
2. $\frac{9!}{2!}$
3. 100
4. 45
5. 120
6. n = 6
7. 120
8. 35
9. 360
10. 63
11. 3^6=729
12. 66
13. (i) 40_{C_5} (ii) 40_{P_5}
14. $\frac{3!}{2!}$ 4!
15. 1

16. 120

17. $90 \times {}^{10}P_8$

18 56,19.350

20. 2772

21. 266

22. 36

23. PAANVR

24. $13C_{10} + 13C_8$

25. 5040

26. 10

27. ${}^{n}C_2 - n$

28. 4560

29. 576

30. 2454

31. 31

32. (i) 21;

(ii) 91;

(iii) 44133.19

34. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

35. 48,144

36. 420

37. (a) 91 (b) 435

38. (i) $7! \times {}^{8}P_5$ (ii) $12! - 8! \times 5!$

39. 24480

40. 52C4

(a) ${}^{26}C_1 \times {}^{26}C_3$ (b) $(13)^4$

(c) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(d) $2 \times {}^{26}C_4$

41. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different

(ii) 2 are identical 1 different

(iv) All are identical, then form the words.)

42. 59

43. 43. 309

44. ${}^{14}P_{12} 2(2\times 3!)^{11}P_9$

45. $4!\,{}^{6}C_4\,{}^{3}C_1$

46. ${}^{12}C_4\,{}^{8}C_4\,{}^{4}C_4$

47. ${}^{56}C_4$

48. ${}^{m+n}C_3 - {}^{m}C_3 - {}^{n}C_3 \;\; or \;\; {}^{m}C_2\,{}^{n}C_1 + {}^{m}C_1\,{}^{n}C_2$