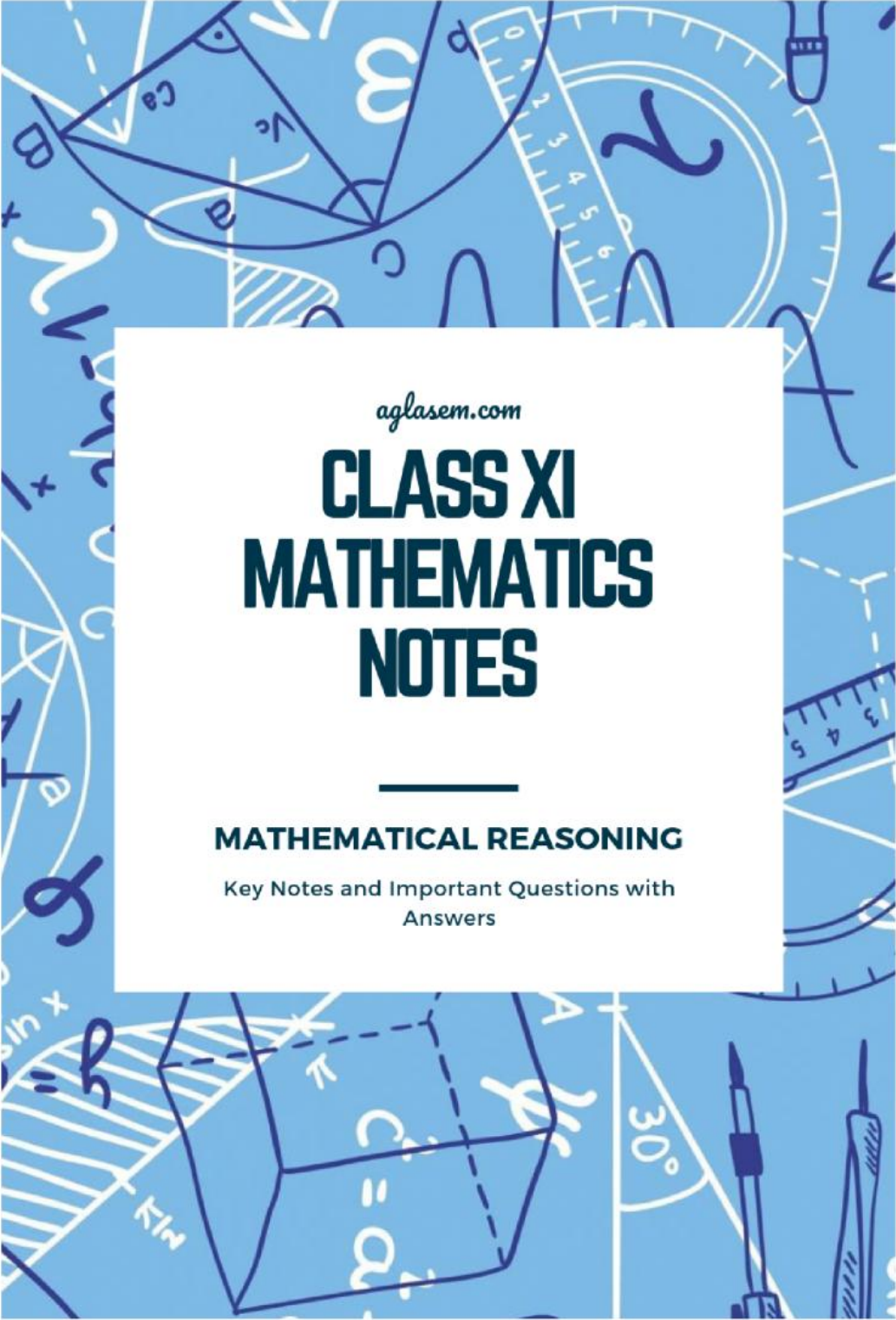




aglasem.com

CLASS XI MATHEMATICS NOTES

MATHEMATICAL REASONING

Key Notes and Important Questions with
Answers

CHAPTER – 14

MATHEMATICAL REASONING

KEY POINTS

- A sentence is called a statement if it is either true or false but not both simultaneously.
- The denial of a statement p is called its negation and is written as $\sim p$ and read as not p .
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If-then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.
- Two simple statements p and q connected by the word 'and' namely ' p and q ' is called a conjunction of p and q and is written as $p \wedge q$.
- Compound statement with '**And**' is
 - (a) true if all its component statements are true
 - (b) false if any of its component statement is false

p	q	$p \wedge q$
T	T	T

T	F	F
F	T	F
F	F	F

- Two simple statements p and q connected by the word 'or' the resulting compound statement ' p or q ' is called disjunction of p and q and is written as $p \vee q$
- Compound statement with '**Or**' is
 - (a) true when at least one component statement is true
 - (b) false when both the component statements are false

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The negation of the compound statement ' p or q ' is ' $\sim p$ and $\sim q$ '
 $\sim (p \vee q) = \sim p \wedge \sim q$.
- The negation of the compound statement ' p and q ' is ' $\sim p$ or $\sim q$ '
 $\sim (p \wedge q) = \sim p \vee \sim q$.
- A statement with "**If p then q** " can be rewritten as
 - (a) p implies q
 - (b) p is sufficient condition for q
 - (c) q is necessary condition for p

(d) p only if q

(e) $(\sim q)$ implies $(\sim p)$

- If in a compound statement containing the connective “or” all the alternatives cannot occur simultaneously, then the connecting word “or” is called as exclusive “or”.
- If, in a compound statement containing the connective “or”, all the alternative can occur simultaneously, then the connecting word “or” is called as inclusive “or”.
- Contrapositive of the statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- Converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$
- “For all”, “For every” are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Identify which of the following are statements (Q. No 1 to 5)

1. Prime factors of 6 are 2 and 3.
2. $x^2 + 6x + 3 = 0$
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.

Write negation of the following statements (Q. No 6 to 11)

6. π is not a rational number.
7. Everyone in Spain speaks Spanish.
8. Zero is a positive number.

9. There exists a complex number which is not a real number.
10. For every real number x , either $x > 1$ or $x < 1$.
11. For every positive real number x , the number $x - 1$ is also positive

Check whether the compound statement is true or false. Write the component statements. (Q. No. 12 to 15)

12. A square is a quadrilateral and its four sides are equal.
13. All integers are either even or odd.
14. $\sqrt{3}$ is a rational number or an irrational number.
15. "O" is either a positive number or negative number.

Identify the type 'Or' (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)

16. Students can take French or Spanish as their third language.
17. To enter in a country, you need a visa or citizenship card.
18. Two lines intersect at a point or are parallel.
19. $\sqrt{2}$ is a rational number or an irrational number?

Write the negation of the following compound statements.

(Question No. 20 to 21)

20. It is daylight and all the people have arisen.
21. Square of an integer is positive or negative.

Identify the quantifiers in the following statements (Q. No.22 to 24)

22. For every integer p , $\sqrt{p}$ is a real number.
23. There exists a number which is equal to its square.
24. There exists a capital for every country in the world.

Write the converse of the following statements (Q. No. 25 to 28)

- 25. If a number x is even, then x^2 is also even.
- 26. If $3 \times 7 = 21$ then $3 + 7 = 10$
- 27. If n is a prime number, then n is odd.
- 28. If x is zero, then x is neither positive nor negative.

Write contrapositive of the following statements (Q. No. 29 to 32)

- 29. If $5 > 7$ then $6 > 7$.
- 30. x is even number implies that x^2 is divisible by 4.
- 31. If a triangle is equilateral, it is isosceles.
- 32. Only if he does not tire he will win.
- 33. Verify by the method of contradiction that $\sqrt{7}$ is irrational.
- 34. By giving counter example, show that the following statement is false:
'If n is an odd integer, then n is prime'.
- 35. Show that the following statement is true by method of contrapositive:
'If x is an integer and x^2 is even, then x is also even'.
- 36. Prove by direct method that for any integer ' n ', $n^3 - n$ is always even.'
- 37. Using the word 'necessary and sufficient' rewrite the following statement
' $A \cup B = \phi$ if and only if $A = B = \phi$ '
Check whether the statement is true.
- 38. Let p : ' x and y are integers such that $x > y$ ' q : ' $-x < -y$ ' Describe the following

(i) $p \vee q$

(ii) $p \wedge q$

(iii) $p \rightarrow q$

(iv) $q \rightarrow p$

(v) $q \leftrightarrow p$

ANSWERS

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. π is a rational number
7. Everyone in Spain doesn't speak Spanish
8. Zero is not a positive number
9. For all complex number x , x is a real number.
10. There exists a real number x such that $x \leq 1$ and $x \geq 1$.
11. There exists a positive real number x such that $x - 1$ is not positive.
12. True; p : A square is a quadrilateral
 q : All the four sides of a square are equal.
13. False; p : All integers are even.
 q : All integers are odd.
14. True; p : $\sqrt{3}$ is a rational number.
 q : $\sqrt{3}$ is an irrational number.

15. False; p: O is a positive number.
q: O is a negative number
16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. It is not daylight or it is false that all the people have arisen.
21. There exists an integer whose square is neither positive nor negative.
22. For every
23. There exists
24. There exists, For every
25. If x^2 is even then x is even
26. If $3 + 7 = 10$ then $3 \times 7 = 21$
27. If n is odd then n a prime number
28. If x is neither positive nor negative then x is zero.
29. If $6 \leq 7$ then $5 \leq 7$
30. If x^2 is not divisible by 4 then x is not even.
31. If a triangle is not isosceles, then it is not equilateral.
32. If he tires, then he will not win.
37. True
38. (i) x and y are integers $x > y$ or $-x < -y$.
(ii) x and y are integers $x > y$ and $-x < -y$.
(iii) If x and y are integers such that $x > y$ then $-x < -y$.
(iv) If x and y are integers such that $-x < -y$ then $x > y$.
(v) x and y are integers such that $x > y$ if and only if $-x < -y$.