27/10/2020

classmate
Date
Page

Ch-5

Exercise 5.6

Q.1 (i) AP = 50, 46, 42, ...

$a = 50$

$d = 46 - 50 = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 \times 50 + (10-1) - 4]$$

$$= 5[100 + (-36)]$$

$$= 5(100 - 36) = 5(64)$$

$$= \boxed{320}$$

(iii) AP = $3, \frac{9}{2}, 6, \frac{15}{2}$

$a = 3$

$d = \frac{9}{2} - 3 = \frac{3}{2}$

$$S_{25} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{25}{2}\left[2 \times 3 + (25-1)\frac{3}{2}\right]$$

$$= \frac{25}{2}\left[6 + \cancel{24}^{12} \times \frac{3}{\cancel{2}}\right]$$

$$= \frac{25}{2} \times [6 + 36] = \frac{25}{\cancel{2}} \times \cancel{42}\,21$$

$$= \boxed{525}$$

(v) AP = $a+b, a-b, a-3b$

$a = a+b$

$d = a-b-a-b = -2b$

$S_{22} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{22}{2}[2 \times (a+b) + (22-1) - 2b]$

$= 11\ [2a+2b + (21) - 2b]$

$= 11\ [2a+2b - 42b]$

$= 11\ [2a - 40b]$

$= \boxed{22a - 440b}$

(vii) AP = $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-2y}{x+y}$

$a = \frac{x-y}{x+y}$

$d = \frac{3x-2y}{x+y} - \left(\frac{x-y}{x+y}\right)$

$= \frac{3x-2y-x+y}{x+y}$

$= \frac{2x-y}{x+y}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2\left(\frac{x-y}{x+y}\right) + (n-1)\left(\frac{2x-y}{x+y}\right)\right]$

$= \frac{n}{2}\left[\frac{2(x-y)}{x+y} + \frac{(n-1)(2x-y)}{x+y}\right]$

$= \frac{n}{2(x+y)}[2(x-y) + (n-1)(2x-y)]$

$$= \frac{n}{2(x+y)} [2x - 2y + 2nx - ny - 2x + y]$$

$$= \frac{n}{2(x+y)} * [2nx - ny - y]$$

$$= \boxed{\frac{n}{2(x+y)} [n(2x-y) - y]}$$

Q.2 AP = 5, 2, −1, 4, −7

$a = 5$

$d = 2 - 5 = -3$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [(2 \times 5) + (n-1)(-3)]$$

$$= \frac{n}{2} [10 - 3n + 3]$$

$$= \boxed{\frac{n}{2} [13 - 3n]}$$

Q.3 $a_n = 5 - 6n$

$a_1 = 5 - 6(1) = -1$

$a_2 = 5 - 6(2) = -7$

$a_3 = 5 - 6(3) = -13$

$a = -1$

$d = -7 + 1 = -6$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(-1) + (n-1)(-6)]$$

$$= \frac{n}{2}[-2-6n+6]$$

$$= \frac{n}{2}[4-6n]$$

$$= \boxed{n[2-3n]}$$

Q.4 AP = 8, 10, 12, 14, ..., 126

Reverse AP = 126, 124, 122, ..., 8

$a = 126$

$d = 124 - 126 = -2$

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{\cancel{10}5}{\cancel{2}}[2(126) + (10-1)-2]$$

$$= 5[252 - 18]$$

$$= 5 \times 234 = \boxed{1170}$$

Q.6 $a_n = An + B$

$a_1 = A(1) + B = A + B$

$a_2 = A(2) + B = 2A + B$

$a_3 = A(3) + B = 3A + B$

$a = A + B$

$d = 2A + B - A - B = A$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{\cancel{20}^{10}}{\cancel{2}}[2(A+B) + (20-1)A]$$

$$= 10[2A + 2B + 19A]$$

$= 10 [21A + 2B]$

$= \boxed{210A + 20B}$

Q.7 $a_n = 2 - 3n$

$a_1 = 2 - 3(1) = -1$

$a_2 = 2 - 3(2) = -4$

$a_3 = 2 - 3(3) = -7$

$a = -1$

$d = -4 + 1 = -3$

$S_{25} = \frac{n}{2} [2a + (n-1)d]$

$= \frac{25}{2} [2(-1) + (25-1)-3]$

$= \frac{25}{2} [-2 - 72]$

$= \frac{25}{2} [-74]$

$= \boxed{-925}$

5/11/2020

classmate
Date
Page

Ch-5
AP
Exercise 5.6

Q. 8 $a_n = 7 - 3n$

$a_1 = 7 - 3(1) = 4$

$a_2 = 7 - 3(2) = 1$

$a_3 = 7 - 3(3) = -2$

$a = 4$

$d = 1 - 4 = -3$

$S_{25} = a + (n-1)d$

$= 4 + (25-1)-3$

$= 1$

$S_{25} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{25}{2}[2 \times 4 + (25-1)-3]$

$= 25[2 \times 4 \times -72$

$S_{25} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{25}{2}[2 \times 4 + (25-1)-3]$

$= \frac{25}{2}[8 - 72]$

$= \frac{25}{2} \times (-64) = 25 \times (-32)$

$= \boxed{-800}$

Q. 9 AP = 25, 22, 19, ...

$a = 25$

$d = 22 - 25 = -3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$116 = \frac{n}{2}[2 \times 25 + (n-1) - 3]$

$116 \times 2 = n[50 - 3n + 3]$

$232 = n(53 - 3n)$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$3n^2 - 24n - 29n + 232 = 0$

$3n(n-8) - 29(n-8) = 0$

$(n-8)(3n-29) = 0$

$(n-8) = 0$

$\boxed{n = 8}$

$(3n - 29) = 0$

$\boxed{n = \frac{29}{3}}$

(Neglected)

$a_8 = a + (n-1)d$

$= 25 + (8-1) - 3$

$= 25 - 21$

$= \boxed{4}$

Q. 10(i) AP = 18, 16, 14, ...

$a = 18$

$d = 16 - 18 = -2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$$0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)]$$

$$0 \times 2 = n[36 - 2n + 2]$$

$$0 = n(38 - 2n)$$

$\therefore \boxed{n = 0}$ (Neglected)

$38 - 2n = 0$

$n = \frac{38}{2} = 19$

$\boxed{n = 19}$

(iv) AP = 63, 60, 57, ...

$a = 63$

$d = 60 - 63 = -3$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$693 = \frac{n}{2}[2 \times 63 + (n-1)(-3)]$$

$$693 \times 2 = n[126 - 3n + 3]$$

$$1386 = n[129 - 3n]$$

$$1386 = 129n - 3n^2$$

$$3n^2 - 129n + 1386 = 0$$

$$n^2 - 43n + 462 = 0$$

$$n^2 - 21n - 22n + 462 = 0$$

$$n(n-21) - 22(n-21) = 0$$

$$(n-21)(n-22) = 0$$

$n - 21 = 0$

$\boxed{n = 21}$

$n - 22 = 0$

$\boxed{n = 22}$

(vi) AP = 45, 39, 33, ...

$a = 45$

$d = 39 - 45 = -6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$180 = \frac{n}{2}[2(45) + (n-1)(-6)]$

$180 \times 2 = n[90 - 6n + 6]$

$360 = n[96 - 6n]$

$360 = 96n - 6n^2$

$6n^2 - 96n + 360 = 0$

$n^2 - 16n + 60 = 0$

$n^2 - 6n - 10n + 60 = 0$

$n(n-6) - 10(n-6) = 0$

$(n-6)(n-10) = 0$

$n - 6 = 0$ | $n - 10 = 0$

$\boxed{n = 6}$ | $\boxed{n = 10}$

10/11/2020

Ch-5

Exercise 5.6

Q.11(iii) $a_2 = a + d = 2$

$a_4 = a + 3d = 8$

ATQ,

$a + d = 2$

$a = 2 - d$ ——— (1)

$a + 3d = 8$

Putting value of a

$(2 - d) + 3d = 8$

$2 - d + 3d = 8$

$2 + 2d = 8$

$2d = 8 - 2$

$d = \frac{6}{2}$

$\boxed{d = 3}$

Putting value of d in (1)

$a = 2 - d$

$a = 2 - 3$

$\boxed{a = -1}$

$S_{51} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{51}{2}[2(-1) + (51-1)3]$

$= \frac{51}{2}[-2 + 150]$

$= \frac{51}{\cancel{2}} \times \cancel{148}\ 74$

$= \boxed{3774}$

Q.12 (iii) 3-digit no. divisible by 13 = 104, 117, 130, ..., 988

$a = 104$

$d = 13$

$l = 988$

ATQ,

$a_n = a + (n-1)d$

$988 = 104 + (n-1)13$

$988 = 104 + 13n - 13$

$988 - 91 = 13n$

$n = \frac{897}{13}$

$\boxed{n = 69}$

$S_{69} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{69}{2}[2 \times 104 + (69-1)13]$

$= \frac{69}{2}[208 + 884]$

$= \frac{69}{\cancel{2}} \times \cancel{1092}\ 546$

$= \boxed{37674}$

(iv) 3-digit no. which are multiple of 11 = 110, 121, 132, ..., 990

$a = 110$

$d = 11$

$l = 990$

ATQ,

$$a_n = a + (n-1)d$$
$$990 = 110 + (n-1)11$$
$$990 = 110 + 11n - 11$$
$$990 - 99 = 11n$$
$$n = \frac{891}{11} \quad 81$$
$$\boxed{n = 81}$$

$$S_{81} = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{81}{2}[2 \times 110 + (81-1)11]$$
$$= \frac{81}{2}[220 + 880]$$
$$= \frac{81}{2} \times 1100 \quad 550$$
$$= \boxed{44550}$$

(v) 2-digit no. divisible by 4 = 12, 16, 20, ..., 96

$a = 12$
$d = 4$
$l = 96$

ATQ,

$$a_n = a + (n-1)d$$
$$96 = 12 + (n-1)4$$
$$96 = 12 + 4n - 4$$
$$96 = 8 + 4n$$
$$96 - 8 = 4n$$
$$n = \frac{88}{4} \quad 22$$

classmate
Date
Page

$n = 22$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{22}{2}\left[2(12) + (22-1)\,4\right]$$

$$= 11\left[24 + 84\right]$$

$$= 11 \times 108$$

$$= 1188$$

11/10/2020

classmate
Date
Page

Ch-5

Exercise 5.6

Q.13 (ii) AP = $3 + 11 + 19 + \ldots + 803$

$a = 3$

$d = 11 - 8 = 8$

$l = 803$

$a_n = a + (n-1)d$

$803 = 3 + (n-1)8$

$803 = 3 + 8n - 8$

$803 + 5 = 8n$

$n = \frac{808}{8}\ 101$

$\boxed{n = 101}$

$S_{101} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{101}{2}[2(3) + (101-1)8]$

$= \frac{101}{2}[6 + 800]$

$= \frac{101}{2} \times 806\ 403$

$= \boxed{40703}$

(v) AP = $7 + 10\frac{1}{2} + 14 + \ldots + 84$

$a = 7$

$d = 10.5 - 7 = 3.5$

$l = 84$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\,3.5$

$84 = 7 + 3.5n - 3.5$

$84 - 3.5 = 3.5n$

$n = \frac{80.5}{3.5}\ 23$

$\boxed{n = 23}$

$S_{23} = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{23}{2}\left[2(7) + (23-1)\,3.5\right]$

$= \frac{23}{2}\left[14 + 77\right]$

$= \frac{23}{2} \times 91\ 45.5$

$= \boxed{1046.5}$

(vii) AP $= 25 + 28 + 31 + \ldots + 100$

$a = 25$

$d = 28 - 25 = 3$

$l = 100$

$a_n = a + (n-1)d$

$100 = 25 + (n-1)\,3$

$100 = 25 + 3n - 3$

$100 - 22 = 3n$

$n = \frac{78}{3}\ 26$

$\boxed{n = 26}$

$$S_{26} = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{26}{2}13\left[2(25) + (26-1)3\right]$$

$$= 13\left[50 + 75\right]$$

$$= 13\,(125)$$

$$= \boxed{1625}$$

(viii) $AP = 18 + 15\frac{1}{2} + 13 + \ldots + \left(-49\frac{1}{2}\right)$

$a = 18$

$d = 15.5 - 18 = -2.5$

$l = -49.5$

$a_n = a + (n-1)d$

$-49.5 = 18 + (n-1)-2.5$

$-49.5 = 18 - 2.5n + 2.5$

~~$-49.5 - 15.5 = -2.5n$~~

$-49.5 - 20.5 = -2.5n$

$-70 = -2.5n$

$n = \frac{70}{2.5}^{28}$

$\boxed{n = 28}$

Q.15 $a_3 = a + 2d = 7$

$a_7 = a + 6d$

ATQ,

$a + 2d = 7$

$a = 7 - 2d$ ——— (1)

$a_7 = 3(a_3) + 2$

$a + 6d = 3(7) + 2$

$a + 6d = 23$

Putting value of a

$7 - 2d + 6d = 23$

$7 + 4d = 23$

$4d = 23 - 7$

$d = \frac{16}{4}$

$\boxed{d = 4}$

Putting value of d in ①

$a = 7 - 2d$

$a = 7 - 2(4)$

$a = 7 - 8$

$\boxed{a = -1}$

$S_{20} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}^{10}[2(-1) + (20-1)4]$

$= 10[-2 + 76]$

$= 10(74) = \boxed{740}$

Q.17 Also $a_{12} = a + 11d = -13$

$S_4 = 24$

ATQ,

$a + 11d = -13$

$a = -13 - 11d$ —— ①

$$S_4 = \frac{n}{2}[2a + (n-1)d]$$

$$24 = \frac{4}{2}[2a + (4-1)d]$$

$$24 = 2(2a + 3d)$$

$$24 = 4a + 6d$$

$$12 = 2a + 3d$$

Putting value of a

$$12 = 2[-13 - 11d] + 3d$$

$$12 = -26 - 22d + 3d$$

$$12 + 26 = -19d$$

$$d = \frac{38}{-19}$$

$$\boxed{d = -2}$$

Putting value of d in (1)

$$a = -13 - 11d$$

$$a = -13 - 11(-2)$$

$$a = -13 + 22$$

$$\boxed{a = 9}$$

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2(9) + (10-1)(-2)]$$

$$= 5[18 + (-18)]$$

$$= 5 \times 0 = \boxed{0}$$

Ex 5.6

Q.18 $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots$

$a=\left(4-\frac{1}{n}\right)$

$d=\left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)$

$=\frac{4n-2-4n+1}{n}$

$=\left(\frac{-1}{n}\right)$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$=\frac{n}{2}\left[2\left(4-\frac{1}{n}\right)+(n-1)\left(\frac{-1}{n}\right)\right]$

$=\frac{n}{2}\left[8-\frac{2}{n}+\left(-1+\frac{1}{n}\right)\right]$

$=\frac{n}{2}\left[\frac{8n-2-n+1}{n}\right]$

$=\boxed{\left(\frac{7n-1}{2}\right)}$

Q.19 $a=22$

$d=-4$

$S_n=64$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$64=\frac{n}{2}\left[2(22)+(n-1)(-4)\right]$

$64 \times 2 = n\,(44 - 4n + 4)$

$128 = n\,(48 - 4n)$

$128 = 48n - 4n^2$

$4n^2 - 48n + 128 = 0$

$n^2 - 12n + 32 = 0$

$n^2 - 8n - 4n + 32 = 0$

$n(n-8) - 4(n-8) = 0$

$(n-8)(n-4) = 0$

$n - 8 = 0$ | $n - 4 = 0$

$\boxed{n = 8}$ | $\boxed{n = 4}$

$\therefore \boxed{n = 8 \text{ or } 4}$

Q.20 $a_5 = a + 4d = 30$ ——— ①

$a_{12} = a + 11d = 65$ ——— ②

Subtract ① from ②

$$\begin{array}{r} a + 11d = 65 \\ a + 4d = 30 \\ (-)\ \ (-)\ \ (-) \\ \hline 7d = 35 \end{array}$$

$\boxed{d = 5}$

Putting value of d in ①

$a + 4d = 30$

$a + 4(5) = 30$

$a = 30 - 20$

$\boxed{a = 10}$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20^{10}}{\cancel{2}}[2(10) + (20-1)5]$$

$$= 10\ [20 + 95\ ]$$

$$= 10\ (115) = \boxed{1150}$$

Q.21 $a_2 = a + d = 14$ ——— ①

$a_3 = a + 2d = 18$ ——— ②

Subtract ① from ②,

$$\begin{array}{r} a + 2d = 18 \\ a + d = 14 \\ (-)\ \ (-)\ \ (-) \\ \hline d = 4 \end{array}$$

Putting value of d in ①

$$a + d = 14$$

$$a + 4 = 14$$

$$a = 14 - 4$$

$$\boxed{a = 10}$$

$$S_{51} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{51}{2}[2(10) + (51-1)4]$$

$$= \frac{51}{2}[20 + 200]$$

$$= \frac{51}{\cancel{2}} \times \cancel{220}\ 110$$

$$= \boxed{5610}$$

Q. 25 $a = 22$

$a_n = 11$

$S_n = 66$

$$a_n = a + (n-1)d$$

$$-11 = 22 + (n-1)d$$

$$-11 - 22 = (n-1)d$$

$$(n-1)d = -33 \quad \text{——} \quad (1)$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$66 = \frac{n}{2}\left[2(22) + (-33)\right]$$

$$66 \times 2 = n\,[44 - 33]$$

$$132 = 11n$$

$$n = \frac{\cancel{132}}{\cancel{11}}$$

$$\boxed{n = 12}$$

Putting value of n in (1)

$$(n-1)\,d = -33$$

$$(12-1)\,d = -33$$

$$d = \frac{-\cancel{33}}{\cancel{11}}$$

$$\boxed{d = -3}$$

Q.26 $a = 7$

$a_n = 49$

$a_n = a + (n-1)d$

$49 = 7 + (n-1)d$

$(n-1)d = 49 - 7$

$(n-1)d = 42$ ——— (1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$420 = \frac{n}{2}[2 \times 7 + 42]$ From (1)

$420 = \frac{n}{2}[14 + 42]$

$420 \times 2 = n(56)$

$n = \frac{420 \times 1}{56}$

$n = 15$

Putting value of n in (1)

$(n-1)d = 42$

$d = \frac{42}{15-1}$

$d = \frac{42}{14}$

$d = 3$

Q.28 $S_9 = 162$

$a_6 : a_{13} = 1 : 2$

Q.10 (a)

$$S_9 = \frac{9}{2}[2a + (9-1)d]$$

$$162 = \frac{9}{2}[2a + (9-1)d] \quad \text{—— (1)}$$

$$a_6 = a + 5d$$

$$a_{13} = a + 12d$$

$$a_6 : a_{13} = 1 : 2$$

$$\therefore \frac{a_6}{a_{13}} = \frac{1}{2}$$

$$\frac{a+5d}{a+12d} = \frac{1}{2}$$

$$2a + 10d = a + 12d$$

$$2a - a = 12d - 10d$$

$$a = 2d \quad \text{—— (2)}$$

$$162 = \frac{9}{2}[2a + (9-1)d]$$

$$162 \times 2 = 9[2(2d) + 8d] \quad (\text{from } (2))$$

$$162 \times 2 = 9[4d + 8d]$$

$$162 \times 2 = 9(12d)$$

$$d = \frac{162 \times 2}{9 \times 12}$$

$$\boxed{d = 3}$$

Putting value of d in (2)

$$a = 2d$$

$$a = 2(3)$$

$$\boxed{a = 6}$$

$a_{15} = a + 14d$

$= 6 + 14(3)$

$= 6 + 42 = \boxed{48}$

Q.29 $a_{10} = a + 9d = 21$

$a + 9d = 21$

$9d = 21 - a$ ——— (1)

$S_{10} = \frac{n}{2}[2a + (n-1)d]$

$120 = \frac{10}{2}[2a + (10-1)d]$

$120 \times 2 = 10[2a + 9d]$

$120 \times 2 = 10[2a + 21 - a]$ (From (1))

$120 \times 2 = 10[a + 21]$

$120 \times 2 = 10a + 210$

$240 - 210 = 10a$

$a = \frac{30}{10}$

$\boxed{a = 3}$

Putting value of a in (1)

$9d = 21 - a$

$d = \frac{21 - 3}{9}$

$d = \frac{18}{9}^{2}$

$\boxed{d = 2}$

$a_n = a + (n-1)d$

$a_n = 3 + (n-1)2$

$a_n = 3 + 2n - 2$

$\boxed{a_n = 2n + 1}$

Q. 30 $S_7 = 63$

$S_{14} = 63 + 161$

$= 224$

$S_7 = \frac{n}{2}[2a + (n-1)d]$

$63 = \frac{7}{2}[2a + (7-1)d]$

$63 \times 2 = 7[2a + 6d]$

$\frac{63 \times 2}{7} = \cancel{14a + 42d}\ 2a + 6d$

$18 = 2a + 6d$

$9 = a + 3d$ —— (1)

$S_{14} = \frac{n}{2}[2a + (n-1)d]$

$224 = \frac{14}{2}7[2a + (14-1)d]$

$224 = 7[2a + 13d]$

$\frac{224}{7}\ 32 = 2a + 13d$

$32 = 2a + 13d$ —— (2)

Multiply (1) with 2.

$18 = 2a + 6d$ —— (3)

Subtract ③ from ②.

$$2a + 13d = 32$$
$$2a + 6d = 18$$
$$(-) \quad (-) \quad (-)$$
$$7d = 14$$
$$\boxed{d = 2}$$

Putting value of d in ①.

$$9 = a + 3d$$
$$9 = a + 3(2)$$
$$a = 9 - 6$$
$$\boxed{a = 3}$$

$$a_{28} = a + 27d$$
$$= 3 + 27(2)$$
$$= 3 + 54 = \boxed{57}$$

Q.31 $S_7 = 182$

$$182 = \frac{7}{2}\left[2a + (7-1)d\right]$$
$$182 \times 2 = 7\left[2a + 6d\right]$$
$$\frac{26}{182} \times 2 = 2a + 6d$$
$$a + 3d = 26$$
$$a = 26 - 3d \quad \text{——} \quad ①$$

$$a_4 : a_{17} = 1 : 5$$
$$\frac{a_4}{a_{17}} = \frac{1}{5}$$

$$\frac{a+3d}{a+16d} = \frac{1}{5}$$

$$5a+15d = a+16d$$

$$5a-a = 16d-15d$$

$$4a = d$$

$$4(26-3d) = d$$

$$104 - 12d = d$$

$$104 = d+12d$$

$$104 = 13d$$

$$d = \frac{104}{13}\ 8$$

$$\boxed{d = 8}$$

Putting value of d in ①

$$a = 26-3d$$

$$a = 26-3(8)$$

$$a = 26-24$$

$$\boxed{a = 2}$$

$\therefore$ AP = $\boxed{2, 10, 18, 26, 34, \ldots}$

Q.32 $a_n = -4n+15$

$a_1 = -4(1)+15$

$= -4+15$

$= 11$

$a_2 = -4(2)+15$

$= -8+15$

$= 7$

$d = a_2 - a_1 = 7-11 = -4$

$a = 11$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20^{10}}{2}[2(11) + (20-1)-4]$$

$$= 10[22 - 76]$$

$$= 10[-54]$$

$$= \boxed{-540}$$

Q. 33 $S_{10} = -150$

$S_{20} = -150 + (-550)$

$= -150 - 550 = -700$

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$-150 = \frac{10^{5}}{2}[2a + (10-1)d]$$

$$-150 = 5[2a + 9d]$$

$$\frac{-150^{30}}{5} = 2a + 9d$$

$$2a + 9d = -30 \quad \text{——— (1)}$$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$-700 = \frac{20^{10}}{2}[2a + (20-1)d]$$

$$\frac{-700^{70}}{10} = 2a + 19d$$

$$2a + 19d = -70 \quad \text{——— (2)}$$

Subtract (1) from (2)

$2a + 19d = -70$

$2a + 9d = -30$

$(-)\quad (-)\quad (+)$

$10d = -40$

$d = \frac{-40}{10}$

$\boxed{d = -4}$

Putting value of d in ①

$2a + 9d = -30$

$2a + 9(-4) = -30$

$2a - 36 = -30$

$2a = -30 + 36$

$a = \frac{6}{2}$

$\boxed{a = 3}$

$\therefore AP = \boxed{3, -1, -5, -9, -13, \ldots}$

Q. 34 $S_{14} = 1505$

$a = 10$

$S_{14} = \frac{n}{2}[2a + (n-1)d]$

$1505 = \frac{14}{2}[2(10) + (14-1)d]$

$1505 = 7[20 + 13d]$

$1505 = 140 + 91d$

$91d = 1505 - 140$

$d = \frac{1365}{91}\ 15$

$\boxed{d = 15}$

$a_{25} = a + 24d$

$= 10 + 24(15)$

$= 10 + 360$

$= \boxed{370}$

Q.35 $a = 2$

$a_n = 29$

$S_n = 155$

$a_n = a + (n-1)d$

$29 = 2 + (n-1)d$

$29 - 2 = (n-1)d$

$\therefore (n-1)d = 27$ ——— ①

$S_n = \frac{n}{2}[2a + (n-1)d]$

$155 = \frac{n}{2}[2(2) + 27]$ (From ①)

$155 \times 2 = n[4 + 27]$

$310 = 31n$

$n = \frac{310}{31}\ 10$

$\boxed{n = 10}$

Putting value of n in ①

$(n-1)d = 27$

$(10-1)d = 27$

$d = \frac{27}{9}\ 3$

$$\boxed{d = 3}$$

Q.37 AP $= -12, -9, -6, \ldots, 21$

$a = -12$

$d = -9 + 12 = 3$

$l = 21$

$a_n = a + (n-1)d$

$21 = -12 + (n-1)3$

$21 + 12 = 3n - 3$

$33 + 3 = 3n$

$n = \frac{36}{3}12$

$$\boxed{n = 12}$$

$S_{12} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2(-12) + (12-1)3]$

$= 6[-24 + 33]$

$= 6[9]$

$= 54$

New sum $= 54 + 1(12)$

$= 54 + 12$

$= \boxed{66}$

Q.38 $S_n = 3n^2 + 6n$

$a_n = S_n - S_{n-1}$

$= (3n^2 + 6n) - [3(n-1)^2 + 6(n-1)]$

$= (3n^2 + 6n) - [3(n^2 - 2n + 1) + 6n - 6]$

$= (3n^2 + 6n) - [3n^2 - 6n + 3 + 6n - 6]$

$= 3n^2 + 6n - 3n^2 + 6n - 3 - 6n + 6$

$= \boxed{6n + 3}$

Q.40 $S_n = 4n^2 + 2n$

$a_n = S_n - S_{n-1}$

$= (4n^2 + 2n) - [4(n-1)^2 + 2(n-1)]$

$= (4n^2 + 2n) - [4(n^2 - 2n + 1) + 2n - 2]$

$= (4n^2 + 2n) - [4n^2 - 8n + 4 + 2n - 2]$

$= 4n^2 + 2n - 4n^2 + 8n - 4 - 2n + 2$

$= 8n - 2$

$= \boxed{2(4n - 1)}$

Q.41 $S_n = 3n^2 + 4n$

$S_1 = 3(1)^2 + 4(1)$

$= 3 + 4 = 7$

$S_2 = 3(2)^2 + 4(2)$

$= 12 + 8 = 20$

$a_1 = S_1 = 7$

$a_2 = S_2 - S_1 = 20 - 7 = 13$

$d = a_2 - a_1 = 13 - 7 = 6$

$a_{25} = a + 24d$

$= 7 + 24(6)$

$= 7 + 144 = \boxed{151}$

$= 7 + 144 - 151$

Ex 5.6

Q 43 $S_q = 63q - 3q^2$

$S_1 = 63(1) + 3(1)^2$

$= 63 - 3 = 60$

$S_2 = 63(2) - 3(2)^2$

$= 126 - 12 = 114$

$a_1 = S_1 = 60$

$a_2 = S_2 - S_1 = 114 - 60 = 54$

$d = a_2 - a_1 = 54 - 60 = -6$

$a_p = a + (p-1)d$

$-60 = 60 + (p-1)\ -6$

$-60 - 60 = -6p + 6$

$-120 - 6 = -6p$

$p = \frac{-126}{-6}$

$\boxed{p = 21}$

$a_{11} = a + 10d$

$= 60 + 10(-6)$

$= 60 - 60$

$= \boxed{0}$

Q.44 $S_m = 4m^2 - m$

$S_1 = 4(1)^2 - 1$

$= 4 - 1 = 3$

$S_2 = 4(2)^2 - 2$

$= 16 - 2 = 14$

$a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 14 - 3 = 11$

$d = a_2 - a_1 = 11 - 3 = 8$

$a_n = a + (n-1)d$

$107 = 3 + (n-1)\ 8$

$107 = 3 + 8n - 8$

$107 + 5 = 8n$

$n = \frac{112}{8}$ 14

$\boxed{n = 14}$

$a_{21} = a + (n-1)d$

$= 3 + (21-1)\,8$

$= 3 + (20)8$

$= 3 + 160 = \boxed{163}$

Q.46 $S_n = \frac{1}{2}\left[3n^2 + 7n\right]$

$S_{n-1} = \frac{1}{2}\left[3(n-1)^2 + 7(n-1)\right]$

$a_n = S_n - S_{n-1}$

$= \frac{1}{2}\left[3n^2 + 7n\right] - \frac{1}{2}\left[3(n-1)^2 + 7(n-1)\right]$

$= \frac{1}{2}\left\{(3n^2 + 7n) - \left[3(n^2 - 2n + 1) + 7n - 7\right]\right\}$

$= \frac{1}{2}\left[3n^2 + 7n - 3n^2 + 6n - 3 - 7n + 7\right]$

$= \frac{1}{2}\left[6n + 4\right]$

$= \boxed{3n + 2}$

$a_{20} = 3(20) + 2$

$= 60 + 2 = \boxed{62}$

Q.47(i) $S_n = \frac{3n^2}{2} + \frac{13n}{2}$

$S_1 = \frac{3(1)^2}{2} + \frac{13(1)}{2} = \frac{3 + 13}{2} = \frac{16}{2} = 8$

$S_2 = \frac{3(2)^2}{2} + \frac{13(2)}{2} = \frac{12 + 26}{2} = \frac{38}{2} = 19$

$a_1 = S_1 = 8$

$a_2 = S_2 - S_1 = 19 - 8 = 11$

$d = a_2 - a_1 = 11 - 8 = 3$

$a_{25} = a + 24d$

$= 8 + 24(3)$

$= 8 + 72 = \boxed{80}$

(ii) $S_n = n^2$

$S_1 = (1)^2 = 1$

$S_2 = (2)^2 = 4$

$a_1 = S_1 = 1$

$a_2 = S_2 - S_1 = 4 - 1 = 3$

$d = a_2 - a_1 = 3 - 1 = 2$

$a_{10} = a + 9d$

$= 1 + 9(2)$

$= 1 + 18 = \boxed{19}$

Q.49 AP = 1, 3, 5, 7, ..., n

$a = 1$

$d = 3 - 1 = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(1) + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n = \boxed{n^2}$

Q. 50 (i) 0 and 50

AP = 1, 3, 5, 7, ..., 49

$a = 1$

$d = 3 - 1 = 2$

$a_n = 49$

$a_n = a + (n-1)d$

$49 = 1 + (n-1)2$

$49 = 1 + 2n - 2$

$49 + 1 = 2n$

$n = \frac{50}{2}\ 25$

$n = 25$

$S_{25} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{25}{2}[2(1) + (25-1)2]$

$= \frac{25}{2}[2 + 48]$

$= \frac{25 \times 50}{2}\ 25$

$= \boxed{625}$

(ii) 100 and 200

AP = 101, 103, 105, ... 199

$a = 101$

$d = 103 - 101 = 2$

$a_n = 199$

$a_n = a + (n-1)d$

$199 = 101 + (n-1)2$

$199 = 101 + 2n - 2$

$199 - 99 = 2n$

$n = \frac{100}{2}\ 50$

$n = 50$

$S_{50} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}\ 25\ [2(101) + (50-1)2]$

$= 25\ [202 + 98]$

$= 25 \times 300 = \boxed{7500}$

Q.51 AP = 3, 9, 15, 21, ..., 999

$a = 3$

$d = 9 - 3 = 6$

$a_n = 999$

$a_n = a + (n-1)d$

$999 = 3 + (n-1)6$

$999 = 3 + 6n - 6$

$999 + 3 = 6n$

$\frac{1002}{6} = 167 = n$

$n = 167$

$S_{167} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{167}{2}[2(3) + (167-1)6]$

$= \frac{167}{2} [6 + 996]$

$= \frac{167 \times 1002}{2} 501$

$= 83667$

Hence, proved

Q.53 AP = 56, 63, 70, 77, ..., 497

$a = 56$

$d = 63 - 56 = 7$

$a_n = 497$

$a_n = a + (n-1)d$

$497 = 56 + (n-1)7$

$497 = 56 + 7n - 7$

$497 - 49 = 7n$

$n = \frac{448}{7} 64$

$n = 64$

$S_{64} = \frac{n}{2} [2a + (n-1)d]$

$= \frac{64}{2} 32 [2(56) + (64-1)7]$

$= 32 [112 + 441]$

$= 32 (553)$

$= 17696$

Q.55(ii) Sum of integers between 100 & 550 not divisible by 9 = Sum of total no. − Sum of no. divisible by 9.

AP = 101, 102, 103, ..., 549

$a = 101$

$d = 102 - 101 = 1$

$a_n = 549$

$a_n = a + (n-1)d$

$549 = 101 + (n-1)1$

$549 = 101 + n - 1$

$549 - 100 = n$

$n = 449$

$S_{449} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{449}{2}[2(101) + (449-1)1]$

$= \frac{449}{2}[202 + 448]$

$= \frac{449}{2} \times 650$ 325

$= 145925$

AP = 108, 117, 126, 135, ..., 549

$a = 108$

$d = 117 - 108 = 9$

$a_n = 549$

$a_n = a + (n-1)d$

$549 = 108 + (n-1)9$

$549 = 108 + 9n - 9$

$549 - 99 = 9n$

$n = \frac{450}{9}$ 50

$\boxed{n = 50}$

$S_{50} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}25 [2(108) + (50-1)9]$

$= 25 [216 + 441]$

$= 25 \times 657 = 16425$

$\therefore$ Sum of integers between 100 & 550 not divisible by 9 $= 145925 - 16425$

$= \boxed{129500}$

(iv) AP = 10, 20, 30, 40, ..., 500

$a = 10$

$d = 20 - 10 = 10$

$a_n = 500$

$a_n = a + (n-1)d$

$500 = 10 + (n-1)10$

$500 = 10 + 10n - 10$

$n = \frac{500}{10}$

$n = 50$

$S_{50} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}25 [2(10) + (50-1)10]$

$= 25 [20 + 490]$

$= 25 \times 510$

$= \boxed{12750}$

Q. 56

(ii) $a_n = 4$

$d = 2$

$S_n = -14$

$a_n = a + (n-1)d$

$4 = a + (n-1)2$

$4 = a + 2n - 2$

$4 + 2 = a + 2n$

$a = 6 - 2n$ ——— (1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2a + (n-1)2]$

$= \frac{n}{2}[2a + 2n - 2]$

$S_n = \frac{n}{2}[a + l]$

$-14 = \frac{n}{2}[a + 4]$

$-28 = n(a+4)$ (From (1))

$-28 = n(6 - 2n + 4)$

$-28 = n(10 - 2n)$

$-28 = 10n - 2n^2$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7) + 2(n-7) = 0$

$(n-7)(n+2) = 0$

$n - 7 = 0$

$\boxed{n = 7}$

$n + 2 = 0$

$\boxed{n = -2}$

(Neglected)

Putting value of n in (i)

$a = 6 - 2n$

$a = 6 - 2(7)$

$a = 6 - 14$

$\boxed{a = -8}$

(iv) $a_n = 28$

$S_n = 144$

$n = 9$

$a_n = a + (n-1)d$

$28 = a + (9-1)d$

$28 = a + 8d$ ——— (i)

$S_n = \frac{n}{2}[a + l]$

$144 = \frac{9}{2}[a + 28]$

$\frac{\cancel{144}^{16} \times 2}{\cancel{9}} = a + 28$

$a = 32 - 28$

$\boxed{a = 4}$

Putting value of a in (i)

$28 = a + 8d$

$28 = 4 + 8d$

$28 - 4 = 8d$

$d = \frac{\cancel{24}^{3}}{\cancel{8}}$

$\boxed{d = 3}$

(vii) $S_n = 3n^2 + 5n$

$a_k = 164$

$S_1 = 3(1)^2 + 5(1)$

$= 3 + 5 = 8$

$S_2 = 3(2)^2 + 5(2)$

$= 12 + 10 = 22$

$a_1 = S_1 = 8$

$a_2 = S_2 - S_1 = 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

$a_k = a + (k-1)d$

$164 = 8 + (k-1)6$

$164 = 8 + 6k - 6$

$164 - 2 = 6k$

$k = \frac{162}{6}\ 27$

$\boxed{k = 27}$

Q.57 $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2a + (12-1)d]$

$= 6[2a + 11d]$

$= 12a + 66d$ ——— (1)

$S_8 = \frac{8}{2}[2a + (8-1)d]$

$= 4[2a + 7d]$

$= 8a + 28d$

$$S_4 = \frac{4}{2}[2a + (4-1)\cdot d]$$

$$= 2[2a + 3d]$$

$$= 4a + 6d$$

$$3(S_8 - S_4) = 3(8a + 28d - 4a - 6d)$$

$$= 3(4a + 22d)$$

$$= 12a + 66d$$

$$= \boxed{S_{12}} \quad \text{(From ①)}$$

Hence, proved

Q. 59 $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_1 = \frac{n}{2}[2(5) + (n-1)2]$$

$$= \frac{n}{2}[10 + 2n - 2]$$

$$= \frac{n[8 + 2n]}{2}$$

$$= n[4 + n]$$

$$S_2 = \frac{n}{2}[2(5) + (n-1)4]$$

$$= \frac{n}{2}[10 + 4n - 4]$$

$$= \frac{n}{2}[6 + 4n]$$

$$= n[3 + 2n] \longrightarrow ①$$

$$S_3 = \frac{n}{2}\left[2(5) + (n-1)6\right]$$

$$= \frac{n}{2}\left[10 + 6n - 6\right]$$

$$= \frac{n}{2}\left[4 + 6n\right]$$

$$= n\left[2 + 3n\right]$$

$$S_1 + S_3 = n(4+n) + n(2+3n)$$

$$= n(4 + n + 2 + 3n)$$

$$= n(6 + 4n)$$

$$= 2n(3 + 2n)$$

$$= \boxed{2S_2} \quad \text{(From ①)}$$

Hence, proved

Q.69 $S_{n_7} = 700$

$n = 7$

$a_1 = a$

$a_2 = a - 20$

$d = a - 20 - a = -20$

$$S_7 = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$700 = \frac{7}{2}\left[2(a) + (7-1)(-20)\right]$$

$$700 \times 2 = 7\left[2a - 120\right]$$

$$\frac{700 \times 2}{7} = 2a - 120$$

$$200 = 2a - 120$$

$$2a = 200 + 120$$

$$a = \frac{320}{2} \quad 160$$

$$\boxed{a = 160}$$

∴ Therefore, the prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 & ₹40.

3. To prove – $S_{30} = 3(S_{20} - S_{10})$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{30} = \frac{30}{2}\,15\,[2a + (30-1)d]$$

$$= 15(2a + 29d) \longleftarrow (1)$$

$$S_{20} = \frac{20}{2}\,10\,[2a + (20-1)d]$$

$$= 10(2a + 19d) \longleftarrow (2)$$

$$S_{10} = \frac{10}{2}\,5\,[2a + (10-1)d]$$

$$= 5(2a + 9d) \longleftarrow (3)$$

$$3(S_{20} - S_{10}) = 3[10(2a + 19d) - 5(2a + 9d)]$$
$$= 3[20a + 190d - 10a - 45d]$$
$$= 3[10a + 145d]$$
$$= 3 \times 5[2a + 29d]$$
$$= 15(2a + 29d)$$
$$= \boxed{S_{30}} \quad (\text{from } (1))$$

Hence, proved.

Q. 71 $(-4) + (-1) + 2 + 5 + \dots + x = 437$

$a = -4$

$d = -1 + 4 = 3$

$a_n = x$

$a_n = a + (n-1)d$

$x = -4 + (n-1)3$

$x = -4 + 3n - 3$

$x = -7 + 3n$

$n = \frac{x+7}{3}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$437 = \frac{x+7}{2\times 3}\left[2(-4) + \left(\frac{x+7}{3} - 1\right)3\right]$

$437 = \frac{x+7}{2\times 3}\left[-8 + \left(\frac{x+4}{3}\right)3\right]$

$437 = \frac{x+7}{2\times 3}(-8 + x + 4)$

$437 = \frac{(x+7)(x-4)}{2\times 3}$

$x^2 + 7x - 4x - 28 = 2622$

$x^2 + 3x - 2650 = 0$

$x^2 + 53x - 50x - 2650 = 0$

$x(x+53) - 50(x+53) = 0$

$(x+53)(x-50) = 0$

$x + 53 = 0$

$\boxed{x = -53}$

(Neglected)

$x - 50 = 0$

$\boxed{x = 50}$

Q. 72 AP = $-2, -7, -12, \ldots$

$a = -2$

$d = -7 + 2 = -5$

$a_n = a + (n-1)d$

$-77 = -2 + (n-1)(-5)$

$-77 = -2 - 5n + 5$

$-77 - 3 = -5n$

$n = \frac{-80}{-5}$ 16

$\boxed{n = 16}$

$S_{16} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{16}{2} 8 [2(-2) + (16-1)(-5)]$

$= 8[-4 - 75]$

$= 8(-79)$

$= \boxed{-632}$

Q. 73 CASE I

$a = 8$

$d = 20$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(8) + (n-1)20]$

$= \frac{n}{2}[16 + 20n - 20]$

$= \frac{n}{2}[20n - 4]$

$= n(10n - 2)$ ——— (1)

CASE II

$a = -30$

$d = 8$

$$S_{2n} = \frac{2n}{2}\left[2a + (2n-1)d\right]$$

$$= n\left[2(-30) + (2n-1)\cdot 8\right]$$

$$= n\left[-60 + 16n - 8\right]$$

$$= n\left[16n - 68\right] \quad \text{——} \quad (2)$$

ATQ,

$$(10n - 2) = (16n - 68)$$

$$68 - 2 = 16n - 10n$$

$$66 = 6n$$

$$n = \frac{66}{6} = 11$$

$$\boxed{n = 11}$$