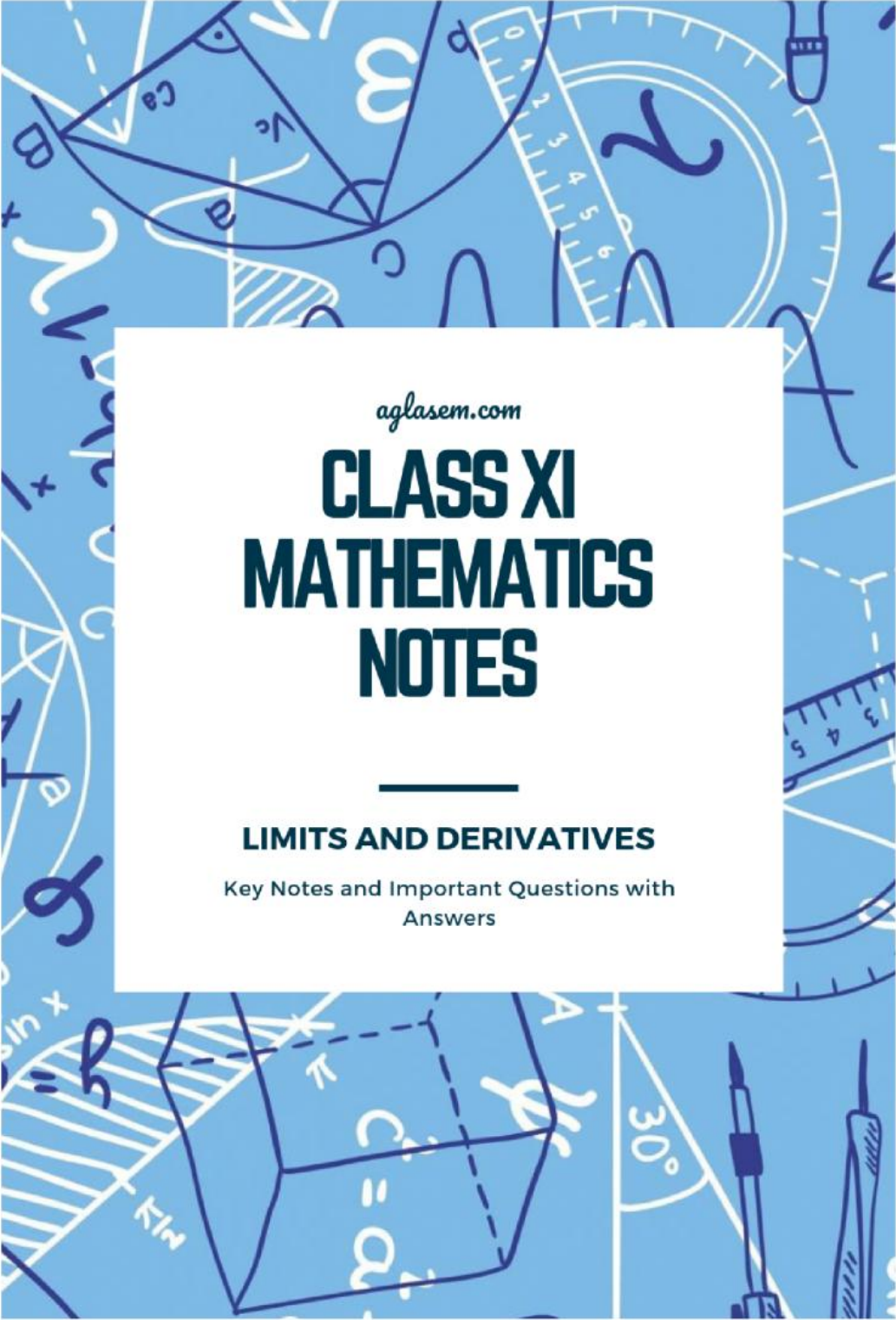


The background is a vibrant blue with a complex pattern of white mathematical sketches. These include geometric shapes like triangles with labeled vertices (A, B, C, D), circles with radii and angles, a 3D cube with dashed lines, a 30-degree angle, a ruler with markings, and various mathematical symbols such as  $\pi$ ,  $\phi$ ,  $\psi$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ ,  $\zeta$ ,  $\eta$ ,  $\theta$ ,  $\iota$ ,  $\kappa$ ,  $\lambda$ ,  $\mu$ ,  $\nu$ ,  $\xi$ ,  $\omicron$ ,  $\pi$ ,  $\rho$ ,  $\sigma$ ,  $\tau$ ,  $\upsilon$ ,  $\phi$ ,  $\chi$ ,  $\psi$ ,  $\omega$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ ,  $\zeta$ ,  $\eta$ ,  $\theta$ ,  $\iota$ ,  $\kappa$ ,  $\lambda$ ,  $\mu$ ,  $\nu$ ,  $\xi$ ,  $\omicron$ ,  $\pi$ ,  $\rho$ ,  $\sigma$ ,  $\tau$ ,  $\upsilon$ ,  $\phi$ ,  $\chi$ ,  $\psi$ ,  $\omega$ .

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# CLASS XI MATHEMATICS NOTES

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## LIMITS AND DERIVATIVES

Key Notes and Important Questions with  
Answers

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## CHAPTER - 13

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### LIMITS AND DERIVATIVES

#### KEY POINTS

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- $\lim_{x \rightarrow c} f(x) = l$  if and only if
- $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = l$
- $\lim_{x \rightarrow c} \alpha = \alpha$ , where  $\alpha$  is a fixed real number.
- $\lim_{x \rightarrow c} x^n = c^n$ , for all  $n \in \mathbb{N}$
- $\lim_{x \rightarrow c} f(x) = f(c)$ , where  $f(x)$  is a real polynomial in  $x$ .

#### Algebra of limits

Let  $f, g$  be two functions such that  $\lim_{x \rightarrow c} f(x) = l$  and  $\lim_{x \rightarrow c} g(x) = m$ , then

- $\lim_{x \rightarrow c} [\alpha f(x)] = \alpha \lim_{x \rightarrow c} f(x)$   
 $= \alpha l$  for all  $\alpha \in \mathbb{R}$

- $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x) = l \pm m$
- $\lim_{x \rightarrow c} [f(x).g(x)] = \lim_{x \rightarrow c} f(x). \lim_{x \rightarrow c} g(x) = l.m$
- $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{l}{m}, m \neq 0, g(x) \neq 0$
- $\lim_{x \rightarrow c} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow c} f(x)} = \frac{1}{l} \text{ provided } l \neq 0, f(x) \neq 0$
- $\lim_{x \rightarrow c} [(f(x))^n] = \left[ \lim_{x \rightarrow c} f(x) \right]^n = l^n, \text{ for all } n \in \mathbb{N}$

## Some important theorems on limits

- $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(-x)$
- $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  where  $x$  is measured in radians.
- $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \left[ \text{Note that } \lim_{x \rightarrow 0} \frac{\cos x}{x} \neq 1 \right]$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$
- $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

- $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$

- $\lim_{x \rightarrow 0} \frac{\log(1 + x)}{x} = 1$

- $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(a^x)}{dx} = a^x \cdot \log a$$

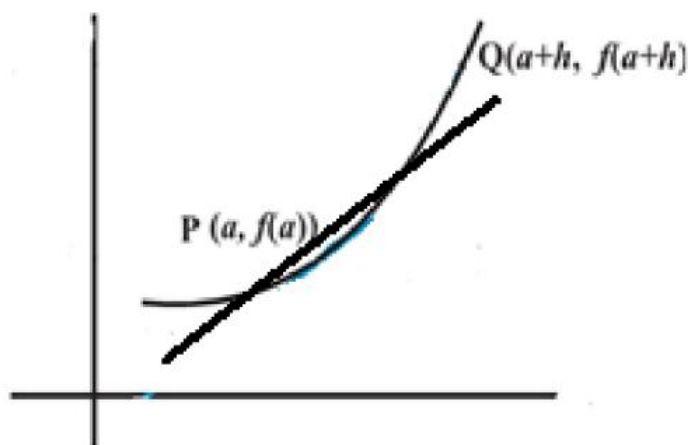
$$\frac{d(\log_e x)}{dx} = \frac{1}{x}$$

$$\frac{d(\text{constant})}{dx} = 0$$

### Laws of Logarithm

- ❖  $\log_e A + \log_e B = \log_e (AB)$
- ❖  $\log_e A - \log_e B = \log_e \left( \frac{A}{B} \right)$
- ❖  $\log_e A^m = m \log_e A$
- ❖  $\log_a 1 = 0$
- ❖ If  $\log_B A = x$  then  $B^x = A$

Let  $y = f(x)$  be a function defined in some neighbourhood of the point 'a'. Let  $P(a, f(a))$  and  $Q(a + h, f(a + h))$  are two points on the graph of  $f(x)$  where  $h$  is very small and  $h \neq 0$ .



$$\text{Slope of PQ} = \frac{f(a+h) - f(a)}{h}$$

If  $h \rightarrow 0$ , point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  (if exists) is called derivative of  $f(x)$  at the point 'a'. It is denoted by  $f'(a)$ .

### Algebra of derivatives

- $\frac{d}{dx}(cf(x)) = c \cdot \frac{d}{dx}(f(x))$  where c is a constant
- $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$
- $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \frac{d}{dx}(f(x)) - f(x) \frac{d}{dx}(g(x))}{(g(x))^2}$
- If  $y = f(x)$  is a given curve then slope of the tangent to the curve at the point (h,k) is given by  $\left. \frac{dy}{dx} \right|_{(h,k)}$  and is denoted by 'm'.

# Important Questions with Solutions

### Section-A

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

**Evaluate the following Limits:**

1.  $\lim_{x \rightarrow 3} \frac{\sqrt{2x+3}}{x+3}$

2.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

3.  $\lim_{x \rightarrow 1} \frac{\sqrt{1+x} - \sqrt{1-x}}{1+x}$

4.  $\lim_{x \rightarrow \infty} \frac{1 + 2 + 3 + \dots + n}{n^2}$

5. Find  $n$ , if  $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$ .  $n \in N$

6.  $\lim_{x \rightarrow 0} \frac{\tan^2 3x}{x^2}$

7.  $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x^2 - a^2}$

## Section-B

### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Differentiate the following functions with respect to  $x$  :

8.  $\frac{x}{2} + \frac{2}{n}$

9.  $x^2 \tan x$

10.  $\frac{x}{\sin x}$

11.  $2^x$

12.  $3^x + x^3 + 4x - 5$

13.  $(x^2 - 3x + 2)(x + 2)$

14.  $e^x \sin x + x^n \cdot \cos x.$

## Section-C

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

16. Evaluate  $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$

17. If  $f(x) = \begin{cases} \frac{|x-1|}{2(x-1)} & x \neq 1 \\ 3 & x = 1 \end{cases}$  show that  $\lim_{x \rightarrow 1} f(x)$  does not exist.

18. If  $f(x) = \begin{cases} 5x - 4 & 0 \leq x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$  show that  $\lim_{x \rightarrow 1} f(x)$  exists.

19. Let  $f(x) = \begin{cases} \cos x & x \geq 1 \\ x + k & , x < 0 \end{cases}$  find k if  $\lim_{x \rightarrow 0} f(x)$  exists.

20. Find  $\lim_{x \rightarrow 5} \left( [x + 3] - \frac{2|x - 5|}{x - 5} + 4x^2 \right)$ , if exists.

21. Find the value of 'k' if  $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$ .

**Evaluate the following Limits:**

22.  $\lim_{x \rightarrow 0} \frac{\sin(x^\circ)}{x}$

23.  $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 - 1} + \sqrt{x - 1}}{\sqrt{x^2 - 1}}, \quad x > 1$

24.  $\lim_{x \rightarrow 9} \frac{x^{\frac{3}{2}} - 27}{x^2 - 81}$

25.  $\lim_{x \rightarrow a} \frac{(x + 2)^{\frac{5}{2}} - (a + 2)^{\frac{5}{2}}}{x - a}$

26.  $\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{\cos x - 1}$

27.  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x \sin^2 3x}$

28.  $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$

$$29. \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

$$30. \quad \lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a}$$

$$31. \quad \lim_{x \rightarrow \pi} \frac{1 + \sec^3 x}{\tan^2 x}$$

$$32. \quad \frac{e^x - e^{\sin x}}{\sin x - x}$$

$$33. \quad \lim_{x \rightarrow 1} \frac{x-1}{\log_e x}$$

$$34. \quad \lim_{x \rightarrow e} \frac{\log_e x - 1}{x - e}$$

$$35. \quad \lim_{x \rightarrow 2} \left[ \frac{4}{x^3 - 2x^2} + \frac{1}{2 - x} \right]$$

$$36. \quad \lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$$

$$37. \quad \lim_{x \rightarrow 0} \frac{\sin(2+x) - \sin(2-x)}{x}$$

$$38. \quad \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$$

$$39. \quad \lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$$

$$40. \quad \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{\sin^2 x}$$

$$41. \quad \lim_{x \rightarrow 0} \frac{\sin^2(x+a) - \sin^2 a}{x}$$

**Differentiate the following functions with respect to x by first principle:**

$$42. \quad f(x) = \frac{x^2 + 1}{x}$$

$$43. \quad f(x) = \sqrt{2x+3}$$

$$44. \quad f(x) = \cos(2x-7)$$

$$45. \quad f(x) = \tan \sqrt{x}$$

$$46. \quad f(x) = e^{\tan x}$$

$$47. \quad f(x) = \cos(2x^2 - 3)$$

**Differentiate the following functions with respect to x:**

$$48. \quad f(x) = \frac{(3x+1)(2\sqrt{x}-1)}{\sqrt{x}}$$

$$49. \quad f(x) = \left(x - \frac{1}{x}\right) \left(x^2 - \frac{1}{x^2}\right)$$

$$50. \quad f(x) = \frac{\sin x - x \cos x}{x \sin x + \cos x}$$

$$51. \quad f(x) = x^3 e^x \sin x$$

$$52. \quad f(x) = x^5 e^x + x^3 \log x - 2^x$$

$$53. \quad f(x) = \frac{2^x \cdot \cot x}{\sqrt{x}}$$

54.  $f(x) = e^{2x} \sin x + x^n \cdot \cos x$

55.  $f(x) = \frac{x^2}{\cos x} - 2 \log(\cos x) + xe^x.$

56.  $f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$

57. If  $f(x) = x - [x]$  then find  $f'\left(\frac{1}{2}\right).$

58. If  $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ , prove that  $2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}.$

59. If  $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$ , prove that  $(2xy) \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}.$

60. For the curve  $f(x) = (x^2 + 6x - 5)(1 - x)$ , find the slope of the tangent at  $x = 3.$

### Section-D

#### LONG ANSWER TYPE QUESTIONS (6 MARKS)

**Differentiate the following functions with respect to x from first principle:**

61.  $f(x) = \sqrt{\cos x}$

62.  $f(x) = xe^x$

63.  $f(x) = e^{\sqrt{ax+b}}$

64.  $f(x) = \cos^2(3x - 7)$

**Evaluate the following Limits:**

$$65. \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\sqrt{2} - \sqrt{1 + \sin x}}$$

$$66. \quad \lim_{x \rightarrow 0} \frac{8}{x^8} \left[ 1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right) \cdot \cos\left(\frac{x^2}{4}\right) \right]$$

$$67. \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\sin x + \cos x)^5}{1 - \sin 2x}$$

$$68. \quad \lim_{x \rightarrow 0} \frac{9^x - 2 \cdot 6^x + 4^x}{x^2}$$

$$69. \quad \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}$$

$$70. \quad \lim_{x \rightarrow 1} \frac{(7+x)^{\frac{1}{3}} - (3+x^2)^{\frac{1}{2}}}{x-1}$$

71. Find the value of 'a' and 'b' if  $\lim_{x \rightarrow 2} f(x)$  and  $\lim_{x \rightarrow 4} f(x)$  exists where

$$f(x) = \begin{cases} x^2 + ax + b & 0 \leq x < 2 \\ 3x + 2 & 2 \leq x \leq 4 \\ 2ax + 5b & 4 < x \leq 8 \end{cases}$$

72. Evaluate  $\lim_{x \rightarrow 3} f(x)$  where

$$f(x) = \begin{cases} x - [x] & x < 2 \\ 4 & x = 3 \\ 3x - 8 & x > 3 \end{cases}$$

## ANSWERS

1.  $\frac{1}{2}$

2. 3

$$3. \quad \frac{1}{\sqrt{2}}$$

$$4. \quad \frac{1}{2}$$

$$5. \quad n = 5$$

$$6. \quad 9$$

$$7. \quad \frac{-2 \sin a}{3\sqrt{a}}$$

$$8. \quad \frac{1}{2} - \frac{2}{x^2}$$

$$9. \quad x^2 \sec^2 x + 2x \tan x$$

$$10. \quad \operatorname{cosec} x - x \cot x \cdot \operatorname{cosec} x \text{ or } \operatorname{cosec} x (1 - x \cot x)$$

$$11. \quad 2^x \log_e 2$$

$$12. \quad 3^x \log_e 3 + 3x^2 + 4$$

$$13. \quad 3x^2 - 2x - 4$$

$$14. \quad e^x (\cos x + \sin x) + x^{n-1} [n \cdot \cos x - x \sin x]$$

$$15. \quad \frac{\sqrt{2}}{8}$$

$$16. \quad \frac{3 + \sqrt{3}}{2}$$

$$19. \quad k = 1$$

$$20. \quad \text{LHL} = 109, \text{RHL} = 106 \text{ so, lim does not exist.}$$

$$21. \quad k = \frac{8}{3}$$

$$22. \quad \frac{\pi}{180}$$

$$23. \quad \frac{\sqrt{2} + 2}{2}$$

$$24. \quad \frac{1}{4}$$

$$25. \quad \frac{5}{2} (a + 2)^{\frac{3}{2}}$$



$$53. \quad \frac{2^x}{\sqrt{x}} \left[ \log 2 \cdot \cot x - \cos e c^2 x - \frac{\cot x}{2 \cdot x} \right]$$

$$54. \quad e^{2x} [\cos x + 2 \sin x] + x^{n-1} [n \cos x - x \sin x]$$

$$55. \quad x \sec x [2 - x \tan x] + 2 \tan x + e^x (x + 1)$$

$$56. \quad \sec^2 x$$

$$57. \quad 1$$

$$60. \quad 46.$$

$$61. \quad \frac{-\sin x}{2\sqrt{\cos x}}$$

$$62. \quad e^x (x + 1)$$

$$63. \quad \frac{a \cdot e^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$$

$$64. \quad -3 \cdot \sin(6x - 14)$$

$$65. \quad 4\sqrt{2}$$

$$66. \quad \frac{1}{32}$$

$$67. \quad 5\sqrt{2}$$

$$68. \quad \left( \log \frac{3}{2} \right)^2$$

$$69. \quad \frac{3}{2}$$

$$70. \quad \frac{-1}{4}$$

$$71. \quad a = -1, \quad b = 6$$

$$72. \quad 1$$