Ch-14
Surface Areas & Volumes
Exercise 14.1

Q.1 r_1 of ball $= 1$ cm
r_2 of sphere $= 8$ cm
Let the no. of balls be x.
ATQ,

$$\frac{4}{3}\pi r_2^3 = x \times \frac{4}{3}\pi r_1^3$$

$$8 \times 8 \times 8 = x \times 1 \times 1 \times 1$$

$$x = \boxed{512 \text{ balls}}$$

Q.2 d of bullet $= 5$ cm
r of bullet $= \frac{5}{2}$ cm

dimensions of cuboid = 11 dm × 1 m × 5 dm

Let the no. of bullets be x.
ATQ,

$$x \times \frac{4}{3}\pi r^3 = l \times b \times a$$

$$x \times \frac{4}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = 110 \times 100 \times 50$$

$$x = \frac{110 \times 100 \times 50 \times 3 \times 7 \times 2 \times 2 \times 2}{4 \times 22 \times 5 \times 5 \times 5}$$

$$x = \boxed{8400 \text{ bullets}}$$

Q.3 R of big ball $= 3$ cm
$r_1 = 1.5$ cm
$r_2 = 2$ cm

ATQ,

Vol of 1st ball + Vol of 2nd ball + Vol of 3rd ball = Vol of big ball

$$\frac{4}{3}\pi(1.5)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi(3)^3$$

$$3.375 + 8 + r_3^3 = 27$$

$$r_3^3 = 27 - 11.375$$

$$r_3 = \sqrt[3]{15.625}$$

$$r_3 = \boxed{2.5 \text{ cm}}$$

d of 3rd ball $= 2.5 \times 2 = \boxed{5 \text{ cm}}$ ✓

Q.4 Volume of brass $= 2.2 \text{ dm}^3 = 2200 \text{ cm}^3$

d of cylindrical wire $= 0.25$ cm

r of wire $= \frac{0.25 \text{ cm}}{2}$

ATQ,

Vol. of wire = Vol. of brass

$$\pi r^2 h = 2200$$

$$\frac{22}{7} \times \left(\frac{0.25}{2}\right)^2 \times h = 2200$$

$$h = \frac{2200 \times 7 \times 2 \times 2}{22 \times 0.25 \times 0.25}$$

$$h = 44800 \text{ cm}$$

$$h = \boxed{448 \text{ m}}$$ ✓

Q.5 d of cylinder $= 2$ cm

r of cylinder $= \frac{x}{x} = 1$ cm

In hollow cylinder,

$l = 16$ cm

external $d = 20$ cm

external $r = \frac{20}{2} = 10$ cm

thickness $= 2.5$ mm $= 0.25$ cm

internal $r = 10 - 0.25 = 9.75$ cm

ATQ,

Vol. of cylinder = Vol. of hollow cylinder

$\pi r^2 h = \pi (R_1^2 - R_2^2) h$

$(1)^2 \times h = (10^2 - 9.75^2) \times 16$

$h = (100 - 95.0625) \times 16$

$h = 4.9375 \times 16$

$\boxed{h = 79 \text{ cm}}$

$(10+9.75) \times (10-9.75) \times 16$

Q.6 Let the radius of cylindrical vessel be R.

d of vessel $= 2R$

$=$ height of vessel

d of small vessel $= 42$ cm

r of small vessel $= 21$ cm

h of small vessel $= 21$ cm

ATQ,

Vol of big vessel $= 2 \times$ Vol. of small vessel

$\pi R^2 H = 2(\pi r^2 h)$

$\pi \times (R)^2 \times (2R) = 2 \times \pi \times (21)^2 \times 21$

$2R^3 = 2 \times 21^3$

$R = \sqrt[3]{21 \times 21 \times 21}$

$\boxed{R = 21 \text{ cm}}$

d of vessel $= 2R$

$= \boxed{42 \text{ cm}}$

Q.7 No. of plates = 50

d of one plate = 14 cm

r of one plate = 7 cm

Thickness of one plate = 0.5 cm

h of cylinder formed = 0.5×50

= 25 cm

TSA of cylinder = $2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times 7 (25+7)$

$= 44 \times 32 = \boxed{1408 \text{ cm}^2}$ ✓

Q.8 No. of plates = 25

r of one plate = 10.5 cm

Thickness of one plate = 1.6 cm

h of cylinder formed = 1.6×25

= 40 cm

CSA of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \cancel{10.5}^{1.5} \times 40$

$= \boxed{2640 \text{ cm}^2}$ ✓

Vol. of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \cancel{10.5}^{1.5} \times 10.5 \times 40$

$= \boxed{13860 \text{ cm}^3}$ ✓

Q.9 r of disc = $\frac{1.5}{2} = 0.75$ cm

h of disc = 0.2 cm

R of cylinder = $\frac{4.5}{2} = 2.25$ cm

H of cylinder = 10 cm

No. of discs = $\frac{\text{Vol. of cylinder}}{\text{Vol. of one disc}}$

$= \frac{\pi R^2 H}{\pi r^2 h}$

$= \frac{2.25 \times 2.25 \times 10}{0.75 \times 0.75 \times 0.2}$

$= \boxed{450 \text{ discs}}$ ✓

Q.10 r of spherical shot = $\frac{4.2}{2} = 2.1$ cm

Dimensions of rectangular lead piece = 66 cm × 42 cm × 21 cm

No. of lead shots = $\frac{\text{Vol. of cuboid}}{\text{Vol. of one shot}}$

$= \frac{l \times b \times h}{\frac{4}{3}\pi r^3} = \frac{66 \times 42 \times 21}{\frac{4}{3} \times \frac{22}{7} \times 0.3 \times 0.7 \times 2.1}$

$= \frac{66 \times 42 \times 21 \times 10 \times 5}{4 \times 22 \times 3 \times 7 \times 21} = \frac{6930}{4.62}$

$= \boxed{1500 \text{ shots}}$ ✓

Q.11 r of spherical shot = $\frac{4}{2} = 2$ cm

Edge of cube = 44 cm

No. of shots = $\frac{\text{Vol. of cube}}{\text{Vol. of each shot}}$

$= \frac{a^3}{\frac{4}{3}\pi r^3} = \frac{44 \times 44 \times 44}{\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2}$

$$= \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 7 \times 7 \times 7}$$

$$= \boxed{2541 \text{ shots}}$$ ✓

Q.12 Let the edges of three cubes be $3x$, $4x$ & $5x$ respectively.

Total volume of 3 cubes $= (3x)^3 + (4x)^3 + (5x)^3$

$= 27x^3 + 64x^3 + 125x^3$

$= 216x^3$

ATQ,

Volume of new cube $= 216x^3$

$a^3 = 216x^3$

$a = \sqrt[3]{216x^3}$

$\boxed{a = 6x}$ ——— (1)

Diagonal of a cube $= \sqrt{3} \times$ side

$12\sqrt{3} = \sqrt{3}a$

$12\sqrt{3} = 6\sqrt{3}\,x$ (Putting (1))

$x = \frac{12\sqrt{3}}{6\sqrt{3}}$

$\boxed{x = 2}$

∴ The sides of cubes are -

$3x = 3 \times 2 = \boxed{6 \text{ cm}}$

$4x = 4 \times 2 = \boxed{8 \text{ cm}}$

$5x = 5 \times 2 = \boxed{10 \text{ cm}}$ ✓

Q.13 R of metallic sphere $= 10.5$ cm

R of each cone $= 3.5$ cm

H of each cone $= 3$ cm

Total no. of cones $= \frac{\text{Vol. of metallic sphere}}{\text{Vol. of cone}}$

$$= \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi R^2 H}$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{10.5 \times 10.5 \times 10.5}{\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3}$$

$$= \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3}$$

$$= \boxed{126 \text{ cones}}$$ ✓

Q.14 r of metallic sphere $= \frac{9}{2}$ cm

R of wire $= \frac{2}{2} = 1 \text{ mm} = 0.1 \text{ cm}$

ATQ,

Volume of wire = Volume of sphere

$$\pi (R)^2 h = \frac{4}{3} \pi r^3$$

$$(0.1)^2 \times h = \frac{4}{3} \times \frac{9}{2} \times \frac{9}{2} \times \frac{9}{2}$$

$$h = \frac{22 \times 9 \times 9 \times 9}{3 \times 2 \times 2 \times 2 \times 0.01}$$

$$h = \frac{243}{0.02}$$

$$h = \boxed{12,150 \text{ cm}}$$ ✓

Q.15 Let radius (r) of big ball be x.

radius (R) of small ball $= \frac{x}{4}$

Let the no. of balls be y.

ATQ,

y (Vol. of one small ball) = Vol. of big ball

$y \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$y \times \left(\frac{x}{4}\right)^3 = x^3$

$y \times \frac{x^3}{64} = x^3$

$y = \frac{x^3 \times 64}{x^3}$

$y = \boxed{64 \text{ balls}}$ ✓

$\frac{\text{SA of all small balls}}{\text{SA of big ball}} = \frac{64 \times 4\pi R^2}{4\pi r^2}$

$= \frac{64 \times \left(\frac{x}{4}\right)^2}{x^2} = \frac{64 \times \frac{x^2}{16}}{x^2}$

$= \frac{4}{1}$

Ratio = $\boxed{4:1}$ ✓

Q.16 R of copper sphere = 3 cm

r of cone = 3 cm

Let the radius of the cone be R.

ATQ,

Vol. of sphere = Vol. of cone

$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$

$4 \times 3 \times 3 \times 3 = R^2 \times 3$

$R = \sqrt{4 \times 3 \times 3}$

$\boxed{R = 6 \text{ cm}}$ ✓

Q.17 r of rod $= \frac{1}{2}$ cm

h of rod $= 8$ cm

H of wire $= 18$ m $= 1800$ cm

ATQ,

Vol. of wire = Vol. of rod

$\pi R^2 H = \pi r^2 h$

$R^2 \times 1800 = \left(\frac{1}{2}\right)^2 \times 8$

$R^2 \times 1800 = \frac{1}{\pi} \times 8 \times 2$

$R^2 = \frac{\pi}{1800\ 900}$

$R = \sqrt{\frac{1}{900}}$

$R = \frac{1}{30}$ cm

Thickness of wire $= d$

$= \pi \times \frac{1}{30\ 15} = \boxed{\frac{1}{15}\ \text{cm}}$

$= \boxed{0.06\ \text{cm}}$

Q.18 Internal r $(r_1) = \frac{6}{\pi} 3 = 3$ cm

External r $(r_2) = \frac{10}{\pi} 5 = 5$ cm

H of cylinder $= 2\frac{2}{3} = \frac{8}{3}$ cm

Let the radius of cylinder be R.

ATQ, Vol. of cylinder = Vol. of hollow sphere

$$\pi R^2 H = \frac{4}{3}\pi (R_2^3 - R_1^3)$$

$$R^2 \times \frac{22}{7} = \frac{4}{3} \times \frac{22}{7} (5^3 - 3^3)$$

$$R^2 \times 8 = 125 - 27$$

$$R^2 = \frac{98}{2}$$

$$R^2 = 49$$

$$R = \sqrt{49}$$

$$R = 7 \text{ cm}$$

diameter = 2R

= 2 × 7 = 14 cm

Q.19 Dimensions of cuboid = 11 cm × 10 cm × 7 cm

r of coin = $\frac{1.75}{2}$ = 0.875 cm

Thickness of coin = (NCERT)

Q.20 SA of metallic sphere = 616 cm^2

ATQ,

SA of sphere = 616 cm^2

$4\pi r^2 = 616$

$r^2 = \frac{616 \times 7}{4 \times 22}$

$r = \sqrt{7 \times 7}$

$r = 7$ cm

h of cone = 28 cm

ATQ,

Vol. of cone = Vol. of sphere

$$\frac{1}{3}\pi R^2 h = \frac{4}{3}\pi r^3$$

$$R^2 \times 28 = 4 \times 7 \times 7 \times 7$$

$$R^2 = \frac{4 \times 7 \times 7 \times 7}{28}$$

$$R = \sqrt{7 \times 7}$$

$$R = 7 \text{ cm}$$

Diameter base of cone = $2R$

$= 7 \times 2 = \boxed{14 \text{ cm}}$

Ch-14

Exercise 14.1

Q.21 h of bucket = 32 cm
r of bucket = 18 cm
H of conical heap = 24 cm
ATQ,

Vol. of bucket = Vol. of cone

$\pi r^2 h = \frac{1}{3}\pi R^2 H$

$18 \times 18 \times 32 = \frac{1}{3} \times R^2 \times 24$

$R^2 = \frac{18 \times 18 \times 32 \times 3}{24}$

$R = \sqrt{18 \times 18 \times 4}$

$R = 18 \times 2$

$\boxed{R = 36 \text{ cm}}$ ✓

Slant height $= \sqrt{R^2 + H^2}$

$= \sqrt{36^2 + 24^2} = \sqrt{1296 + 576}$

$= \sqrt{1872} = \boxed{12\sqrt{13} \text{ cm}}$ ✓

Q.22 ~~Let the no. of cones~~
r of sphere = 5.6 cm
R of cone = 2.8 cm
H of cone = 3.2 cm

No. of cones $= \frac{\text{Vol. of sphere}}{\text{Vol. of 1 cone}} = \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi R^2 H} = \frac{4 \times 5.6 \times 5.6 \times 5.6}{2.8 \times 2.8 \times 3.2}$

$= \frac{89.6}{3.2} 28 = \boxed{28 \text{ cones}}$ ✓

Q. 23 Dimensions of cuboid = 53 cm × 40 cm × 15 cm

Internal r_1 of pipe = $\frac{7}{2}$ cm

External r_2 of pipe = $\frac{8}{2}4 = 4$ cm

ATQ,

Vol. of cuboid = Vol. of pipe

$l \times b \times h = \pi H (r_2^2 - r_1^2)$

$53 \times 40 \times 15 = \frac{22}{7} \times H \left(4^2 - \frac{7^2}{2^2}\right)$

$\frac{53 \times 40 \times 15 \times 7}{22} = H \left(16 - \frac{49}{4}\right)$

$\frac{53 \times 40 \times 15 \times 7}{22} = H \left(\frac{64 - 49}{4}\right)$

$\frac{53 \times 40 \times 15 \times 7}{22} = H \left(\frac{15}{4}\right)$

$H = \frac{53 \times 20 \times 15 \times 7 \times 4}{11 \times 15}$

$H = \frac{29680}{11}$

$\boxed{H = 2698.18 \text{ cm}}$

Q. 24 Internal r_1 of sphere = $\frac{6}{2}3 = 3$ cm

External r_2 of sphere = $\frac{10}{2}5 = 5$ cm

R of cylinder = $\frac{14}{2}7 = 7$ cm

ATQ,

Vol. of cylinder = Vol. of sphere

$\pi R^2 H = \frac{4}{3} \pi (r_2^3 - r_1^3)$

$$7 \times 7 \times H = \frac{4}{3}\left(5^3 - 3^3\right)$$

$$7 \times 7 \times H = \frac{4}{3}\left(125 - 27\right)$$

$$7 \times 7 \times H = \frac{4}{3}\left(98\right)$$

$$H = \frac{4 \times 98}{3 \times 7 \times 7}$$

$$H = \frac{8}{3}\,2.66$$

$$\boxed{H = 2.66\ cm}$$

Q.25 Internal r_1 of sphere $= \frac{4}{2} = 2$ cm

External r_2 of sphere $= \frac{8}{2}4 = 4$ cm

R of cone $= \frac{8}{2}4 = 4$ cm

ATQ,

$$\text{Vol. of cone} = \text{Vol. of hollow sphere}$$

$$\frac{1}{3}\pi R^2 H = \frac{4}{3}\pi (r_2^3 - r_1^3)$$

$$4 \times 4 \times H = 4\left(4^3 - 2^3\right)$$

$$4 \times 4 \times H = 4\left(64 - 8\right)$$

$$H = \frac{56}{4}14$$

$$\boxed{H = 14\ cm}$$

Q.26 Internal r_1 of sphere = 2 cm
external r_2 of sphere = 4 cm
R of cone = 4 cm (same as Q.25)

Q.27 R of big ball = 3 cm
r_1 (first small ball) = 1.5 cm = $\frac{3}{2}$ cm
r_2 (second small ball) = 2 cm
ATQ,
Vol of big ball = Vol. of first ball + Vol. of second ball + Vol. of third ball

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi\left[(1.5)^3 + (2)^3 + (r_3)^3\right]$$

$$27 = 3.375 + 8 + r_3^3$$

$$27 - 11.375 = r_3^3$$

$$r_3 = \sqrt[3]{15.625}$$

$$r_3 = \boxed{2.5 \text{ cm}}$$

Diameter = $2r_3 = 2.5 \times 2$
= 5 cm

Q.28 r of pond = $\frac{40}{2} = 20$ m

Width of path = 2 m
External R = 20 + 2 = 22 m
Depth of path = 20 cm = 0.2 m

Gravel required = volume of path
$= \pi h (R^2 - r^2)$
$= \frac{22}{7} \times 0.2 (22^2 - 20^2)$

$$= \frac{22}{7} \times 0.2 (22+20)(22-20)$$

$$= \frac{22}{7} \times 0.2 \times 42^{6} \times 2$$

$$= \boxed{52.8\ m^3}$$ ✓

Q.29 r of well $= \frac{3.5}{2}$ m

Depth of well = 16 m

l of platform = 27.5 m

b of platform = 7 m

ATQ,

Vol. of earth on platform = Vol. of earth dug out

$l \times b \times h = \pi r^2 H$

$27.5 \times 7 \times h = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 16$

$$h = \frac{\cancel{22}^{11} \times \cancel{3.5}^{1} \times \cancel{3.5}^{1} \times \cancel{16}^{8}}{\cancel{7} \times \cancel{2} \times \cancel{2} \times \cancel{7} \times 27.5}$$

$$h = \frac{22}{27.5}$$

$h = 0.8$ m

$\boxed{h = 80\ cm}$ ✓

Q.30 r of well $= \frac{x}{x} = 1$ m ~~= 100 cm~~

Depth of well = 14 m ~~= 1400 cm~~

H of embankment = 40 cm = 0.4 m

Width of embankment = R embankment − r well

ATQ,

Vol. of earth dug out = Vol. of embankment

$\pi r^2 h = \pi (R^2 - r^2) H$

$100 \times 100 \times 1400 = (R^2 - 100^2) \times 0.4$

$\frac{100 \times 100 \times 1400}{0.4} = R^2 - 10000$

$\frac{100 \times 100 \times 1400 \times 10}{4} = R^2 - 10000$

$350000000 - 10000 = R^2$

$R =$

$1 \times 1 \times 14 = (R^2 - 1) \times 0.4$

$\frac{14}{0.4} = R^2 - 1$

$R^2 - 1 = 35$

$R^2 = 35 + 1$

$R^2 = 36$

$R = \sqrt{36}$

$\boxed{R = 6\ m}$ ✓

Width of embankment $= R - r$

$= R - 1 = 6 - 1$

$= \boxed{5\ m}$ ✓

Q. 31 r of well = 4 m

Width of embankment = 3 m

External R = 3 + 4 = 7 m

Depth of well = 14 m

ATQ,

Vol. of embankment = Vol. of earth dug out

$\pi (R^2 - r^2) H = \pi r^2 h$

$(7^2 - 4^2) H = 4 \times 4 \times 14$

$$(49-16)\,H = 16 \times 14$$

$$H = \frac{16 \times 14}{33}$$

$$\boxed{H = 6.78\ m}$$ ✓

Q.32 r of well $= \frac{3}{2}$ m

Width of embankment = 4m

External $R = \frac{3}{2} + 4 = \frac{3+8}{2} = \frac{11}{2}$ m

Depth of well = 14m

ATQ,

Vol. of embankment = Vol. of earth dug out

$$\pi (R^2 - r^2) H = \pi r^2 h$$

$$\left(\frac{11^2}{2^2} - \frac{3^2}{2^2}\right) H = \frac{3}{2} \times \frac{3}{2} \times 14$$

$$\left(\frac{121}{4} - \frac{9}{4}\right) H = \frac{9}{4} \times 14$$

$$\left(\frac{112}{4}\right) H = \frac{9}{4} \times 14$$

$$H = \frac{9 \times 14}{4 \times 28_2}$$ ✓

$$\boxed{H = 1.125\ m}$$

Q.33 Side of cube = 9 cm

For the largest cone,

Side of cube = Diameter of cone

$$9 = 2r$$

$$r = \frac{9}{2}\ cm$$

Side of cube = Height of cone = 9 cm

Volume of largest cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3$

$= \frac{2673}{14} = \boxed{190.92\ cm^3}$

Q.34 h of cylindrical bucket = 32 cm
r of cylindrical bucket = 18 cm
H of conical heap = 24 cm
ATQ,
Vol. of conical heap = Vol. of bucket

$\frac{1}{3}\pi R^2 H = \pi r^2 h$

$\frac{1}{3} \times R^2 \times 24 = 18 \times 18 \times 32$

$R^2 = \frac{18 \times 18 \times 32 \times 4}{24}$

$R^2 = \sqrt{18 \times 18 \times 4}$

$R = 18 \times 2$

$\boxed{R = 36\ cm}$

Slant height $= \sqrt{R^2 + H^2}$

$= \sqrt{36^2 + 24^2} = \sqrt{1296 + 576}$

$= \sqrt{1872} = \boxed{43.26\ cm}$

Q.35 l of roof = 6 m = 600 cm
b of roof = 4m = 400 cm
r of cylindrical vessel = 20 cm
h of rainfall = 1 cm

ATQ, Vol. of water in vessel = Vol. of water in roof

$\pi r^2 H = l \times b \times h$

$\frac{22}{7} \times 20 \times 20 \times H = 400 \times 600 \times 1$

$\frac{22}{7} \times 400 \times H = 400 \times 600$

$H = \frac{600 \times 7}{22}$

$\boxed{H = 190.9 \text{ cm}}$ ✓

Q.36 r of cylindrical vessel = $\frac{2}{2}$ = 1m

H of cylindrical vessel = 3.5 m

Dimensions of roof = 22m × 20 m × h

ATQ,

Vol. of water in vessel = Vol. of water in roof

$\pi r^2 H = l \times b \times h$

$\frac{22}{7} \times 1 \times 1 \times 3.5 = 22 \times 20 \times h$

$h = \frac{1}{2 \times 20}$

$h = \frac{1}{40} = 0.025 \text{ m}$

$\boxed{h = 2.5 \text{ cm}}$ ✓

Q.37 Radius of conical flask = r

Height of conical flask = h

Radius of cylindrical flask = mr

ATQ,

Vol. of water in cylindrical flask = Vol. of water in conical flask

$$\pi r^2 H = \frac{1}{3}\pi r^2 h$$

$$(mr)^2 H = \frac{1}{3} r^2 h$$

$$m^2 r^2 H = \frac{1}{3} r^2 h$$

$$\boxed{H = \frac{h}{3m^2}}$$

Q.38 r of cylindrical tank $= \frac{21}{2}$ m

h of cylindrical tank $= 5$ m

Dimensions of rectangular tank $= 15\text{m} \times 11\text{ m} \times H$

-ATQ,

Vol. of rectangular tank = Vol. of cylindrical tank

$$l \times b \times H = \pi r^2 h$$

$$15 \times 11 \times H = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 5$$

$$H = \frac{11 \times 3 \times 21 \times 5}{2 \times 15 \times 11}$$

$$\boxed{H = 10.5\text{ m}}$$

Q.39 r of hemispherical bowl $= 9$ cm

R of cylindrical bottle $= \frac{3}{2}$ cm

H of cylindrical bottle $= 4$ cm.

$\therefore$ No. of bottles $= \frac{\text{Vol. of bowl}}{\text{Vol. of 1 bottle}}$

$$= \frac{\frac{2}{3}\pi R^3}{\pi R^2 H}$$

$$= \frac{\frac{2}{3} \times 9 \times 9 \times 9}{\frac{3}{2} \times \frac{3}{2} \times 4} = \frac{2 \times 9 \times 9 \times 9 \times 2 \times 2}{3 \times 3 \times 3 \times 4 \times 2}$$

$$= \boxed{54 \text{ bottles}}$$ ✓

Q.40 R of cylindrical tub = 12 cm
Depth of cylindrical tub = 20 cm
Rise of level of water = 6.75 cm
Let the radius of ball = R
ATQ,

Vol. of ball = Vol. of water rise in tub

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\frac{4}{3} \times R^3 = 12 \times 12 \times 6.75$$

$$R^3 = \frac{12 \times 12^3 \times 6.75 \times 3}{4}$$

$$R = \sqrt[3]{729}$$ ✓

$$\boxed{R = 9 \text{ cm}}$$

Ch-14
Exercise 14.1

Q.41 l of tank $= 80\ m$
b of tank $= 50\ m$
Water displaced by 1 person $= 0.04\ m^3$
Water displaced by 500 persons $= 0.04 \times 500$
$= 20\ m^3$

ATQ,
Vol. of water rise in tank = Vol. of water displacement
$l \times b \times h = 20\ m^3$
$80 \times 50 \times h = 20$
$h = \frac{20}{4000}$
$h = \frac{1}{200} = \frac{5}{1000}$
$h = 0.005\ m$
$h = 0.5\ cm$

Q.42 r of cylindrical jar $= 6\ cm$
Rise of oil level $= 2\ cm$
R of iron sphere $= 1.5\ cm = \frac{3}{2}\ cm$

No. of iron spheres $= \frac{\text{Vol. of raised oil}}{\text{Vol. of 1 iron sphere}}$
$= \frac{\pi r^2 h}{\frac{4}{3}\pi R^3} = \frac{6 \times 6 \times 2}{\frac{4}{3} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}$
$= \frac{6 \times 6 \times 2 \times 3 \times 2 \times 2 \times 2}{4 \times 3 \times 3 \times 3} = 16$ spheres

Q.43 r of cylindrical tub = 12 cm
Depth of water in tub = 20 cm
Let the level of water rise by h.
R of spherical ball = 9 cm
ATQ,

Vol. of water rise in tub = Vol. of ball

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$12 \times 12 \times h = \frac{4}{3} \times 3 \times 9 \times 9$$

$$h = \frac{4 \times 3 \times 9 \times 9}{144}$$

$$h = 6.75 \text{ cm}$$

Q.44 r of sphere = 2 cm
No. of spheres = 16
Dimensions of rectangular box = 16 cm × 8 cm × 8 cm
ATQ,

Vol. of liquid = Vol. of box − Vol. of 16 spheres

$$= (l \times b \times h) - \left(16 \times \frac{4}{3}\pi r^3\right)$$

$$= (16 \times 8 \times 8) - \left(16 \times \frac{4}{3} \times \frac{669}{213} \times 2 \times 2 \times 2\right)$$

$$= (1024) - \left(\frac{342528}{639}\right) = 1024 - 536.03$$

$$= 487.97 \text{ cm}^3$$

Q.45 Area of base of cuboid = l × b = 160 cm²
h of cuboid = 12 cm
Rise of water level = 2 cm
Vol. of raised water = 160 × 2 = 320 cm³

ATQ,

Vol. of 3 spheres = Vol. of water raised in cuboid

$$3\left(\frac{4}{3}\pi r^3\right) = 320 \text{ cm}^3$$

$$4 \times \frac{22}{7} \times r^3 = 320$$

$$r^3 = \frac{320 \times 7}{4 \times 22}$$

$$r^3 = \frac{280}{11}$$

$$r = \sqrt[3]{25.454}$$

$$r = 2.94 \text{ cm}$$

Q.46 r of marble $= \frac{1.4}{2} = 0.7$ cm

R of vessel $= \frac{7}{2}$ cm

ATQ,

Vol. of 150 marbles = Vol. of water displaced

$$150 \times \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$150 \times \frac{4}{3} \times 0.7 \times 0.7 \times 0.7 = \frac{7}{2} \times \frac{7}{2} \times h$$

$$h = \frac{125 \times 4 \times 0.7 \times 0.7 \times 0.7 \times 2 \times 2}{5 \times 7 \times 7}$$

$$h = \frac{150 \times 4 \times 0.7 \times 0.7 \times 0.7 \times 2 \times 2}{15 \times 7 \times 7}$$

Vol. of 150 marbles $= 150 \times \frac{4}{3}\pi r^3 = \frac{50}{150} \times 4 \times \frac{22}{7} \times (0.7)$

$$= 50 \times 4 \times \frac{22}{7} \times 0.343 \quad 0.049 = 216 \text{ cm}^3$$

ATQ,

Vol. of water displaced = Vol. of 150 marbles

$\pi R^2 h = 216 \text{ cm}^3$

$\frac{22}{7} \times 3.5 \times 3.5 \times h = 216$

$h = \frac{216 \times 7}{11 \times 3.5 \times 3.5}$

$\boxed{h = 5.6 \text{ cm}}$

Q.47 h of cone = 11 cm

r of cone = 2.5 cm

R of sphere = $\frac{0.5}{2} = 0.25$ cm

No. of balls = $\frac{\text{Vol. of cone}}{\text{Vol. of 1 sphere}} \times \text{Water flowed}$

$= \frac{\frac{1}{3}\pi r^2 h \times \frac{2}{5}}{\frac{4}{3}\pi R^3}$

$= \frac{2.5 \times 2.5 \times 11 \times 2}{0.25 \times 0.25 \times 0.25 \times 4 \times 5} = \boxed{440 \text{ balls}}$

Q.48 r of each sphere = 2 cm

No. of spheres = 16

Dimensions of cuboidal box = 16 cm × 8 cm × 8 cm

ATQ,

Vol. of water = Vol of box − Vol. of 16 spheres

$= (l \times b \times h) - 16\left(\frac{4}{3}\pi r^3\right)$

$= (16 \times 8 \times 8) - \left(16 \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2\right)$

$= 1024 - \frac{704}{21} = 1024 - 536.38$

$= \boxed{487.62 \text{ cm}^3}$

Q. 49 Internal r of pipe = 1 cm
Length of water flowing in one sec (h) = 80 cm
Vol. of water flowing in one sec = $\pi r^2 h$
$= \pi \times 1 \times 1 \times 80$
$= 80\pi \text{ cm}^3$

Vol of water flowing in 30 mins $= 80\pi \times 60 \times 30$
$= 144000\pi \text{ cm}^3$

R of cylindrical tank = 40 cm
Vol. of water in tank $= \pi R^2 H$
$= \pi \times 40 \times 40 \times H = 1600\pi H \text{ cm}^3$

ATQ,

Vol. of water in tank = Vol. of water flowing in 30 mins

$1600\pi H = 144000\pi$

$H = \frac{144000}{1600} 90$

$\boxed{H = 90 \text{ cm}}$

Q. 50 b of canal = 1.5 m
h of canal = 6 m
Area of cross section = $1.5 \times 6 = 9 \text{ m}^2$
Speed of water = 10 km/h = $\frac{10000}{60}$ m/min

Vol. of water that flows in 30 mins $= 30 \times \pi \times \frac{1000}{60\pi}$

$= 45000 \text{ m}^3$

ATQ,

Vol. of water that flows = Vol. of field
45000 m^3 = (Area of field) $\times h$
45000 = Area of field $\times 0.08$
Area of field $= \frac{45000}{0.08} = \boxed{562500 \text{ m}^2}$

Q.51 NCERT

Q.52 r of tank $= 1.5\ m$

h of tank $= 3.5\ m$

Vol. of half of tank $= \frac{\pi r^2 h}{2}$

$= \frac{22}{7} \times \frac{1}{2} \times 1.5 \times 1.5 \times 3.5$

$= \frac{173.25}{14}\ m^3$

$1\ m^3 = 1000\ l$

Vol. of half of tank in $l = \frac{173.25}{14} \times 1000$

Time taken to half empty the tank @ 225 l/min $= \frac{173.25 \times 1000}{14 \times 225}$

$= \boxed{55 \text{ mins}}$

Q.53 Increase in the level (h) of water in $1/2$ hr $= 3.15\ m = 315\ cm$

r of water tank $= 40\ cm$

Vol. of water in 30 mins $= \pi r^2 h$

$= \pi \times 40 \times 40 \times 315$

$= 504000\pi\ cm^3$

Rate of flow of water $= 2.52\ km/h$

Length of water in $1/2$ hr (pipe) $= 2.52 \times \frac{1}{2} = 1.26\ km$

$= 126000\ cm$

Vol. of water that flows in $1/2$ (pipe) $= \pi R^2 H = \pi \times R^2 \times 126000\ cm^3$

ATQ,

Vol. of water that flows in 1/2 hr (pipe) = Vol of water in 30 mins

$\pi R^2 \times 126000 = 5040000\pi$

$R^2 = \frac{504000}{126000}$

$R = \sqrt{4}$

$R = 2$ cm

Diameter $= 2R$

$= 2 \times 2 = \boxed{4 \text{ cm}}$

Q.54 l of cuboidal pond $= 50$ m

b of cuboidal pond $= 44$ m

Rise in water level $= 21$ cm $= \frac{21}{100}$ m

Vol. of water in pond $= l \times b \times h$

$= 50 \times 44 \times \frac{21}{100} = 462 \text{ m}^3$

r of pipe $= \frac{14}{2} 7 = 7$ cm $= \frac{7}{100}$ m

Length of water flowing in 1 hr $= 15$ km

$= 15000$ m

Vol. of water flowing in 1 hr $= \pi r^2 H$

$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000$

$= 231 \text{ m}^3$

Required time $= \frac{\text{Vol. of water in pond}}{\text{Vol. of water flowing in 1 hr}}$

$= \frac{462}{231} 2 = \boxed{2 \text{ hrs}}$

Q.55 b of canal $= 300\ cm = 3\ m$

h of canal $= 120\ cm = 1.2\ m$

Speed of canal water $= 20\ km/h = 20000\ m/h$

Vol. of water flown in 1 hr $= l \times b \times a$

$= 3 \times 1.2 \times 20000$

$= 72000\ m^3$

Vol. of water flown in 20 mins $= \frac{72000 \times 20}{60}$

$= 24000\ m^3$

Standing water required $= 8\ cm = 0.08\ m$

Area irrigated in 20 mins $= \frac{24000}{0.08} = \frac{2400000}{8} = 300000$

$= 300000\ m^3$

$= \boxed{30 \text{ hectares}}$ [1 hectare $= 10000\ m^3$]

Q.56 $r + h$ (in cylinder) $= 37\ cm$

TSA of cylinder $= 1628\ cm^2$

ATQ,

$2\pi r(r+h) = 1628\ cm^2$

$2 \times \frac{22}{7} \times r\,(37) = 1628$

$r = \frac{1628 \times 7}{2 \times 22 \times 37}$

$\boxed{r = 7\ cm}$

$h = 37 - r = 37 - 7$

$= \boxed{30\ cm}$

Vol. of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30$

$= \boxed{4620\ cm^3}$

Q.57 r of cylinder = ~~18 m~~ $\frac{36}{2}$ = 18 m

= R of cone

h of tent = 77 dm = 7.7 m

h of cylinder = 44 dm = 4.4 m

H of cone = 7.7 − 4.4 = 3.3 m

L of cone = $\sqrt{R^2 + H^2}$

$= \sqrt{(18)^2 + (3.3)^2}$

$= \sqrt{324 + 10.89}$

$= \sqrt{334.89}$

= 18.3 m

Canvas required for tent = CSA of cone + CSA of cylinder

$= \pi RL + 2\pi rh$

$= \pi (RL + 2rh)$

$= \frac{22}{7}[(18 \times 18.3) + (2 \times 18 \times 4.4)]$

$= \frac{22}{7} \times 18 [18.3 + 8.8]$

$= \frac{22}{7} \times 18 \times 27.1$

$= \frac{10731.6}{7}\ m^2$

Cost of canvas @ ₹ 3.5/m^2 = $\frac{10,731.6}{2} \times 3.5$

= ₹ 5365.8

Q.58 r of cylinder = 7 cm

h of cylinder = 14 cm

R of largest sphere carved out = r of cylinder = 7 cm

Vol. of sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = \frac{4312}{3}$

$= \boxed{1437.3\ cm^3}$

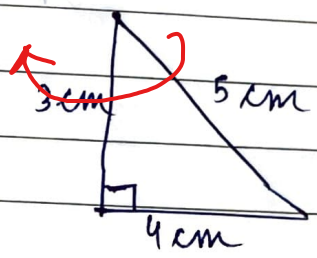
Q.59 Sides of right angled Δ = 3 cm, 4 cm, 5 cm

In first case,

r of cone = 4 cm

h of cone = 3 cm

l of cone = 5 cm



Vol of cone$_1 = \frac{1}{3}\pi r^2 h$

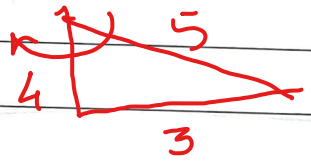
$= \frac{1}{3} \times \pi \times 4 \times 4 \times 3 = 16\pi\ cm^3$

In second case,

r of cone = 3 cm

h of cone = 4 cm

l of cone = 5 cm



Vol. of cone$_2 = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3 \times 3 \times 4$

$= 12\pi\ cm^3$

Difference in volumes = Vol$_2$ − Vol$_1$ = $16\pi - 12\pi$

$= \boxed{4\pi\ cm^3}$

CSA of cone$_1$ = $\pi r l = \pi \times 4 \times 5 = \boxed{20\pi\ cm^2}$

CSA of cone$_2$ = $\pi r l = \pi \times 3 \times 5 = \boxed{15\pi\ cm^2}$

Q.60 r of cone $= \frac{14}{2} = 7$ m

h of cone $= 24$ m

l of cone $= \sqrt{h^2 + r^2}$

$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$

$= \sqrt{625} = 25$ m

ATQ,

Area of cloth = CSA of tent

$l \times b = \pi r l$

$l \times 5 = \frac{22}{7} \times 7 \times 25$

$l = \frac{22 \times 25}{5}$

$\boxed{l = 110 \text{ m}}$

Cost of cloth @ ₹25/m^2 $= 110 \times 25$

$= \boxed{₹2750}$

Date
Page

Ch-14

Exercise 14.1

Q.61 Vol. of hemisphere = $2425\frac{1}{2}$ cm^3

ATQ,

$$\frac{2}{3}\pi r^3 = 2425\frac{1}{2} \text{ cm}^3$$

$$\frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$$

$$r^3 = \frac{4851 \times 7 \times 3}{2 \times 22 \times 2}$$

$$r^3 = 1157.625$$

$$r = \sqrt[3]{1157.625}$$

$$r = 10.5 \text{ cm}$$

CSA of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= \boxed{693 \text{ cm}^2}$$

Q.62 Let the external & internal radius of hollow cylinder be R & r respectively.

$\therefore$ r of cylinder = 14 cm

ATQ, CSA of hollow cylinder = 88 cm^2

$$2\pi(R-r)h = 88$$

$$2 \times \frac{22}{7} \times 14(R-r) = 88$$

$$R - r = \frac{88 \times 7}{7 \times 22 \times 7}$$

$$R - r = 1 \quad \text{---(1)}$$

Vol. of metal in hollow cylinder $= 176\ cm^3$

$$\pi (R^2 - r^2) h = 176$$

$$\frac{22}{7} (R^2 - r^2) \times 14 = 176$$

$$R^2 - r^2 = \frac{176 \times 4}{2 \times 22}$$

$$(R+r)(R-r) = 4$$

$$(R+r)\,1 = 4 \quad \text{(from ①)}$$

$$R + r = 4 \quad \text{—— ②}$$

Add ① & ②

$$R - r = 1$$

$$(+)\ R + r = 4$$

$$2R = 5$$

$$R = 2.5\ cm$$

Putting value R in ①

$$R - r = 1$$

$$2.5 - r = 1$$

$$r = 2.5 - 1$$

$$r = 1.5\ cm$$

External diameter $= 2R = 2 \times 2.5 = \boxed{5\ cm}$

Internal diameter $= 2r = 2 \times 1.5 = \boxed{3\ cm}$

Q.63 Internal r of hemisphere $= \frac{21}{2}$ cm

External R of hemisphere $= \frac{25.2}{2} = \frac{252}{10} \times \frac{63}{5} = \frac{63}{5}$ cm

SA of hollow hemispherical vessel = Outer CSA + Inner CSA + area of upper ring

$= 2\pi R^2 + 2\pi r^2 + (\pi R^2 - \pi r^2)$

$= 2\pi R^2 + 2\pi r^2 + \pi R^2 - \pi r^2$

$= 3\pi R^2 + \pi r^2$

$= \left(\frac{3 \times 22}{7} \times \frac{63}{5} \times \frac{63}{5}\right) + \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right)$

$= 1496.88 + 346.5$

$= 1843.38 \text{ cm}^2$

Cost of painting @ ₹0.10 / cm² = 1843.38 × 0.10

= ₹184.338

Q.64 To prove – SA of sphere = CSA of circumscribed cylinder

Proof – Radius of sphere = r

Radius of cylinder = r

Height of cylinder = Diameter = 2r

SA of sphere $= 4\pi r^2$ —— ①

CSA of cylinder $= 2\pi r h = 2\pi r(2r)$

$= 4\pi r^2$ —— ②

From ① & ②

SA of sphere = CSA of circumscribed cylinder

Hence, proved

Q.65 TSA of solid hemisphere = 462 cm²

ATQ,

$3\pi r^2 = 462 \text{ cm}^2$

$3 \times \frac{22}{7} \times r^2 = 462$

$r^2 = \frac{462 \times 7}{3 \times 22}$

$r = \sqrt{7 \times 7}$

$r = 7\,cm$

Vol of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{2156}{3} = \boxed{718.66\,cm^3}$

Q.66 R of cylindrical pipe $= \frac{5}{2}\,mm = \frac{5}{2} \times \frac{1}{10}\,cm$

$= \frac{1}{4}\,cm$

Speed of water (H) $= 10\,m/min = 1000\,cm/min$

Vol. of water flowing in 1 min $= \pi R^2 H = \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 1000$

$= \frac{1375}{7}\,cm^3$

r of conical vessel $= \frac{40}{2} = 20\,cm$

h of vessel $= 24\,cm$

Vol. of vessel $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 24$

$= \frac{70400}{7}\,cm^3$

Time required to fill vessel $= \frac{\text{Vol of vessel}}{\text{Vol. of water flowing in 1 min}}$

$= \frac{\frac{70400}{7}}{\frac{1375}{7}} = \frac{70400}{1375} = \frac{256}{5}$

$= \boxed{51.2\ mins}$

Q.67 r of cone = 60 cm
h of cone = 120 cm
R of cylinder = 60 cm
H of cylinder = 180 cm

Vol. of water left in cylinder = Vol. of water in cylinder − Vol. of cone

$= \pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \left(\frac{22}{7} \times 60 \times 60 \times 180\right) - \left(\frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times 120\right)$

$= \frac{22}{7} \times 60 \times 60 \left[180 - \left(\frac{1}{3} \times 120\right)\right]$

$= \frac{22}{7} \times 60 \times 60 \times 140$

$= 1584000 \text{ cm}^3 = \boxed{1.584 \text{ m}^3}$

Q.68 r of cone $= \frac{9}{2} = 4.5$ m

h of cone = 3.5 m

Vol. of rice = Vol. of cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{3.5}{1} \times 3.5$

$= \boxed{74.25 \text{ m}^3}$

l of cone $= \sqrt{r^2 + h^2} = \sqrt{(4.5)^2 + (3.5)^2}$

$= \sqrt{20.25 + 12.25} = \sqrt{32.5} = 5.7$ m

Canvas required = CSA of cone

$= \pi r l = \frac{22}{7} \times \frac{9}{2} \times 5.7$

$= \boxed{80.61 \text{ m}^2}$

Q.69 r of cylindrical bucket = 18 cm
h of bucket = 32 cm
H of conical heap = 24 cm
ATQ
Vol. of conical heap = Vol. of cylindrical bucket

$\frac{1}{3} \times R^2 H = \pi r^2 h$

$\frac{1}{3} \times R^2 \times 24 = 18 \times 18 \times 32$

$R^2 = \frac{18 \times 18 \times 32 \times 8}{24}$

$R^2 = \sqrt{18 \times 18 \times 4}$

$R = 18 \times 2$

$R = 36$ cm

L of cone $= \sqrt{H^2 + R^2}$

$= \sqrt{24^2 + 36^2} = \sqrt{576 + 1296}$

$= \sqrt{1872} = 43.26$ cm

Q.70 r of hemispherical bowl = 9 cm
R of bottle = 1.5 cm
H of bottle = 4 cm

No. of bottles required $= \frac{\text{Vol. of bowl}}{\text{Vol. of one bottle}}$

$= \frac{\frac{2}{3}\pi r^3}{\pi R^2 H} = \frac{\frac{2}{3} \times 9 \times 9 \times 9}{1.5 \times 1.5 \times 4}$

= 54 bottles

Q.71 h of pencil = 25 cm

circumference of base of pencil = $2\pi r$

$= 1.5$ cm

CSA of pencil = $2\pi r h$

$= 1.5 \times 25$

$= 37.5 \text{ cm}^2$

CSA of 120000 pencils $= 37.5 \times 120000$

$= 4500000 \text{ cm}^2$

$= 45000 \text{ dm}^2$

cost of colouring @ ₹ 0.05/dm² $= 45000 \times 0.05$

$= \boxed{₹ 2250}$

Q.72 r of conical vessel = 5 cm

h of vessel = 24 cm

Vol of vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5 \times 5 \times 24 = 200\pi \text{ cm}^3$

Vol. of water in vessel $= \frac{3}{4}$ (Vol. of vessel)

$= \frac{3}{4} \times 200\pi = 150\pi \text{ cm}^3$

R of cylindrical vessel = 10 cm

ATQ,

Vol. of water in vessel = Vol. of cylindrical vessel

$150\pi \text{ cm}^3 = \pi R^2 H$

$10 \times 10 \times H = 150$

$H = \frac{150}{10 \times 10}$

$\boxed{H = 1.5 \text{ cm}}$