

NCERT SOLUTIONS

CLASS - 12th



aglasem.com

Class : 12th
Subject : Maths
Chapter : 3
Chapter Name : Matrices

Q1 In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write

- (i) The order of the matrix,
- (ii) The number of elements,
- (iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Answer.

(i) In the given matrix, the number of rows is 3 and the number of columns is 4.

Therefore, the order of the matrix is 3×4 .

(ii) since the order of the matrix is 3×4 , there are $3 \times 4 = 12$ elements in it.

(iii) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$

Page : 64 , Block Name : Exercise 3.1

Q2 If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Answer. We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and

Hence, the possible orders of a matrix having 24 elements are:

$1 \times 24, 24 \times 1, 2 \times 12, 12 \times 2, 3 \times 8, 8 \times 3, 4 \times 6$, and 6×4

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1×13 and 13×1 .

Page : 64 , Block Name : Exercise 3.1

Q3 If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer. We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural

numbers whose product is 18. The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6), and (6, 3). Hence, the possible orders of a matrix having 18 elements are: 1×18 , 18×1 , 2×9 , 9×2 , 3×6 and 6×3 . (1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5. Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1 .

Page : 64 , Block Name : Exercise 3.1

Q4 Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

$$(A) \quad a_{ij} = \frac{(i+j)^2}{2}$$

$$(B) \quad a_{ij} = \frac{i}{j} a_{ji} = \frac{i}{j}$$

$$(C) \quad a_{ij} = \frac{(i+2j)^2}{2}$$

Answer. In general, a 2×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$a_{ij} = \frac{(i+j)^2}{2}; i, j = 1, 2$$

$$\therefore a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2 \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2} \quad a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Therefore, the required matrix is $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

(ii)

$$a_{ij} = \frac{i}{j}, i, j = 1, 2$$

$$\therefore a_{11} = \frac{1}{1} = 1 \quad a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2 \quad a_{22} = \frac{2}{2} = 1$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii)

$$a_{ij} a_{ji} = \frac{(i+2j)^2}{2}, i, j = 1, 2$$

$$\therefore a_{11} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} \quad a_{12} = \frac{(1+4)^2}{2} = \frac{5^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = 8 \quad a_{22} = \frac{(2+4)^2}{2} = \frac{6^2}{2} = 18$$

Therefore, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

Page : 64 , Block Name : Exercise 3.1

Q5 Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2} | -3i + j |$

(ii) $a_{ij} = 2i - j$

Answer. In general, a 3×4 matrix is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

(i)

$$a_{ij} = \frac{1}{2} | -3i + j |, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

$$\therefore a_{11} = \frac{1}{2} | -3 \times 1 + 1 | = \frac{1}{2} | -3 + 1 | = \frac{1}{2} | -2 | = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2} | -3 \times 2 + 1 | = \frac{1}{2} | -6 + 1 | = \frac{1}{2} | -5 | = \frac{5}{2}$$

$$a_{31} = \frac{1}{2} | -3 \times 3 + 1 | = \frac{1}{2} | -9 + 1 | = \frac{1}{2} | -8 | = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2} | -3 \times 1 + 2 | = \frac{1}{2} | -3 + 2 | = \frac{1}{2} | -1 | = \frac{1}{2}$$

$$a_{22} = \frac{1}{2} | -3 \times 2 + 2 | = \frac{1}{2} | -6 + 2 | = \frac{1}{2} | -4 | = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2} | -3 \times 3 + 2 | = \frac{1}{2} | -9 + 2 | = \frac{1}{2} | -7 | = \frac{7}{2}$$

$$a_{13} = \frac{1}{2} | -3 \times 1 + 3 | = \frac{1}{2} | -3 + 3 | = 0$$

$$a_{23} = \frac{1}{2} | -3 \times 2 + 3 | = \frac{1}{2} | -6 + 3 | = \frac{1}{2} | -3 | = \frac{3}{2}$$

$$a_{33} = \frac{1}{2} | -3 \times 3 + 3 | = \frac{1}{2} | -9 + 3 | = \frac{1}{2} | -6 | = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2} | -3 \times 1 + 4 | = \frac{1}{2} | -3 + 4 | = \frac{1}{2} | 1 | = \frac{1}{2}$$

$$a_{24} = \frac{1}{2} | -3 \times 2 + 4 | = \frac{1}{2} | -6 + 4 | = \frac{1}{2} | -2 | = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2} | -3 \times 3 + 4 | = \frac{1}{2} | -9 + 4 | = \frac{1}{2} | -5 | = \frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii)

$$a_{ij} = 2i - j, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

$$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

$$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$$

$$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$$

$$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$$

$$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$$

$$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$$

$$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Page : 64 , Block Name : Exercise 3.1

Q6 Find the values of x, y and z from the following equations

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

$$(ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Answer. (i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$x = 1, y = 4, \text{ and } z = 3$$

(ii) $\begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$x + y = 6, xy = 8, 5 + z = 5$$

$$\text{Now, } 5 + z = 5 \Rightarrow z = 0$$

We know that:

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$\Rightarrow (x - y)^2 = 36 - 32 = 4$$

$$\Rightarrow x - y = \pm 2$$

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$$\therefore x = 4, y = 2, \text{ and } z = 0 \text{ or } x = 2, y = 4, \text{ and } z = 0$$

(iii) $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y + z = 9 \dots (1)$$

$$x + z = 5 \dots (2)$$

$$y + z = 7 \dots (3)$$

From (1) and (2), we have:

$$y + 5 = 9$$

$$\Rightarrow y = 4$$

Then, from (3), we have:

$$4 + z = 7$$

$$\Rightarrow z = 3$$

$$\therefore x + z = 5$$

$$\therefore x = 2$$

$$\therefore x = 2, y = 4, \text{ and } z = 3$$

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Answer. $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

As the two matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$a - b = -1 \dots (1)$$

$$2a - b = 0 \dots (2)$$

$$2a + c = 5 \dots (3)$$

$$3c + d = 13 \dots (4)$$

From (2), we have:

$$b = 2a$$

Then, from (1), we have:

$$a - 2a = -1$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

Now, from (3), we have:

$$2 \times 1 + c = 5$$

$$\Rightarrow c = 3$$

From (4) we have:

$$3 \times 3 + d = 13$$

$$\Rightarrow 9 + d = 13 \Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3, \text{ and } d = 4$$

Page : 64 , Block Name : Exercise 3.1

Q8

$A = [a_{ij}]_{m \times m}$ is a square matrix, if

(A) $m < n$

(B) $m > n$

(C) $m = n$

(D) none of these

Answer.

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A = [a_{ij}]_{m \times n}$ is a square matrix, if $m = n$

Page : 64 , Block Name : Exercise 3.1

Q9 Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$

(B) Not Possible to find

(C) $y = 7, x = \frac{-2}{3}$

(D) $x = \frac{-1}{3}, y = \frac{-2}{3}$

Answer. The correct answer is B

It is given that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get,

$$3x+7=0 \Rightarrow x = -\frac{7}{3}$$

$$5=y-2 \Rightarrow y=7$$

$$y+1=8 \Rightarrow y=7$$

$$2-3x=4 \Rightarrow x = -\frac{2}{3}$$

We find that on comparing the corresponding elements Of the two matrices, we get two different values Of x, which is not possible.

Hence, it is not possible to find the values Of x and y for which the given matrices are equal.

Page : 64 , Block Name : Exercise 3.1

Q10 The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

27

28

81

512

Answer.

The correct answer is D.

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is 2^9
 $= 512$

Page : 64 , Block Name : Exercise 3.1

Q1 Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following:

(i) $A+B$

(ii) $A-B$

(iii) $3A-C$

(iv) AB

(v) BA

$$\text{Answer. (i) } A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

$$\text{(ii) } A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii)

$$\begin{aligned} 3A - C &= 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 2 & 3 \times 4 \\ 3 \times 3 & 3 \times 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix} \end{aligned}$$

(iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B

Therefore, AB is defined as:

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2(1) + 4(-2) & 2(3) + 4(5) \\ 3(1) + 2(-2) & 3(3) + 2(5) \end{bmatrix} \\ &= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix} \end{aligned}$$

(v) Matrix B has 2 columns. This number is equal to the number of rows in matrix A .

Therefore, BA is defined as:

$$\begin{aligned} BA &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 11 & -2(2) + 5(3) & -2(4) + 5(2) \end{bmatrix} \\ &= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix} \end{aligned}$$

Q2 Compute the following:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$(iv) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$\text{Answer. (i)} \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 + 2ab & b^2 + c^2 + 2bc \\ a^2 + c^2 - 2ac & a^2 + b^2 - 2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

$$\begin{aligned}
 \text{(iv)} & \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \left(\because \sin^2 x + \cos^2 x = 1 \right)
 \end{aligned}$$

Page : 64 , Block Name : Exercise 3.2

Q3 Compute the indicated products.

$$\text{(i)} \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\text{(ii)} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$\text{(iii)} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\text{(iv)} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\text{(v)} \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\text{(vi)} \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Answer. (i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} a(a) + b(b) & a(-b) + b(a) \\ -b(a) + a(b) & -b(-b) + a(a) \end{bmatrix} \\
&= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}
\end{aligned}$$

$$\text{(ii)} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

$$\begin{aligned}
\text{(iii)} \quad & \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1(1) - 2(2) & 1(2) - 2(3) & 1(3) - 2(1) \\ 2(1) + 3(2) & 2(2) + 3(3) & 2(3) + 3(1) \end{bmatrix} \\
&= \begin{bmatrix} 1 - 4 & 2 - 6 & 3 - 2 \\ 2 + 6 & 4 + 9 & 6 + 3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad & \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} \\
&= \begin{bmatrix} 2(1) + 3(0) + 4(3) & 2(-3) + 3(2) + 4(0) & 2(5) + 3(4) + 4(5) \\ 3(1) + 4(0) + 5(3) & 3(-3) + 4(2) + 5(0) & 3(5) + 4(4) + 5(5) \\ 4(1) + 5(0) + 6(3) & 4(-3) + 5(2) + 6(0) & 4(5) + 5(4) + 6(5) \end{bmatrix} \\
&= \begin{bmatrix} 2 + 0 + 12 & -6 + 6 + 0 & 10 + 12 + 20 \\ 3 + 0 + 15 & -9 + 8 + 0 & 15 + 16 + 25 \\ 4 + 0 + 18 & -12 + 10 + 0 & 20 + 20 + 30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{(v)} \quad & \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 2(1) + 1(-1) & 2(0) + 1(2) & 2(1) + 1(1) \\ 3(1) + 2(-1) & 3(0) + 2(2) & 3(1) + 2(1) \\ -1(1) + 1(-1) & -1(0) + 1(2) & -1(1) + 1(1) \end{bmatrix}
\end{aligned}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\begin{aligned} \text{(vi)} \quad & \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3(2) - 1(1) + 3(3) & 3(-3) - 1(0) + 3(1) \\ -1(2) + 0(1) + 2(3) & -1(-3) + 0(0) + 2(1) \end{bmatrix} \\ &= \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix} \end{aligned}$$

Page : 64 , Block Name : Exercise 3.2

$$\text{Q4 if } A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, (B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}), \text{ then compute}$$

$(A + B)$ and $(B - C)$. Also, verify that $A + (B - C) = (A + B) - C$

Answer.

$$\begin{aligned} A + B &= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B - C &= \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \end{aligned}$$

$$A + (B - C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (-1) & 2 + (-2) & -3 + 0 \\ 5 + 4 & 0 + (-1) & 2 + 3 \\ 1 + 1 & -1 + 2 & 1 + 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$(A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 4 & 1 - 1 & -1 - 2 \\ 9 - 0 & 2 - 3 & 7 - 2 \\ 3 - 1 & -1 - (-2) & 4 - 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence we have verified that $A + (B - C) = (A + B) - C$

Page : 64 , Block Name : Exercise 3.2

$$\text{Q5 if } A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} \text{ then compute } 3A - 5B.$$

Answer.

$$3A - 5B = 3 \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Page : 64 , Block Name : Exercise 3.2

$$\text{Q6 Simplify } \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

Answer.

$$\begin{aligned} & \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \left(\because \cos^2\theta + \sin^2\theta = 1 \right) \end{aligned}$$

Page : 64 , Block Name : Exercise 3.2

Q7 Find X and Y, if

$$(i) X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$(ii) 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{ and } 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Answer. (i)

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \dots (1)$$

..... (2)

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Adding equation (1) and (2)

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\text{Now, } X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$(ii) 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots(3)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \dots(4)$$

Multiplying equations (3) with (2), we get:

$$2(2X + 3Y) = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \dots(5)$$

Multiplying equation (4) and (3), we get :

$$3(3X + 2Y) = 3 \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \dots(6)$$

From (5) and (6), we have

$$(4X + 6Y) - (9X + 6Y) = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore X = -\frac{1}{5} \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$\text{Now, } 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Page : 64 , Block Name : Exercise 3.2

Q8 Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Answer. $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Page : 64 , Block Name : Exercise 3.2

Q9 Find x and y, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Answer.

$$2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have

$$2 + y = 5$$

$$\Rightarrow y = 3$$

$$2x + 2 = 8$$

$$\Rightarrow x = 3$$

$$\therefore x = 3 \text{ and } y = 3$$

Page : 64 , Block Name : Exercise 3.2

Q10 Solve the equation for x, y, z and t, if $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Answer.

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x + 3 = 9$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$2y = 12$$

$$\Rightarrow y = 6$$

$$2z - 3 = 15$$

$$\Rightarrow 2z = 18$$

$$\Rightarrow z = 9$$

$$2t + 6 = 18$$

$$\Rightarrow 2t = 12$$

$$\Rightarrow t = 6$$

$$\therefore x = 3, y = 6, z = 9, \text{ and } t = 6$$

Page : 64 , Block Name : Exercise 3.2

Q11 If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ find the values of x and y.

Answer.

$$x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x - y = 10 \text{ and } 3x + y = 5$$

Adding these two equations, we have:

$$5x = 15$$

$$x = 3$$

$$\text{Now, } 3x + y = 5$$

$$y = 5 - 3x$$

$$\Rightarrow y = 5 - 9 = -4$$

$$\therefore x = 3 \text{ and } y = -4$$

Page : 64 , Block Name : Exercise 3.2

Q12 Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.

Answer.

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x + y$$

$$\Rightarrow y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3w = 2w + 3$$

$$\Rightarrow w = 3$$

$$3z = -1 + z + w$$

$$\Rightarrow 2z = -1 + w = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } w = 3$$

Page : 64 , Block Name : Exercise 3.2

Q13 If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ show that $F(x)F(y) = F(x+y)$.

Answer.

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$F(x)F(y)$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= F(x + y)$$

$$\therefore F(x)F(y) = F(x + y)$$

Page : 64 , Block Name : Exercise 3.2

Q14 Show that(i)

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Answer.

(i)

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(5) + 1(6) & 2(-1) + 1(7) \\ 3(5) + 4(6) & 3(-1) + 4(7) \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1) + 2(0) + 3(2) & 1(1) + 2(-1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(-1) + 1(0) + 0(2) & 0(1) + 1(-1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(-1) + 1(0) + 0(2) & 1(1) + 1(-1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) + (-1)(0) + 1(1) & 0(2) + (-1)(1) + 1(1) & 0(3) + (-1)(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Page : 64 , Block Name : Exercise 3.2

Q15 Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Answer. We have $A^2 = A \times A$

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2) + 0(2) + 1(1) & 2(0) + 0(1) + 1(-1) & 2(1) + 0(3) + 1(0) \\ 2(2) + 1(2) + 3(1) & 2(0) + 1(1) + 3(-1) & 2(1) + 1(3) + 3(0) \\ 1(2) + (-1)(2) + 0(1) & 1(0) + (-1)(1) + 0(-1) & 1(1) + (-1)(3) + 0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 + 1 & 0 + 0 - 1 & 2 + 0 + 0 \\ 4 + 2 + 3 & 0 + 1 - 3 & 2 + 3 + 0 \\ 2 - 2 + 0 & 0 - 1 + 0 & 1 - 3 + 0 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Q16 If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Answer. $A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + 2I$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$$\therefore A^3 - 6A^2 + 7A + 2I = O$$

Page : 82 , Block Name : Exercise 3.2

Q17 If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$?

$$\text{Answer. } A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$\text{Now } A^2 = kA - 2I$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$$3k - 2 = 1$$

$$\Rightarrow 3k = 3$$

$$\Rightarrow k = 1$$

Thus the value of k is 1

Page : 82 , Block Name : Exercise 3.2

Q18 If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, , show that

$$I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Answer. On the L.H.S

$$I+A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \dots\dots(1)$$

On the R.H.S

$$(I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha \tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha \tan\frac{\alpha}{2} \\ -\cos\alpha \tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha \tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix} \dots\dots(2)$$

$$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2} \end{bmatrix} \quad 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2}$$

$$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from (1) and (2) ,we get L.H.S=R.H.S

Page : 82 , Block Name : Exercise 3.2

Q19 A trust fund has ₦ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₦ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) ₦ 1800

(b) ₦ 2000

Answer. (a) Let ₦ x be invested in the first bond. Then, the sum of money invested in the second bond will be ₦ (30000 - x).

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of ₦ 1800, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$

$$\Rightarrow 5x + 21000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest Of ₦ 1800, the trust fund should invest ₦ 15000 in the first bond and the remaining ₦ 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum Of money invested in the second bond will be ₦ (30000 – x).

Therefore, in order to obtain an annual total interest of ₦ 2000, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of ₦ 2000, the trust fund should invest ₦ 5000 in the first bond and the remaining ₦ 25000 in the second bond.

Page : 82 , Block Name : Exercise 3.2

Q20 The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₦80, ₦60 and ₦40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. Choose the correct answer in Exercises 21 and 22

Answer. The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books. The selling prices Of a chemistry book, a physics book, and an economics book are respectively given as ₦ 80, ₦ 60, and ₦ 40. The total amount of money that will be received from the sale of all these books can be represented in the form Of a matrix as:

$$12 \begin{bmatrix} 10 & 8 & 10 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$\begin{aligned}
&= 12[10 \times 80 + 8 \times 60 + 10 \times 40] \\
&= 12(800 + 480 + 400) \\
&= 12(1680) \\
&= 20160
\end{aligned}$$

Thus, the bookshop will receive ₦ 20160 from the sale Of all these books.

Page : 83 , Block Name : Exeercise 3.2

Q21 The restriction on n , k and p so that $PY + WY$ will be defined are:

- (A) $k = 3$, $p = n$
- (B) k is arbitrary, $p = 2$
- (C) p is arbitrary, $k = 3$
- (D) $k = 2$, $p = 3$

Answer.

Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

Therefore, matrix PY will be defined if $k = 3$. Consequently, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y , matrix WY is well-defined and is of the order $n \times k$.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$ are the restrictions on n , k , and p so that $PY + WY$ will be defined.

Page : 83 , Block Name : Exercise 3.2

Q22 If $n = p$, then the order of the matrix $7X - 5Z$ is:

- (A) $p \times 2$
- (B) $2 \times n$
- (C) $n \times 3$
- (D) $p \times n$

Answer. The correct answer is B.

Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$, i.e., $2 \times n$ [Since $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Page : 83 , Block Name : Exercise 3.2

Q1 Find the transpose of each of the following matrices:

$$(i) \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$$

$$\text{Answer. (i) Let } A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$$

$$(ii) \text{ Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$(iii) \text{ Let } A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}, \text{ then } A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 6 \\ 6 & 6 & -1 \end{bmatrix}$$

Page : 88 , Block Name : Exercise 3.3

$$\text{Q2 If } A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \text{ then verify that,}$$

$$(i) (A + B)' = A' + B'$$

$$(ii) (A - B)' = A' - B'$$

$$\text{Answer. } A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$(i) A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A + B)' = A' + B'$

$$(ii) A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence we have verified that $(A - B)' = A' - B'$

Page : 88 , Block Name : Exercise 3.3

$$Q3 \text{ If } A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \text{ then verify that}$$

$$(i) (A + B)' = A' + B'$$

$$(ii) (A - B)' = A' - B'$$

Answer.

$$(i) \text{ It is known that } A = (A')'$$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Hence we have verified that $(A + B)' = A' + B'$

$$(ii) A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Thus, we have verified that $(A - B)' = A' - B'$

Page : 88 , Block Name : Exercise 3.3

Q4 If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$

Answer.

We know that $A = (A')$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

Page : 88 , Block Name : Exercise 3.3

Q5 For the matrices A and B, verify that $(AB)' = B'A'$, where

$$(i) \quad A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$$

$$(ii) \quad A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$$

$$\text{Answer. (i) } AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$\text{Now } A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$(ii) \quad AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$

Q6 If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

Answer. (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos\alpha)(\cos\alpha) + (-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha) + (-\sin\alpha)(\cos\alpha) \\ (\sin\alpha)(\cos\alpha) + (\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha) + (\cos\alpha)(\cos\alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$

(ii)

$$A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = I \end{aligned}$$

Hence, we have verified that $A'A = I$

Q7 (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Answer.

(i) We have:

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$$\therefore A' = A$$

Hence, A is a symmetric matrix.

(ii) We have

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$$\therefore A' = -A$$

Hence, A is a skew-symmetric matrix.

Page : 89 , Block Name : Exercise 3.3

Q8 For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Answer.

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$(i) A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

$$(ii) A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = - (A - A')$$

Hence, $(A - A')$ is a skew-symmetric matrix.

Page : 89 , Block Name : Exercise 3.3

$$Q9 \text{ Find } \frac{1}{2}(A + A') \text{ and } \frac{1}{2}(A - A'), \text{ when } A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\text{Answer. The given matrix is } A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Page : 89 , Block Name : Exercise 3.3

Q10 Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$(i) \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Choose the correct answer in the Exercises 11 and 12.

$$\text{Answer. Let } A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

$$\text{Thus, } P = \frac{1}{2}(A + A') \text{ is a symmetric matrix.}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

$$\text{Thus, } Q = \frac{1}{2}(A - A') \text{ is a skew-symmetric matrix.}$$

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

$$(ii) \text{ Let } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

$$\text{Now, } Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

$$\text{(iii) let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix .

$$\text{Now, } A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$

(iv)

$$\text{Let } A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\text{Now } A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$$

Page : 89 , Block Name : Exercise 3.3

Q11 If A, B are symmetric matrices of same order, then $AB - BA$ is a

- (A) Skew symmetric matrix
- (B) Symmetric matrix
- (C) Zero matrix
- (D) Identity matrix

Answer. The correct answer is A.

A and B are symmetric matrices ,therefore , we have :

$$A' = A \text{ and } B' = B \dots(1)$$

$$\text{Consider } (AB - BA)' = (AB)' - (BA)' \quad \left[(A - B)' = A' - B' \right]$$

$$= B'A' - A'B' \quad \left[(AB)' = B'A' \right]$$

$$= BA - AB \text{ [by (1)]}$$

$$= -(AB - BA)$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus ,($AB - BA$) is a skew-symmetric matrix.

Page : 90 , Block Name : Exercise 3.3

Q12 If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{3}$
- (C) π
- (D) $\frac{3\pi}{2}$

Answer. The correct answer is B.

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\text{Now, } A + A' = I$$

$$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \alpha = \frac{\pi}{3}$$

Page : 90 , Block Name : Exercise 3.3

Q1 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad (R_2 \rightarrow \frac{1}{5}R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q2 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q3 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q4 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow R_2 - 5R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow R_1 + 3R_2 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad \left(R_2 \rightarrow -2R_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q5 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow R_2 - 7R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow R_1 - R_2 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad \left(R_2 \rightarrow 2R_2 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q6 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow R_2 - R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_2 - 5R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow 2R_2)$$

Page : 97 , Block Name : Exercise 3.4

Q7 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q8 Using elementary transformations, find the inverse of the matrix.if it is exist-

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

We know that $A = IA$

$$\begin{aligned}
\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \\
\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2) \\
\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 3R_1) \\
\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2) \\
\therefore A^{-1} &= \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}
\end{aligned}$$

Page : 97 , Block Name : Exercise 3.4

Q9 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\begin{aligned}
\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \\
\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2) \\
\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1) \\
\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 3R_2) \\
\therefore A^{-1} &= \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}
\end{aligned}$$

Page : 97 , Block Name : Exercise 3.4

Q10 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \quad (C_2 \rightarrow \frac{1}{2}C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q11 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

We know that $A=AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \quad (C_2 \rightarrow C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \quad \left(C_1 \rightarrow \frac{1}{2}C_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q12 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left(R_1 \rightarrow \frac{1}{6}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow R_2 + 2R_1 \right)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S. Therefore, A^{-1} does not exist.

Page : 97 , Block Name : Exercise 3.4

Q13 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

We know that $A = 1A$

$$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q14 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Answer.

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_2$, we have

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S. Therefore, A^{-1} does not exist.

Page : 97 , Block Name : Exercise 3.4

Q15 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\text{Answer. } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow \frac{1}{5})$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ -\frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2 \text{ and } R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A \quad (R_3 \rightarrow R_3 + 3R_1)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left(R_3 \rightarrow \frac{1}{5}R_3 \right)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & 0 & -\frac{5}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -2 \end{bmatrix} A \quad \left(R_1 \rightarrow R_1 - R_3 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left(R_1 \rightarrow (-1)R_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q16 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Answer. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 3R_3$ and $R_2 \rightarrow R_2 + 8R_3$, we have

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{1}{25}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 10R_3$, and $R_2 \rightarrow R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q17 Using elementary transformations, find the inverse of the matrix. if it is exist-

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Answer. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, and $R_2 \rightarrow R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Page : 97 , Block Name : Exercise 3.4

Q18 Matrices A and B will be inverse of each other only if

- A. $AB = BA$
- C. $AB = 0, BA = I$
- B. $AB = BA = 0$
- D. $AB = BA = I$

Answer. D

We know that if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A. In this case, it is clear that A is the inverse of B.

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

Page : 97 , Block Name : Exercise 3.4

Q1 $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and

$n \in \mathbb{N}$.

Answer.

It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show : $P(n): (aI + bA)^n = a^n I + na^{n-1}bA, n \in \mathbb{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): (aI + bA) = aI + ba^0A = aI + bA$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k(aI + bA) \\ &= (a^k I + ka^{k-1}bA)(aI + bA) \quad \dots\dots(1) \\ &= a^{k+1}I + ka^k bAI + a^k bLA + ka^{k-1}b^2A^2 \\ &= a^{k+1}I + (k+1)a^k bA + ka^{k-1}b^2A^2 \end{aligned}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

From (1), we have:

$$\begin{aligned} (aI + bA)^{k+1} &= a^{k+1}I + (k+1)a^k bA + 0 \\ &= a^{k+1}I + (k+1)a^k bA \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI + bA)^n = a^n I + na^{n-1}bA \text{ where } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, n \in \mathbb{N}$$

Page : 100 , Block Name : Miscellaneous Exercise

Q2

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \text{ prove that } A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbb{N}$$

Answer. It is given that

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To show: $P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbb{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{-1} & 3^{1-1} & 3^{1-1} \\ 3^{-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$\text{That is } P(k): A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Now, we prove that the result is true for $n = k + 1$.

Now, $A^{k+1} = A \cdot A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbb{N}$$

Q3

$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer

Answer. It is given that

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

To prove: $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbb{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in \mathbb{N}$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$A^{k+1} = A^k \cdot A$$

$$\begin{aligned} &= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3(1+2k) - 4k & -4(1+2k) + 4k \\ 3k + 1 - 2k & -4k - 1(1-2k) \end{bmatrix} \\ &= \begin{bmatrix} 3 + 6k - 4k & -4 - 8k + 4k \\ 3k + 1 - 2k & -4k - 1 + 2k \end{bmatrix} \\ &= \begin{bmatrix} 3 + 2k & -4 - 4k \\ 1 + k & -1 - 2k \end{bmatrix} \\ &= \begin{bmatrix} 1 + 2(k+1) & -4(k+1) \\ 1 + k & 1 - 2(k+1) \end{bmatrix} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbb{N}$$

Q4 If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

Answer. It is given that A and B are symmetric matrices. Therefore, we have:

$$\text{Now } (AB - BA)' = (AB)' - (BA)' \quad \left[(A - B)' = A' - B' \right]$$

$$= B'A' - A'B' \quad \left[(AB)' = B'A' \right]$$

$$= BA - AB$$

$$= -(AB - BA) \quad [\text{Using (1)}]$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

Q5 Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric

Answer. We suppose that A is a symmetric matrix, then $A' = A \dots (1)$

Consider

$$\begin{aligned} (B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B')' \quad \left[(AB)' = B'A' \right] \end{aligned}$$

$$= B'A'(B) \quad \left[(B')' = B \right]$$

$$= B'(A'B) \quad [\text{Using (1)}]$$

$$= B'(AB)$$

$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then,

$$A' = -A$$

Consider

$$\begin{aligned} (B'AB)' &= [B'(AB)]' = (AB)'(B')' \\ &= (B'A')B = B'(-A)B \\ &= -B'AB \end{aligned}$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if A is a skew-symmetric matrix, then $B'AB$ is a skew-symmetric matrix.

Hence, if A is a symmetric or skew-symmetric matrix, then $B'AB$ is a symmetric or skew-symmetric matrix accordingly.

Q6 Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & -z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation

$$A'A = I$$

Answer.

It is given that $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Now $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 + x^2 + x^2 & 0 + xy - xy & 0 - xz + xz \\ 0 + xy - xy & 4y^2 + y^2 + y^2 & 2yz - yz \\ 0 - xz + zx & 2yz - yz - yz & z^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$6y^2 = 1 \Rightarrow y = \pm \frac{1}{\sqrt{6}}$$

$$3z^2 = 1 \Rightarrow z = \pm \frac{1}{\sqrt{3}}$$

Q7 For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

Answer. We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = 0$$

$$\Rightarrow [4 + 4x] = [0]$$

$$\therefore 4 + 4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of x is -1 .

Page : 100 , Block Name : Miscellaneous Exercise

Q8 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

Answer.

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3) + 1(-1) & 3(1) + 1(2) \\ -1(3) + 2(-1) & -1(1) + 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0 = \text{R.H.S}$$

$$\therefore A^2 - 5A + 7I = 0$$

Page : 100 , Block Name : Miscellaneous Exercise

Q9 Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Answer. We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x + 0 - 2 & 0 - 10 + 0 & 2x - 5 - 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x - 2 & -10 & 2x - 8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x(x - 2) - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0]$$

$$\Rightarrow [x^2 - 48] = [0]$$

$$\therefore x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Page : 100 , Block Name : Miscellaneous Exercise

Q10 A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

- (a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.
- (b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Answer. (a) The unit sale prices of x, y, and z are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00. Consequently, the total revenue in market I can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$= 25000 + 2.50 + 2000 \times 1.50 + 18000 \times 1.0$$

$$= 25000 + 3000 + 18000$$

$$= 46000$$

The total revenue in market II can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$$

$$= 15000 + 30000 + 8000$$

$$= 53000$$

Therefore, the total revenue in market I is Rs 46000 and the same in market II is Rs 53000.

- (b) The unit cost prices of x, y, and z are respectively given as Rs 2.00, Rs 1.00, and 50 paise. Consequently, the total cost prices of all the products in market I can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$= 20000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50$$

$$= 20000 + 2000 + 9000$$

$$= 31000$$

Since the total revenue in market I is Rs 46000, the gross profit in this market is (Rs 46000 – Rs 31000) Rs 15000.

The total cost prices of all the products in market II can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50$$

$$= 12000 + 20000 + 4000$$

$$= \text{Rs}36000$$

Since the total revenue in market II is Rs 53000, the gross profit in this market is (Rs 53000 – Rs 36000) Rs 17000.

Page : 101 , Block Name : Miscellaneous Exercise

Q11

$$X \text{ so that } X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Answer. It is given that:

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

$$\text{Now, let } X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a + 4c = -7, \quad 2a + 5c = -8,$$

$$b + 4d = 2, \quad 2b + 5d = 4, \quad 3b + 6d = 6$$

$$\text{Now, } a + 4c = -7 \Rightarrow a = -7 - 4c$$

$$\therefore 2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$$

$$\Rightarrow -3c = 6$$

$$\Rightarrow c = -2$$

$$\therefore a = -7 - 4(-2) = -7 + 8 = 1$$

$$\text{Now, } b + 4d = 2 \Rightarrow b = 2 - 4d$$

$$\therefore 2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$$

$$\Rightarrow -3d = 0$$

$$\Rightarrow d = 0$$

$$\therefore b = 2 - 4(0) = 2$$

Thus, $a = 1, b = 2, c = -2, d = 0$

$$\text{Hence, the required matrix } X \text{ is } \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

Page : 101 , Block Name : Miscellaneous Exercise

Q12 If A and B are square matrices of the same order such that $AB = BA$, then prove by induction

that $tAB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in \mathbb{N}$

Answer. A and B are square matrices of the same order such that $AB = BA$.

To prove: $P(n): AB^n = B^nA, n \in \mathbb{N}$

For $n = 1$, we have:

$$P(1): AB = BA \quad [\text{Given}]$$

$$\Rightarrow AB^1 = B^1A$$

Therefore, the result is true for $n = 1$

Let the result be true for $n = k$.

$$P(k): AB^k = B^kA \quad \dots\dots\dots (1)$$

Now, we prove that the result is true for $n = k + 1$

$$AB^{k+1} = AB^k \cdot B$$

$$= (B^kA)B \quad [\text{By (1)}]$$

$$= B^k(AB) \quad [\text{Associative law}]$$

$$= B^k(BA) \quad [AB = BA \text{ (Given)}]$$

$$= (B^kB)A \quad [\text{Associative law}]$$

$$= B^{k+1}A$$

Therefore, the result is true for $n = k + 1$

Thus, by the principle of mathematical induction, we have $AB^n = B^nA, n \in \mathbb{N}$.

Now, we prove that $(AB)^n = A^nB^n$ for all $n \in \mathbb{N}$

For $n = 1$, we have:

$$(AB)^1 = A^1B^1 = AB$$

Therefore, the result is true for $n = 1$

Let the result be true for $n = k$.

$$(AB)^k = A^kB^k \dots (2)$$

Now, we prove that the result is true for $n = k + 1$.

$$AB^{k+1} = AB^k \cdot B$$

$$= (B^kA)B \quad [\text{By (1)}]$$

$$= B^k(AB) \quad [\text{Associative law}]$$

$$= B^k(BA) \quad [AB = BA \text{ (Given)}]$$

$$= (B^kB)A \quad [\text{Associative law}]$$

$$= B^{k+1}A$$

Therefore, the result is true for $n = k + 1$

Thus, by the principle of mathematical induction, we have $(AB)^n = A^nB^n$, for all natural numbers.

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

Answer.

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} \text{Now,}$$

$$A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$\alpha^2 + \beta\gamma = 1$$

$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Page : 101 , Block Name : Miscellaneous Exercise

Q14 If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix (B) A is a zero matrix

(C) A is a square matrix (D) None of these

Answer.B

If A is both symmetric and skew-symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = O$$

$$\Rightarrow 2A = O$$

$$\Rightarrow A = O$$

Therefore, A is a zero matrix.

Page : 101 , Block Name : Miscellaneous Exercise

Q15 If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) I - A (C) I (D) 3A

Answer.

$$\begin{aligned}
(I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\
&= I + A^3 + 3A + 3A^2 - 7A \\
&= I + A^2 \cdot A + 3A + 3A - 7A \quad \left[A^2 = A \right] \\
&= I + A \cdot A - A \\
&= I + A^2 - A \\
&= I + A - A \\
&= I \\
\therefore (I + A)^3 - 7A &= I
\end{aligned}$$

Page : 101 , Block Name : Miscellaneous Exercise