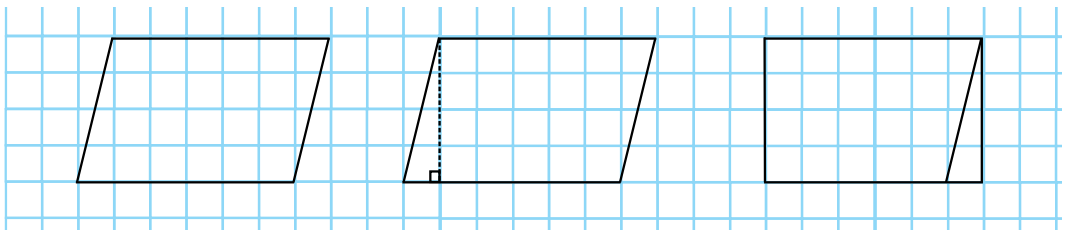
Chapter 9

Perimeter and Area

9.1 Area of a Parallelogram

We come across many shapes other than squares and rectangles.
How will you find the area of a land which is a parallelogram in shape?
Let us find a method to get the area of a parallelogram.
Can a parallelogram be converted into a rectangle of equal area?

Draw a parallelogram on a graph paper as shown in Fig 9.1(i). Cut out the parallelogram. Draw a line from one vertex of the parallelogram perpendicular to the opposite side [Fig 9.1(ii)]. Cut out the triangle. Move the triangle to the other side of the parallelogram.



(i) (ii) (iii)

Fig 9.1

What shape do you get? You get a rectangle.
Is the area of the parallelogram equal to the area of the rectangle formed?
Yes, area of the parallelogram = area of the rectangle formed
What are the length and the breadth of the rectangle?

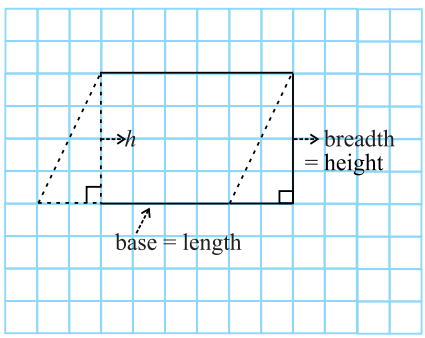


Fig 9.2

We find that the length of the rectangle formed is equal to the base of the parallelogram and the breadth of the rectangle is equal to the height of the parallelogram (Fig 9.2).

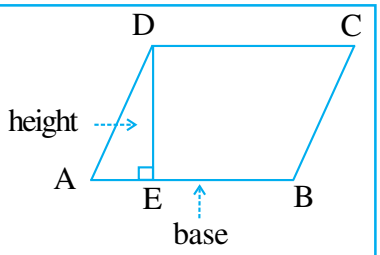
Now, Area of parallelogram = Area of rectangle

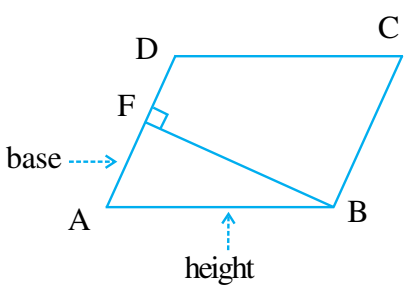
= length × breadth = $l \times b$

But the length l and breadth b of the rectangle are exactly the base b and the height h, respectively of the parallelogram.

Thus, the area of parallelogram = base × height = $b \times h$.

Any side of a parallelogram can be chosen as **base** of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as **height** (altitude). In the parallelogram ABCD, DE is perpendicular to AB. Here AB is the base and DE is the height of the parallelogram.





In this parallelogram ABCD, BF is the perpendicular to opposite side AD. Here AD is the **base** and BF is the **height**.

Consider the following parallelograms (Fig 9.2).

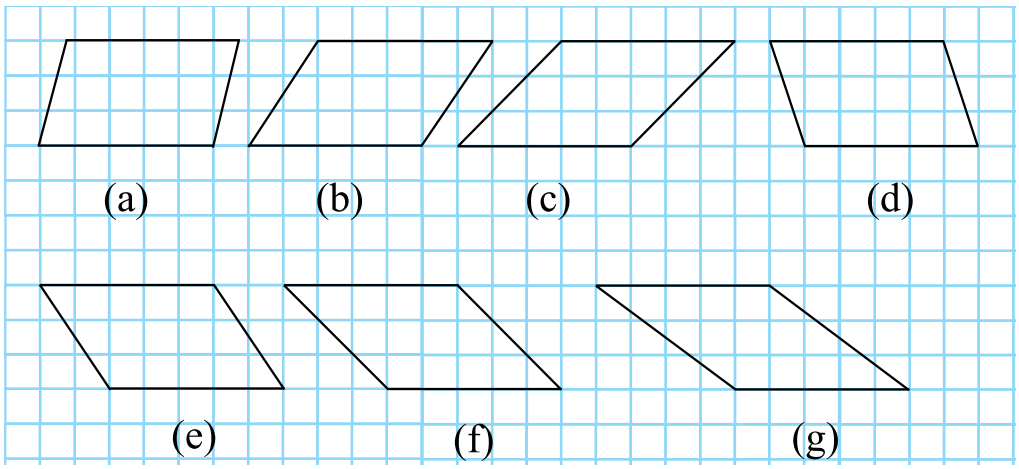


Fig 9.3

Find the areas of the parallelograms by counting the squares enclosed within the figures and also find the perimeters by measuring the sides.

Complete the following table:

Parallelogram	Base	Height	Area	Perimeter
(a)	5 units	3 units	15 sq units	
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				

You will find that all these parallelograms have equal areas but different perimeters. Now, consider the following parallelograms with sides 7 cm and 5 cm (Fig 9.4).

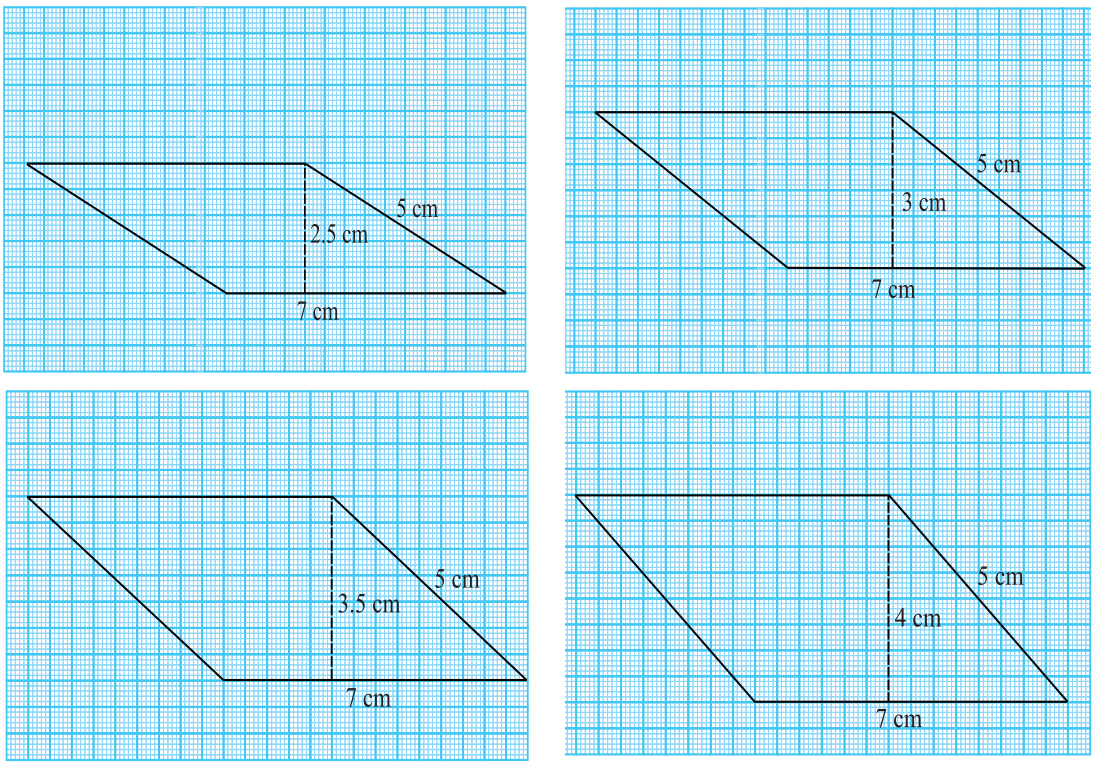


Fig 9.4

Find the perimeter and area of each of these parallelograms. Analyse your results.

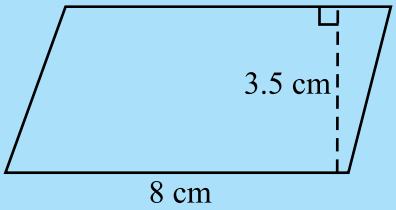
You will find that these parallelograms have different areas but equal perimeters.

To find the area of a parallelogram, you need to know only the base and the corresponding height of the parallelogram.

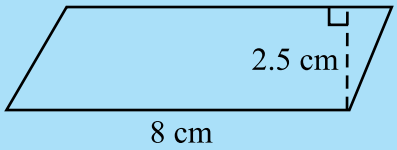
Try These

Find the area of following parallelograms:

(i)

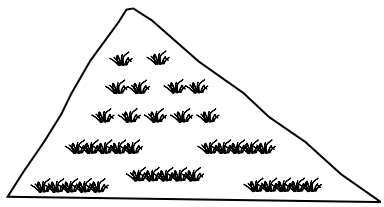


(ii)



(iii) In a parallelogram ABCD, AB = 7.2 cm and the perpendicular from C on AB is 4.5 cm.

9.2 Area of a Triangle



A gardener wants to know the cost of covering the whole of a triangular garden with grass.

In this case we need to know the area of the triangular region.

Let us find a method to get the area of a triangle.

Draw a scalene triangle on a piece of paper. Cut out the triangle. Place this triangle on another piece of paper and cut out another triangle of the same size.

So now you have two scalene triangles of the same size.

Are both the triangles congruent?

Superpose one triangle on the other so that they match. You may have to rotate one of the two triangles.

Now place both the triangles such that a pair of corresponding sides is joined as shown in Fig 9.5.

Is the figure thus formed a parallelogram?

Compare the area of each triangle to the area of the parallelogram.

Compare the base and height of the triangles with the base and height of the parallelogram.

You will find that the sum of the areas of both the triangles is equal to the area of the parallelogram. The base and the height of the triangle are the same as the base and the height of the parallelogram, respectively.

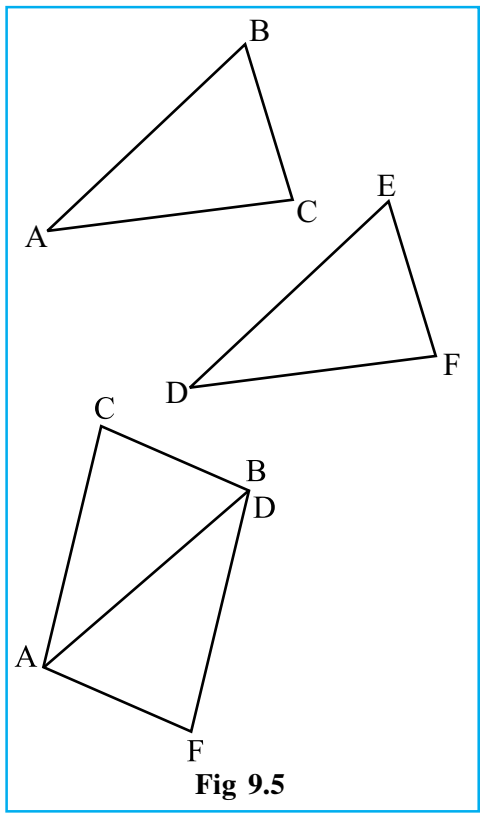


Fig 9.5

$$\text{Area of each triangle} = \frac{1}{2}\,(\text{Area of parallelogram})$$

$$= \frac{1}{2}\,(\text{base} \times \text{height}) \quad (\text{Since area of a parallelogram} = \text{base} \times \text{height})$$

$$= \frac{1}{2}(b \times h) \quad \left(\text{or } \frac{1}{2}bh \text{, in short}\right)$$

TRY THESE

1. Try the above activity with different types of triangles.
2. Take different parallelograms. Divide each of the parallelograms into two triangles by cutting along any of its diagonals. Are the triangles congruent?

In the figure (Fig 9.6) all the triangles are on the base AB = 6 cm.

What can you say about the height of each of the triangles corresponding to the base AB?

Can we say all the triangles are equal in area? Yes.

Are the triangles congruent also? No.

We conclude that **all the congruent triangles are equal in area but the triangles equal in area need not be congruent.**

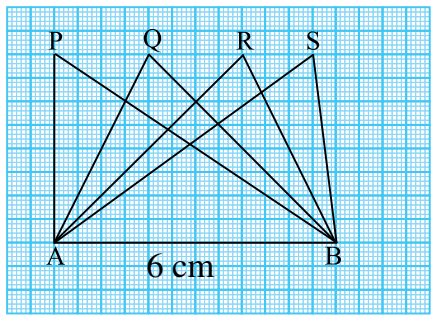


Fig 9.6

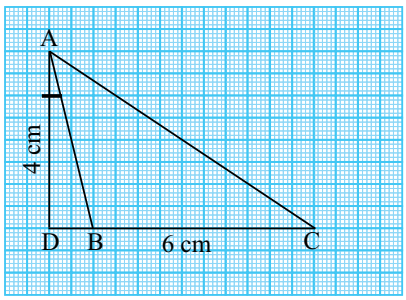


Fig 9.7

Consider the obtuse-angled triangle ABC of base 6 cm (Fig 9.7).

Its height AD which is perpendicular from the vertex A is outside the triangle.

Can you find the area of the triangle?

EXAMPLE 1 One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram (Fig 9.8).

SOLUTION Given that length of base (b) = 4 cm, height (h) = 3 cm

Area of the parallelogram = $b \times h$

= 4 cm × 3 cm = 12 cm^2

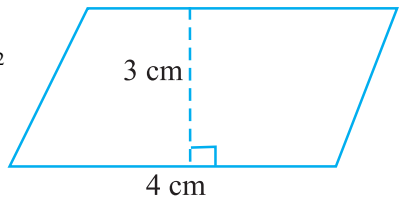


Fig 9.8

EXAMPLE 2 Find the height 'x' if the area of the parallelogram is 24 cm^2 and the base is 4 cm.

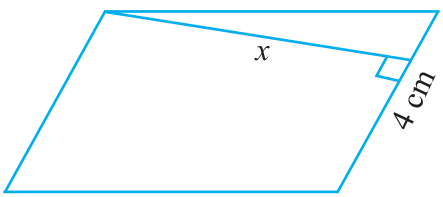


Fig 9.9

SOLUTION Area of parallelogram = $b \times h$

Therefore, 24 = 4 × x (Fig 9.9)

or $\frac{24}{4} = x$ or $x = 6$ cm

So, the height of the parallelogram is 6 cm.

EXAMPLE 3 The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm (Fig 9.10). Find the

(i) area of the parallelogram. (ii) the height corresponding to the base AD.

SOLUTION

(i) Area of parallelogram = $b \times h$

$= 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$

(ii) base (b) = 4 cm, height = x (say),

Area = 18 cm^2

Area of parallelogram = $b \times x$

$18 = 4 \times x$

$\frac{18}{4} = x$

Therefore, $x = 4.5$ cm

Thus, the height corresponding to base AD is 4.5 cm.

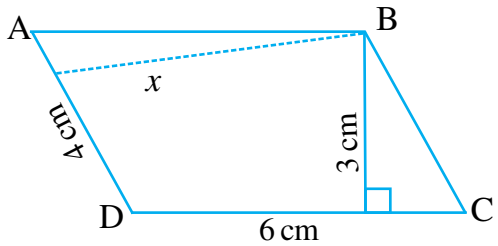
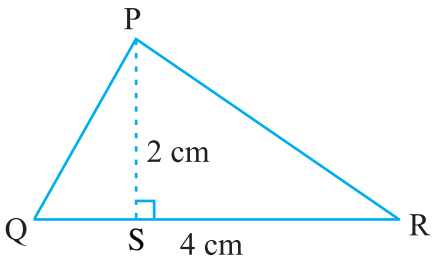
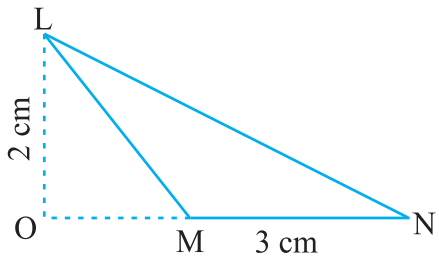


Fig 9.10

EXAMPLE 4 Find the area of the following triangles (Fig 9.11).



(i)



(ii)

Fig 9.11

SOLUTION

(i) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{QR} \times \text{PS}$

$= \frac{1}{2} \times 4 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

(ii) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{MN} \times \text{LO}$

$= \frac{1}{2} \times 3 \text{ cm} \times 2 \text{ cm} = 3 \text{ cm}^2$

EXAMPLE 5 Find BC, if the area of the triangle ABC is 36 cm^2 and the height AD is 3 cm (Fig 9.12).

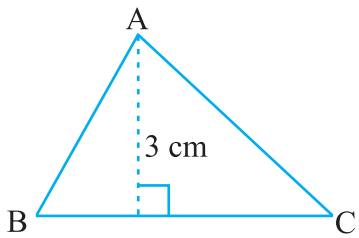


Fig 9.12

SOLUTION Height = 3 cm, Area = 36 cm^2

$$\text{Area of the triangle ABC} = \frac{1}{2}bh$$

or $$36 = \frac{1}{2} \times b \times 3 \text{ i.e., } b = \frac{36 \times 2}{3} = 24 \text{ cm}$$

So, BC = 24 cm

EXAMPLE 6 In ΔPQR, PR = 8 cm, QR = 4 cm and PL = 5 cm (Fig 9.13). Find:

(i) the area of the ΔPQR

(ii) QM

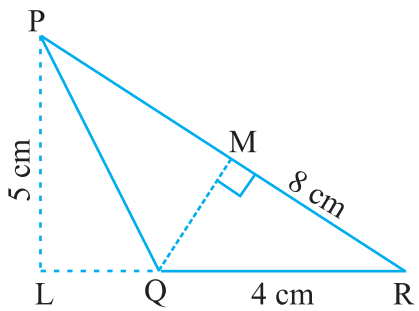


Fig 9.13

SOLUTION

(i) QR = base = 4 cm, PL = height = 5 cm

$$\text{Area of the triangle PQR} = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 4 \text{ cm} \times 5 \text{ cm} = 10 \text{ cm}^2$$

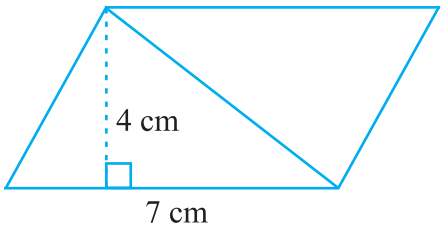
(ii) PR = base = 8 cm QM = height = ? Area = 10 cm^2

$$\text{Area of triangle} = \frac{1}{2} \times b \times h \quad \text{i.e., } 10 = \frac{1}{2} \times 8 \times h$$

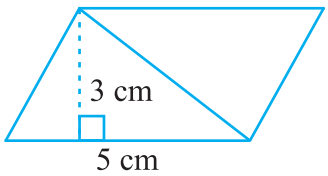
$$h = \frac{10}{4} = \frac{5}{2} = 2.5. \quad \text{So, QM} = 2.5 \text{ cm}$$

EXERCISE 9.1

1. Find the area of each of the following parallelograms:



(a)



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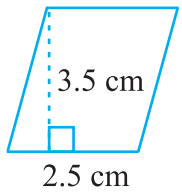

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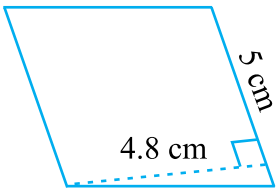
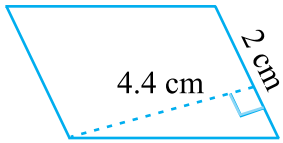
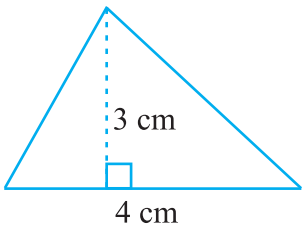


(d)

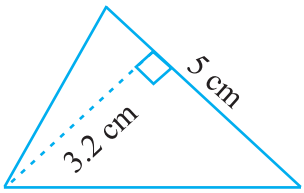


(e)

2. Find the area of each of the following triangles:



(a)



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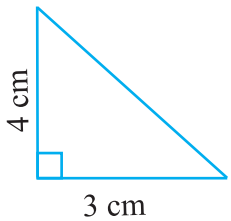

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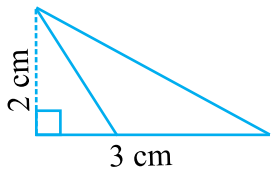


(d)

3. Find the missing values:

S.No.	Base	Height	Area of the Parallelogram
a.	20 cm		246 cm^2
b.		15 cm	154.5 cm^2
c.		8.4 cm	48.72 cm^2
d.	15.6 cm		16.38 cm^2

4. Find the missing values:

Base	Height	Area of Triangle
15 cm	_____	87 cm^2
_____	31.4 mm	1256 mm^2
22 cm	_____	170.5 cm^2

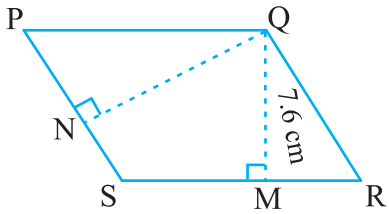


Fig 9.14

5. PQRS is a parallelogram (Fig 9.14). QM is the height from Q to SR and QN is the height from Q to PS. If SR = 12 cm and QM = 7.6 cm. Find:

(a) the area of the parallegram PQRS

(b) QN, if PS = 8 cm

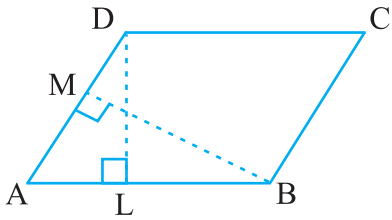


Fig 9.15

6. DL and BM are the heights on sides AB and AD respectively of parallelogram ABCD (Fig 9.15). If the area of the parallelogram is 1470 cm^2, AB = 35 cm and AD = 49 cm, find the length of BM and DL.

7. ΔABC is right angled at A (Fig 9.16). AD is perpendicular to BC. If AB = 5 cm, BC = 13 cm and AC = 12 cm, Find the area of ΔABC. Also find the length of AD.

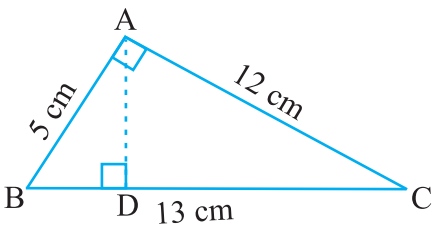


Fig 9.16

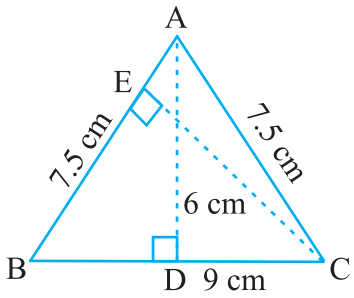


Fig 9.17

8. ΔABC is isosceles with AB = AC = 7.5 cm and BC = 9 cm (Fig 9.17). The height AD from A to BC, is 6 cm. Find the area of ΔABC. What will be the height from C to AB i.e., CE?

9.3 Circles

A racing track is semi-circular at both ends (Fig 9.18).

Can you find the distance covered by an athlete if he takes two rounds of a racing track? We need to find a method to find the distances around when a shape is circular.

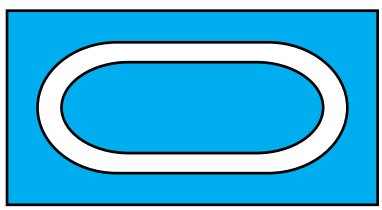
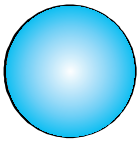
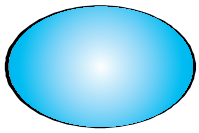


Fig 9.18

9.3.1 Circumference of a Circle

Tanya cut different cards, in curved shape from a cardboard. She wants to put lace around to decorate these cards. What length of the lace does she require for each? (Fig 9.19)


(a)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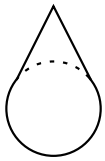

(c)

Fig 9.19

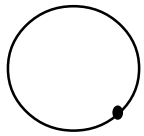


Fig 9.20

You cannot measure the curves with the help of a ruler, as these figures are not "straight". What can you do?

Here is a way to find the length of lace required for shape in Fig 9.19(a). Mark a point on the edge of the card and place the card on the table. Mark the position of the point on the table also (Fig 9. 20).

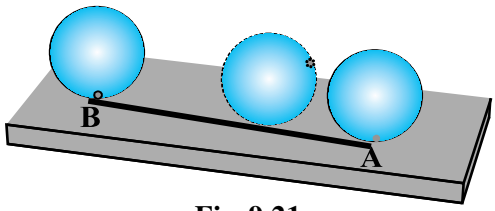


Fig 9.21

Now roll the circular card on the table along a straight line till the marked point again touches the table. Measure the distance along the line. This is the length of the lace required (Fig 9.21). It is also the distance along the edge of the card from the marked point back to the marked point.

You can also find the distance by putting a string on the edge of the circular object and taking all round it.

The distance around a circular region is known as its circumference.

Do This

Take a bottle cap, a bangle or any other circular object and find the circumference.

Now, can you find the distance covered by the athlete on the track by this method?

Still, it will be very difficult to find the distance around the track or any other circular object by measuring through string. Moreover, the measurement will not be accurate.

So, we need some formula for this, as we have for rectilinear figures or shapes.

Let us see if there is any relationship between the diameter and the circumference of the circles.

Consider the following table: Draw six circles of different radii and find their circumference by using string. Also find the ratio of the circumference to the diameter.

Circle	Radius	Diameter	Circumference	Ratio of Circumference to Diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$

2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	$\frac{66}{21} = 3.14$
4.	21.0 cm	42.0 cm	132.0 cm	$\frac{132}{42} = 3.14$
5.	5.0 cm	10.0 cm	32.0 cm	$\frac{32}{10} = 3.2$
6.	15.0 cm	30.0 cm	94.0 cm	$\frac{94}{30} = 3.13$

What do you infer from the above table? Is this ratio approximately the same? Yes.

Can you say that the circumference of a circle is always more than three times its diameter? Yes.

This ratio is a constant and is denoted by π (pi). Its approximate value is $\frac{22}{7}$ or 3.14.

So, we can say that $\frac{C}{d} = \pi$, where 'C' represents circumference of the circle and 'd' its diameter.

or $C = \pi d$

We know that diameter (d) of a circle is twice the radius (r) i.e., $d = 2r$

So, $C = \pi d = \pi \times 2r$ or $C = 2\pi r$.

Try These

In Fig 9.22,

(a) Which square has the larger perimeter?

(b) Which is larger, perimeter of smaller square or the circumference of the circle?

Fig 9.22

Do This

Take one each of quarter plate and half plate. Roll once each of these on a table-top. Which plate covers more distance in one complete revolution? Which plate will take less number of revolutions to cover the length of the table-top?

EXAMPLE 7 What is the circumference of a circle of diameter 10 cm (Take $\pi = 3.14$)?

SOLUTION Diameter of the circle (d) = 10 cm

Circumference of circle = πd

$= 3.14 \times 10$ cm $= 31.4$ cm

So, the circumference of the circle of diameter 10 cm is 31.4 cm.

EXAMPLE 8 What is the circumference of a circular disc of radius 14 cm?

$$\left(\text{Use } \pi = \frac{22}{7}\right)$$

SOLUTION Radius of circular disc (r) = 14 cm

Circumference of disc = $2\pi r$

$$= 2 \times \frac{22}{7} \times 14 \text{ cm} = 88 \text{ cm}$$

So, the circumference of the circular disc is 88 cm.

EXAMPLE 9 The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe ($\pi = 3.14$)?

SOLUTION Radius of the pipe (r) = 10 cm

Length of tape required is equal to the circumference of the pipe.

Circumference of the pipe = $2\pi r$

$= 2 \times 3.14 \times 10$ cm

$= 62.8$ cm

Therefore, length of the tape needed to wrap once around the pipe is 62.8 cm.

EXAMPLE 10 Find the perimeter of the given shape (Fig 9.23) (Take $\pi = \frac{22}{7}$).

SOLUTION In this shape we need to find the circumference of semicircles on each side of the square. Do you need to find the perimeter of the square also? No. The outer boundary, of this figure is made up of semicircles. Diameter of each semicircle is 14 cm.

We know that:

Circumference of the circle = πd

Circumference of the semicircle = $\frac{1}{2}\pi d$

$$= \frac{1}{2} \times \frac{22}{7} \times 14 \text{ cm} = 22 \text{ cm}$$

Circumference of each of the semicircles is 22 cm

Therefore, perimeter of the given figure = 4×22 cm $= 88$ cm

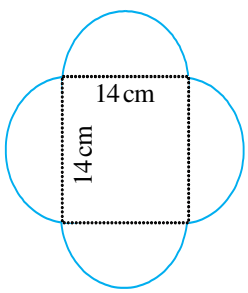


Fig 9.23

Example 11 Sudhanshu divides a circular disc of radius 7 cm in two equal parts. What is the perimeter of each semicircular shape disc? (Use $\pi = \frac{22}{7}$)

Solution To find the perimeter of the semicircular disc (Fig 9.24), we need to find

(i) Circumference of semicircular shape (ii) Diameter

Given that radius (r) = 7 cm. We know that the circumference of circle = $2\pi r$

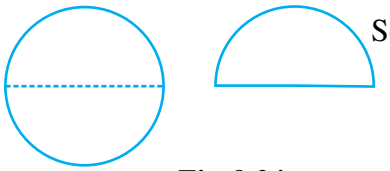


Fig 9.24

So, the circumference of the semicircle $= \frac{1}{2} \times 2\pi r = \pi r$

$= \frac{22}{7} \times 7 \text{ cm} = 22 \text{ cm}$

So, the diameter of the circle $= 2r = 2 \times 7 \text{ cm} = 14 \text{ cm}$

Thus, perimeter of each semicircular disc = 22 cm + 14 cm = 36 cm

9.3.2 Area of Circle

Consider the following:

- A farmer dug a flower bed of radius 7 m at the centre of a field. He needs to purchase fertiliser. If 1 kg of fertiliser is required for 1 square metre area, how much fertiliser should he purchase?
- What will be the cost of polishing a circular table-top of radius 2 m at the rate of ₹ 10 per square metre?

Can you tell what we need to find in such cases, Area or Perimeter? In such cases we need to find the area of the circular region. Let us find the area of a circle, using graph paper.

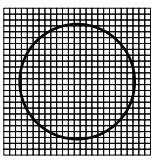


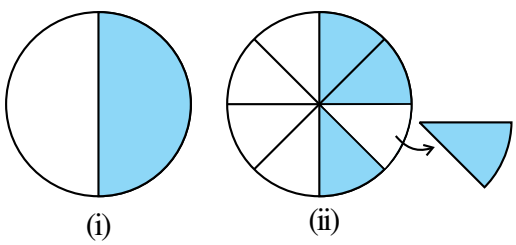
Fig 9.25

Draw a circle of radius 4 cm on a graph paper (Fig 9.25). Find the area by counting the number of squares enclosed.

As the edges are not straight, we get a rough estimate of the area of circle by this method.

There is another way of finding the area of a circle.

Draw a circle and shade one half of the circle [Fig 9.26(i)]. Now fold the circle into **eighths** and cut along the folds [Fig 9.26(ii)].



(i) (ii)

Fig 9.26

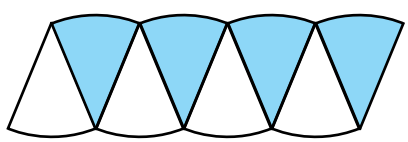


Fig 9.27

Arrange the separate pieces as shown, in Fig 9.27, which is roughly a parallelogram.

The more sectors we have, the nearer we reach an appropriate parallelogram.

As done above if we divide the circle in 64 sectors, and arrange these sectors. It gives nearly a rectangle (Fig 9.28).

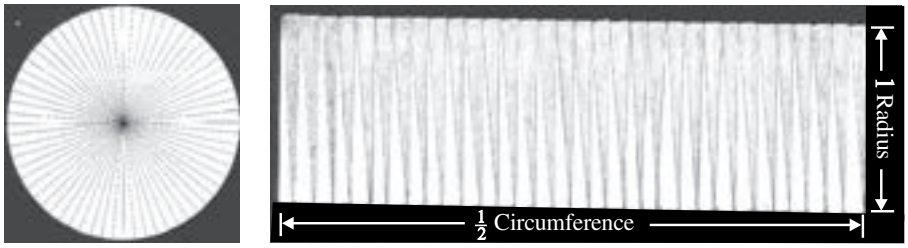


Fig 9.28

What is the breadth of this rectangle? The breadth of this rectangle is the radius of the circle, i.e., 'r'.

As the whole circle is divided into 64 sectors and on each side we have 32 sectors, the length of the rectangle is the length of the 32 sectors, which is half of the circumference. (Fig 9.28)

Area of the circle = Area of rectangle thus formed = $l \times b$

$= (\text{Half of circumference}) \times \text{radius} = \left(\frac{1}{2} \times 2\pi r\right) \times r = \pi r^2$

So, the area of the circle = πr^2

Try These

Draw circles of different radii on a graph paper. Find the area by counting the number of squares. Also find the area by using the formula. Compare the two answers.

Example 12 Find the area of a circle of radius 30 cm (use $\pi = 3.14$).

Solution Radius, $r = 30$ cm

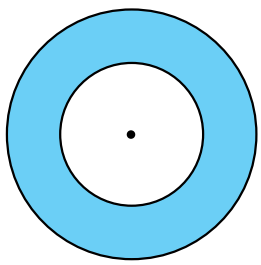
Area of the circle = $\pi r^2 = 3.14 \times 30^2 = 2{,}826$ cm^2

Example 13 Diameter of a circular garden is 9.8 m. Find its area.

Solution Diameter, $d = 9.8$ m. Therefore, radius $r = 9.8 \div 2 = 4.9$ m

Area of the circle = $\pi r^2 = \frac{22}{7} \times (4.9)^2$ m^2 = $\frac{22}{7} \times 4.9 \times 4.9$ m^2 = 75.46 m^2

Example 14 The adjoining figure shows two circles with the same centre. The radius of the larger circle is 10 cm and the radius of the smaller circle is 4 cm.



Find: (a) the area of the larger circle

(b) the area of the smaller circle

(c) the shaded area between the two circles. ($\pi = 3.14$)

SOLUTION

(a) Radius of the larger circle = 10 cm

So, area of the larger circle = πr^2

= $3.14 \times 10 \times 10 = 314$ cm^2

(b) Radius of the smaller circle = 4 cm

Area of the smaller circle = πr^2

= $3.14 \times 4 \times 4 = 50.24$ cm^2

(c) Area of the shaded region = $(314 - 50.24)$ cm^2 = 263.76 cm^2

EXERCISE 9.2

1. Find the circumference of the circles with the following radius: (Take $\pi = \frac{22}{7}$)

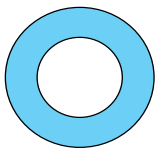
(a) 14 cm (b) 28 mm (c) 21 cm

2. Find the area of the following circles, given that:

(a) radius = 14 mm (Take $\pi = \frac{22}{7}$) (b) diameter = 49 m

(c) radius = 5 cm

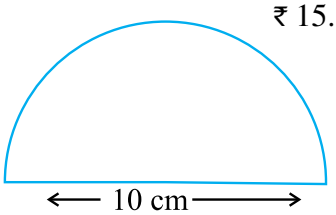
3. If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet. (Take $\pi = \frac{22}{7}$)



4. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ₹ 4 per meter. (Take $\pi = \frac{22}{7}$)

5. From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet. (Take $\pi = 3.14$)

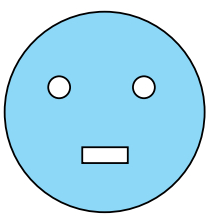
6. Saima wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ₹ 15. (Take $\pi = 3.14$)



7. Find the perimeter of the adjoining figure, which is a semicircle including its diameter.

8. Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹ 15/m^2. (Take $\pi = 3.14$)

9. Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square? (Take $\pi = \frac{22}{7}$)

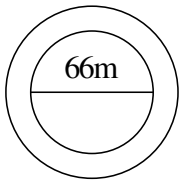


10. From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. (as shown in the adjoining figure). Find the area of the remaining sheet. (Take $\pi = \frac{22}{7}$)

11. A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet? (Take $\pi = 3.14$)

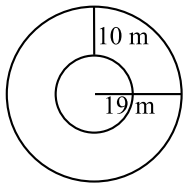
12. The circumference of a circle is 31.4 cm. Find the radius and the area of the circle? (Take $\pi = 3.14$)

13. A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path? ($\pi = 3.14$)



14. A circular flower garden has an area of 314 m^2. A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden? (Take $\pi = 3.14$)

15. Find the circumference of the inner and the outer circles, shown in the adjoining figure? (Take $\pi = 3.14$)



16. How many times a wheel of radius 28 cm must rotate to go 352 m? (Take $\pi = \frac{22}{7}$)

17. The minute hand of a circular clock is 15 cm long. How far does the tip of the minute hand move in 1 hour. (Take $\pi = 3.14$)

WHAT HAVE WE DISCUSSED?

1. Area of a parallelogram = base × height

2. Area of a triangle = $\frac{1}{2}$ (area of the parallelogram generated from it)

$= \frac{1}{2} \times$ base × height

3. The distance around a circular region is known as its circumference.

Circumference of a circle = πd, where d is the diameter of a circle and $\pi = \frac{22}{7}$ or 3.14 (approximately).

4. Area of a circle = πr^2, where r is the radius of the circle.