UNIT 1

INTEGERS

(A) Main Concepts and Results

- Representation of integers on the number line and their addition and subtraction.
- Properties of integers:
  - Integers are closed under addition, subtraction and multiplication.
  - Addition and multiplication are commutative for integers, i.e., $a + b = b + a$ and $a \times b = b \times a$ for any two integers a and b.
  - Addition and multiplication are associative for integers, i.e., $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.
  - Zero (0) is an additive identity for integers, i.e., $a + 0 = 0 + a = a$ for any integer a.
  - 1 is multiplicative identity for integers, i.e., $a \times 1 = 1 \times a = a$ for any integer a.
  - Integers show distributive property of multiplication over addition, i.e., $a \times (b + c) = a \times b + a \times c$ for any three integers a, b and c.
- Product of a positive integer and a negative integer is a negative integer, i.e, $a \times (-b) = -ab$, where a and b are positive integers.
- Product of two negative integers is a positive integer, i.e., $(-a) \times (-b) = ab$, where a and b are positive integers.
- Product of even number of negative integers is positive, where as the product of odd number of negative integers is negative, i.e.,

$\underbrace{(-a)\times(-b)\times\ldots\times(-p)}_{\text{even number } 2m \text{ times}} = a\times b\times\ldots\times p$ and

$\underbrace{(-a)\times(-b)\times\ldots\times(-q)}_{\text{odd number } (2m+1) \text{ times}} = -(a\times b\times\ldots\times q)$, where a, b, ..., p, q and m are positive integers.

- When a positive integer is divided by a negative integer or vice-versa and the quotient obtained is an integer then it is a negative integer, i.e.,

  $a \div (-b) = (-a) \div b = -\frac{a}{b}$, where a and b are positive integers and $-\frac{a}{b}$ is an integer

- When a negative integer is divided by another negative integer to give an integer then it gives a positive integer, i.e., $(-a) \div (-b) = \frac{a}{b}$, where a and b are positive integers and $\frac{a}{b}$ is also an integer.
- For any integer a, $a \div 1 = a$ and $a \div 0$ is not defined.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Write the correct answer.

Example 1: Madhre is standing in the middle of a bridge which is 20 m above the water level of a river. If a 35 m deep river is flowing under the bridge (see Fig. 1.1), then the vertical distance between the foot of Madhre and bottom level of the river is:

(a) 55 m (b) 35 m (c) 20 m (d) 15 m

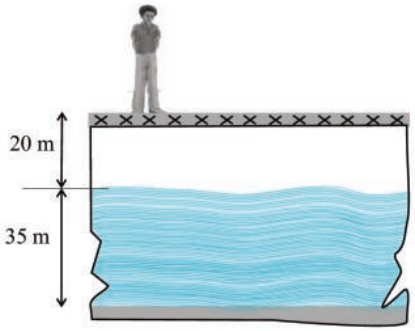


Fig. 1.1

15-04-2018

Solution: The correct answer is (a)
[Vertical distance = 20 m + 35 m = 55 m]

Example 2: [(– 10) × (+ 9)] + (– 10) is equal to
(a) 100 (b) –100 (c) – 80 (d) 80

Solution: Correct answer is (b)

Example 3: –16 ÷ [8 ÷ (–2)] is equal to
(a) –1 (b) 1 (c) 4 (d) –4

Solution: Correct answer is (c),

In Examples 4 and 5, fill in the blanks to make the statements true.

Example 4: (– 25) × 30 = – 30 × _____.

Solution: 25

Example 5: 75 ÷ _____ = – 75

Solution: – 1

ADDING INTEGERS	
If the signs are the same	**If the signs are different**
Find the sum of the values of integers without sign, and then use the same sign as the integers have.	Find the difference of the values of integers without sign (subtract lower value integer from greater value integer) and then use the sign of the integer with the greater value.

In Examples 6 and 7, state whether the statements are True or False.

Example 6: (–5) × (–7) is same as (–7) × (–5)

Solution : True

Example 7: (– 80) ÷ (4) is not same as 80 ÷ (–4)

Solution: False

Example 8: Find the odd one out* of the four options in the following:
(a) (–2, 24) (b) (–3, 10) (c) (–4, 12) (d) (–6, 8)

Solution: Here $-2 \times 24 = -48$,
$-4 \times 12 = -48$ and
$-6 \times 8 = -48$

* To find odd one out, you have to look for a pattern between the numbers and then find out which option is not on that pattern.

All the pairs i.e. (–2, 24); (–4, 12); (–6, 8) give same answer on multiplication, whereas $-3 \times 10 = -30$, gives a different answer. So, odd one is (b).

Example 9: Find the odd one out of the four options given below:

(a) (–3, –6) (b) (+1, –10) (c) (–2, –7) (d) (–4, –9)

Solution: Here $-3 + (-6) = -9$,

$+1 + (-10) = -9$ and

$-2 + (-7) = -9$

All the above pairs i.e. (–3, –6); (+1, –10); (–2, –7) give same answer on adding, whereas $-4 + (-9) = -13$, gives a different answer. So, odd one out is (d).

Example 10: Match the integer in Column I to an integer in Column II so that the sum is between –11 and –4

Column I	**Column II**
(a) –6	(i) –11
(b) +1	(ii) –5
(c) +7	(iii) +1
(d) –2	(iv) –13

Solution: (a) ↔ (iii)

because $-6 + (+1) = -5$, which lies between –11 and –4.

(b) ↔ (i)

because $+1 + (-11) = -10$ which lies between –11 and –4.

(c) ↔ (iv)

because $+7 + (-13) = -6$ which lies between –11 and –4

(d) ↔ (ii)

because $-2 + (-5) = -7$ which lies between –11 and –4.

Example 11: If a is an integer other than 1 and –1, match the following:

Column I	**Column II**
(a) $a \div (-1)$	(i) a
(b) $1 \div (a)$	(ii) 1
(c) $(-a) \div (-a)$	(iii) Not an integer
(d) $a \div (+1)$	(iv) $-a$

Solution: (a) ↔ (iv) (b) ↔ (iii) (c) ↔ (ii) (d) ↔ (i)

Example 12: Write a pair of integers whose sum is zero (0) but difference is 10.

15-04-2018

Solution: Since sum of two integers is zero, one integer is the additive inverse of other integer, like – 3, 3; – 4, 4 etc. But the difference has to be 10. So, the integers are 5 and – 5 as 5–(–5) is 10.

SUBTRACTING INTEGERS

Words	Numbers	Formula
To subtract an integer, add its inverse	3 – 7 = 3 + (–7) 5 – (–8) = 5 + 8	$a - b = a + (-b)$ $a - (-b) = a + b$

Example 13: Write two integers which are smaller than –3, but their difference is greater than –3.

Solution: – 5 and – 4 are smaller than – 3 but their difference is (–4) – (–5) = 1 which is greater than – 3.

or

– 6 and –10 are smaller than – 3 but their difference is (–6) – (–10) = 4 which is greater than – 3.

Example 14: Write a pair of integers whose product is – 15 and whose difference is 8.

Solution: There are few pairs of integers whose product is – 15.

e.g. -1×15

-3×5

$3 \times (-5)$

$15 \times (-1)$

but difference of –3 and 5 or –5 and 3 is 8. So the required pair of integers is – 3, 5 and – 5, 3.

Example 15: If Δ is an operation such that for integers a and b we have $a \Delta b = a \times a + b \times b - a \times b$, then find $(-3) \Delta 2$.

Solution : $-3 \Delta 2 = (-3) \times (-3) + 2 \times 2 - (-3) \times 2$

$= 9 + 4 - (-6) = 13 + 6 = 19.$

Example 16: In an objective type test containing 25 questions. A student is to be awarded +5 marks for every correct answer, –5 for every incorrect answer and zero for not writing any answer. Mention the ways of scoring 110 marks by a student.

Solution: Marks scored = +110

So, minimum correct responses = 110 ÷ (+5) = 22

Case 1

Correct responses = 22
Marks for 1 correct response = + 5
Marks for 22 correct respones = +110 (22 × 5)
Marks scored = +110
Marks obtained for incorrect answer = 0
So, no incorrect response
And, therefore, 3 were unattempted

Case 2

Correct responses = 23
Marks from 23 correct responses = + 115 (23 × 5)
Marks scored = + 110
Marks obtained for incorrect answers = 110 – (+115)
= –5
Marks for 1 incorrect answer = –5
Number of incorrect responses = (–5) ÷ (–5)
= 1
So, 23 correct, 1 incorrect and 1 unattempted.

Case 3

Correct responses = 24
Marks from 24 correct responses = + 120 (24 × 5)
Marks scored = + 110
Marks obtained for incorrect answers = +110 – (+120)
= –10
Number of incorrect responses = (–10) ÷ (–5)
= 2
Thus the number of questions = 24 + 2 = 26. Whereas, total number of questions is 25. So, this case is not possible.
So, the possible ways are:

- 22 correct, 0 incorrect, 3 unattempted
- 23 correct, 1 incorrect, 1unattempted.

MULTIPLYING AND DIVIDING TWO INTEGERS

If the signs are the same, the sign of the answer is **positive.**
If the signs are different, the sign of the answer is **negative.**

15-04-2018

Application on Problem Solving Strategy



Example 17

A boy standing on the third stair on a staircase goes up by five more stairs. Which stair is he standing at now? At which step will he be after he comes down by 2 stairs?

Solution:



Understand and Explore the Problem

- What do you know?

  The current position of the boy.

- What are we trying to find?

  The new position of the boy after he takes five more stairs in the same direction.



Plan a Strategy

- Going up one stairs is equal to + 1 and coming down by one stairs is equal to – 1. Ground level is taken as 0.



Solve

- He is currently at the third stair i.e. at (+3).
- He goes up 5 stairs in the same direction.
- Since 3 + 5 = 8. Therefore, he is at 8th stair on the staircase.

Now, the boy comes down by 2 stairs. Since he comes down in opposite direction i.e. downwards by 2 stairs (i.e. –2), so 8 + (–2) = 8 – 2 = 6. He is at 6th step now.



Revise

Since the boy moves 5 stairs up and then 2 stairs down, therefore, as a result he moves (5 –2) stairs i.e. 3 stairs up. As he was on stair 3 and since he goes up by 3 stairs (i.e. +3), so, he is at (3 + 3) the i.e 6th step now.



Think and Discuss

1. Can you find the position of the boy if he comes down further by 3 more stairs?

(C) Exercise

In the Questions 1 to 25, there are four options, out of which only one is correct. Write the correct one.

1. When the integers 10, 0, 5, – 5, – 7 are arranged in descending or ascending order, them find out which of the following integers always remains in the middle of the arrangement.

(a) 0 (b) 5 (c) – 7 (d) – 5

2. By observing the number line (Fig. 1.2), state which of the following statements is not true.



Fig. 1.2

(a) B is greater than –10 (b) A is greater than 0

(c) B is greater than A (d) B is smaller than 0

3. By observing the above number line (Fig. 1.2), state which of the following statements is true.

(a) B is 2 (b) A is – 4

(c) B is –13 (d) B is – 4

Think and Discuss

1. **Compare** the sums 10 + (–22) and –10 + 22.
2. **Describe** how to add the following addition expressions on a number line, 9 + (– 13) and –13 + 9. Then compare the sums.

4. Next three consecutive numbers in the pattern 11, 8, 5, 2, --, --, -- are

(a) 0, – 3, – 6 (b) – 1, – 5, – 8

(c) – 2, – 5, – 8 (d) – 1, – 4, – 7

15-04-2018

5. The next number in the pattern – 62, – 37, – 12 _____ is

(a) 25 (b) 13 (c) 0 (d) –13

6. Which of the following statements is not true?

(a) When two positive integers are added, we always get a positive integer.

(b) When two negative integers are added we always get a negative integer.

(c) When a positive integer and a negative integer is added we always get a negative integer.

(d) Additive inverse of an integer 2 is (– 2) and additive inverse of (– 2) is 2.

7. On the following number line value 'Zero' is shown by the point



(a) X (b) Y (c) Z (d) W

8. If ⊗, O, ⊘ and • represent some integers on number line, then descending order of these numbers is



(a) •, ⊗, ⊘, ○ (b) ⊗, •, ⊘, ○ (c) ○, ⊘, ⊗, • (d) ○, •, ⊗, ⊘

9. On the number line, the value of (–3) × 3 lies on right hand side of

(a) – 10 (b) – 4 (c) 0 (d) 9

10. The value of 5 ÷ (–1) does not lie between

(a) 0 and – 10 (b) 0 and 10 (c) – 4 and – 15 (d) – 6 and 6

11. Water level in a well was 20m below ground level. During rainy season, rain water collected in different water tanks was drained

into the well and the water level rises 5 m above the previous level. The wall of the well is 1m 20 cm high and a pulley is fixed at a height of 80 cm. Raghu wants to draw water from the well. The minimum length of the rope that he can use is

(a) 17 m (b) 18 m (c) 96 m (d) 97 m

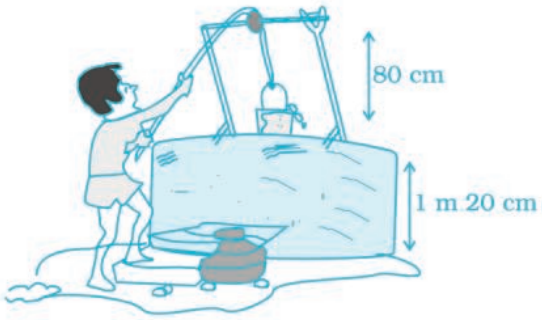


Fig. 1.3

12. (– 11) × 7 is not equal to

(a) 11 × (– 7) (b) – (11 × 7) (c) (– 11) × (– 7) (d) 7 × (– 11)

13. (– 10) × (– 5) + (– 7) is equal to

(a) – 57 (b) 57 (c) – 43 (d) 43

14. Which of the folllowing is not the additive inverse of a ?

(a) $-(-a)$ (b) $a \times (-1)$ (c) $-a$ (d) $a \div (-1)$

Vocabulary

1. _____is the_____ of addition.
2. The expression 3 ×4 and 4 ×3 are equal by the_____.
3. The expressions 1 + (2 + 3) and (1 + 2) + 3 are equal by the_____.
4. Multiplication and _____are opposite operations.
5. _____ and _____are commutative.

- **Addition**
- **Associative Property**
- **Commutative Property**
- **Divison**
- **Multiplication**
- **Opposite Operation**
- **Subtraction**

15-04-2018

15. Which of the following is the multiplicative identity for an integer a?
(a) a (b) 1 (c) 0 (d) – 1

16. [(– 8) × (– 3)] × (– 4) is not equal to
(a) (– 8) × [(– 3) × (– 4)] (b) [(– 8) × (– 4)] × (– 3)
(c) [(– 3) × (– 8)] × (– 4) (d) (– 8) × (– 3) – (– 8) × (– 4)

17. (– 25) × [6 + 4] is not same as
(a) (– 25) × 10 (b) (– 25) × 6 + (– 25) × 4
(c) (– 25) × 6 × 4 (d) – 250

18. – 35 × 107 is not same as
(a) – 35 × (100 + 7) (b) (– 35) × 7 + (– 35) × 100
(c) – 35 × 7 + 100 (d) (– 30 – 5) × 107

19. (– 43) × (– 99) + 43 is equal to
(a) 4300 (b) – 4300 (c) 4257 (d) – 4214

20. (– 16) ÷ 4 is not same as
(a) (– 4) ÷ 16 (b) – (16 ÷ 4) (c) 16 ÷ (– 4) (d) – 4

21. Which of the following does not represent an integer?
(a) 0 ÷ (– 7) (b) 20 ÷ (– 4) (c) (– 9) ÷ 3 (d) (– 12) ÷ 5

22. Which of the following is different from the others?
(a) 20 + (–25) (b) (– 37) – (– 32)
(c) (– 5) × (–1) (d) (45) ÷ (– 9)

23. Which of the following shows the maximum rise in temperature?
(a) 23° to 32° (b) – 10° to + 1° (c) – 18° to – 11° (d) – 5° to 5°

24. If a and b are two integers, then which of the following may not be an integer?
(a) $a + b$ (b) $a - b$ (c) $a \times b$ (d) $a \div b$

25. For a non-zero integer a which of the following is not defined?
(a) $a \div 0$ (b) $0 \div a$ (c) $a \div 1$ (d) $1 \div a$

Encircle the odd one of the following (Questions 26 to 30).

26. (a) (–3, 3) (b) (–5, 5) (c) (–6, 1) (d) (–8, 8)

27. (a) (–1, –2) (b) (–5, +2) (c) (–4, +1) (d) (–9, +7)

28. (a) $(-9) \times 5 \times 6 \times (-3)$ (b) $9 \times (-5) \times 6 \times (-3)$
(c) $(-9) \times (-5) \times (-6) \times 3$ (d) $9 \times (-5) \times (-6) \times 3$

29. (a) $(-100) \div 5$ (b) $(-81) \div 9$
(c) $(-75) \div 5$ (d) $(-32) \div 9$

30. (a) $(-1) \times (-1)$ (b) $(-1) \times (-1) \times (-1)$
(c) $(-1) \times (-1) \times (-1) \times (-1)$ (d) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

In Questions 31 to 71, fill in the blanks to make the statements true.

31. $(-a) + b = b +$ Additive inverse of _____.

32. _____ $\div (-10) = 0$

33. $(-157) \times (-19) + 157 =$ _____

34. $[(-8) +$ _____ $] +$ _____ $=$ _____ $+ [(-3) +$ _____ $] = -3$

35. On the following number line, $(-4) \times 3$ is represented by the point _____.



36. If x, y and z are integers then $(x+$___ $) + z =$ _____ $+ (y +$ _____ $)$

37. $(-43) +$ _____ $= -43$

38. $(-8) + (-8) + (-8) =$ _____ $\times (-8)$

39. $11 \times (-5) = -($ _____ $\times$ _____ $) =$ _____

40. $(-9) \times 20 =$ _____

41. $(-23) \times (42) = (-42) \times$ _____

42. While multiplying a positive integer and a negative integer, we multiply them as _____ numbers and put a _____ sign before the product.

43. If we multiply _____ number of negative integers, then the resulting integer is positive.

44. If we multiply six negative integers and six positive integers, then the resulting integer is _____.

15-04-2018

45. If we multiply five positive integers and one negative integer, then the resulting integer is _____.

46. _____ is the multiplicative identity for integers.

47. We get additive inverse of an integer a when we multiply it by _____.

48. $(-25) \times (-2) =$ _____

49. $(-5) \times (-6) \times (-7) =$ _____

50. $3 \times (-1) \times (-15) =$ _____

51. $[12 \times (-7)] \times 5 =$ _____ $\times [(-7) \times$ _____ $]$

52. $23 \times (-99) =$ _____ $\times (-100 +$ _____$) = 23 \times$ _____ $+ 23 \times$ _____

53. _____ $\times (-1) = -35$

54. _____ $\times (-1) = 47$

Think and Discuss

1. **Explain** why $10 - (-10)$ does not equal $-10 - 10$.
2. **Describe** the answer that you get when you subtract a greater number from a lesser number.

55. $88 \times$ _____ $= -88$

56. _____ $\times (-93) = 93$

57. $(-40) \times$ _____ $= 80$

58. _____ $\times (-23) = -920$

59. When we divide a negative integer by a positive integer, we divide them as whole numbers and put a _____ sign before quotient.

60. When -16 is divided by _____ the quotient is 4.

61. Division is the inverse operation of _____

62. $65 \div (-13) =$ _____

63. $(-100) \div (-10) =$ _____

64. $(-225) \div 5 =$ _____

65. _____ $\div (-1) = -83$

66. _____ $\div (-1) = 75$

67. $51 \div$ _____ $= -51$

68. $113 \div$ _____ $= -1$

69. $(-95) \div$ _____ $= 95$

70. $(-69) \div (69) =$ _____

71. $(-28) \div (-28) =$ _____

In Questions 72 to 108, state whether the statements are True or False.

72. $5 - (-8)$ is same as $5 + 8$.

73. $(-9) + (-11)$ is greater than $(-9) - (-11)$.

74. Sum of two negative integers always gives a number smaller than both the integers.

75. Difference of two negative integers cannot be a positive integer.

76. We can write a pair of integers whose sum is not an integer.

77. Integers are closed under subtraction.

78. $(-23) + 47$ is same as $47 + (-23)$.

79. When we change the order of integers, their sum remains the same.

80. When we change the order of integers their difference remains the same.

81. Going 500 m towards east first and then 200 m back is same as going 200 m towards west first and then going 500 m back.

82. $(-5) \times (33) = 5 \times (-33)$

83. $(-19) \times (-11) = 19 \times 11$

15-04-2018

84. $(-20) \times (5-3) = (-20) \times (-2)$

85. $4 \times (-5) = (-10) \times (-2)$

86. $(-1) \times (-2) \times (-3) = 1 \times 2 \times 3$

87. $-3 \times 3 = -12 - (-3)$

88. Product of two negative integers is a negative integer.

89. Product of three negative integers is a negative integer.

90. Product of a negative integer and a positive integer is a positive integer.

91. When we multiply two integers their product is always greater than both the integers.

92. Integers are closed under multiplication.

93. $(-237) \times 0$ is same as $0 \times (-39)$

94. Multiplication is not commutative for integers.

95. (-1) is not a multiplicative identity of integers.

96. 99×101 can be written as $(100-1) \times (100+1)$

97. If a, b, c are integers and $b \neq 0$ then, $a \times (b-c) = a \times b - a \times c$

98. $(a+b) \times c = a \times c + a \times b$

99. $a \times b = b \times a$

100. $a \div b = b \div a$

101. $a - b = b - a$

Think and Discuss

1. **List** all possible multiplication and division statements for the integers with 5, –5, 6, –6 and 30, –30.

   For example, $5 \times 6 = 30$.

2. **Compare** the sign of the product of two negative integers with the sign of the sum of two negative integers.

3. **Suppose** the product of two integers is positive. What do you know about the signs of the integers?

102. $a \div (-b) = -(a \div b)$

103. $a \div (-1) = -a$

104. Multiplication fact $(-8) \times (-10) = 80$ is same as division fact $80 \div (-8) = (-10)$

105. Integers are closed under division.

106. $[(-32) \div 8] \div 2 = -32 \div [8 \div 2]$

107. The sum of an integer and its additive inverse is zero (0).

108. The successor of $0 \times (-25)$ is $1 \times (-25)$

109. Observe the following patterns and fill in the blanks to make the statements true:

(a) $-5 \times 4 = -20$
$-5 \times 3 = -15 = -20 - (-5)$
$-5 \times 2 =$ _____ $= -15 - (-5)$
$-5 \times 1 =$ _____ $=$ _____
$-5 \times 0 = 0 =$ _____
$-5 \times -1 = 5 =$ _____
$-5 \times -2 =$ _____ $=$ _____

(b) $7 \times 4 = 28$
$7 \times 3 =$ _____ $= 28 - 7$
$7 \times 2 =$ _____ $=$ _____ $- 7$
$7 \times 1 = 7 =$ _____ $- 7$
$7 \times 0 =$ _____ $=$ _____ $-$ _____
$7 \times -1 = -7 =$ _____ $-$ _____
$7 \times -2 =$ _____ $=$ _____ $-$ _____
7×-3 _____ $=$ _____ $-$ _____

110. **Science Application:** An atom consists of charged particles called electrons and protons. Each proton has a charge of +1 and each electron has a charge of –1. Remember number of electrons is equal to number of protons, while answering these questions:

(a) What is the charge on an atom?

(b) What will be the charge on an atom if it loses an electron?

(c) What will be the charge on an atom if it gains an electron?

111. An atom changes to a charged particle called ion if it loses or gains electrons. The charge on an ion is the charge on electrons plus charge

15-04-2018

on protons. Now, write the missing information in the table given below:

Name of Ion	Proton Charge	Electron Charge	Ion Charge
Hydroxide ion	+9	—	–1
Sodium ion	+11	—	+1
Aluminium ion	+13	–10	—
Oxide ion	+8	–10	—



Plan a Strategy

- Some problems contain a lot of information. Read the entire problem carefully to be sure you understand all of the facts. You may need to read it over several times perhaps aloud so that you can hear yourself.
- Then decide which information is the most important (prioritise). Is there any information that is absolutely necessary to solve the problem? This information is the most important.
- Finally, put the information in order (sequence). Use comparison words like *before*, *after*, *longer*, *shorter*, and so on to help you. Write down the sequence before you try to solve the problem.

Read the problem given below and then answer the questions that follow:

- Five friends are standing in a line waiting for the opening of a show. They are in line according to their arrival. Shreya arrived 3 minutes after Sachin. Roy took his place in line at 9:01 P.M. He was 1 minute behind Reena and 7 minutes ahead of Shreya. The first person arrived at 9:00 P.M. Babu showed up 6 minutes after the first person. List the time of each person's arrival.
  (a) Whose arrival information helped you to determine arrival time of each?
  (b) Can you determine the order without the time?
  (c) List the friends' order from the earliest arrival to the last arrival.

15-04-2018

112. Social Studies Application: Remembering that 1AD came immediately after 1BC, while solving these problems take 1BC as –1 and 1AD as +1.

(a) The Greeco-Roman era, when Greece and Rome ruled Egypt started in the year 330 BC and ended in the year 395 AD. How long did this era last?

(b) Bhaskaracharya was born in the year 1114 AD and died in the year 1185 AD. What was his age when he died?

(c) Turks ruled Egypt in the year 1517 AD and Queen Nefertis ruled Egypt about 2900 years before the Turks ruled. In what year did she rule?

(d) Greek mathematician Archimedes lived between 287 BC and 212 BC and Aristotle lived between 380 BC and 322 BC. Who lived during an earlier period?

113. The table shows the lowest recorded temperatures for each continent. Write the continents in order from the lowest recorded temperature to the highest recorded temperature.

The Lowest Recorded Temperatures	
Continent	**Temperature (in Fahrenheit)**
Africa	-11^0
Antarctica	-129^0
Asia	-90^0
Australia	-9^0
Europe	-67^0
North America	-81^0
South America	-27^0

114. Write a pair of integers whose product is –12 and there lies seven integers between them (excluding the given integers).

115. From given integers in Column I match an integer of Column II so that their product lies between –19 and –6:

Column I	**Column II**
– 5	1
6	–1
– 7	3
8	–2

15-04-2018

116. Write a pair of integers whose product is – 36 and whose difference is 15.

117. Match the following

	Column I		Column II
(a)	$a \times 1$	(i)	Additive inverse of a
(b)	1	(ii)	Additive identity
(c)	$(-a) \div (-b)$	(iii)	Multiplicative identity
(d)	$a \times (-1)$	(iv)	$a \div (-b)$
(e)	$a \times 0$	(v)	$a \div b$
(f)	$(-a) \div b$	(vi)	a
(g)	0	(vii)	$-a$
(h)	$a \div (-a)$	(viii)	0
(i)	$-a$	(ix)	–1

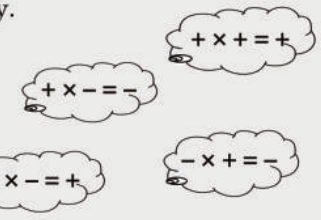
118. You have ₹ 500 in your savings account at the beginning of the month. The record below shows all of your transactions during the month. How much money is in your account after these transactions?

Cheque No.	Date	Transaction Description	Payment	Deposit
384102	4/9	Jal Board	₹ 120	₹ 200
275146	12/9	Deposit		
384103	22/9	LIC India	₹ 240	₹ 150
801351	29/9	Deposit		

Think and Discuss

Is it not true? If + is a friend and – is an enemy.

1. Your friend's friend is your friend.
2. Your friend's enemy is your enemy.
3. Your enemy's friend is your enemy.
4. Your enemy's enemy is your friend.



119. (a) Write a positive integer and a negative integer whose sum is a negative integer.

(b) Write a positive integer and a negative integer whose sum is a positive integer.

(c) Write a positive integer and a negative integer whose difference is a negative integer.

(d) Write a positive integer and a negative integer whose difference is a positive integer.

(e) Write two integers which are smaller than – 5 but their difference is – 5.

(f) Write two integers which are greater than – 10 but their sum is smaller than – 10.

(g) Write two integers which are greater than – 4 but their difference is smaller than – 4.

(h) Write two integers which are smaller than – 6 but their difference is greater than – 6.

(i) Write two negative integers whose difference is 7.

(j) Write two integers such that one is smaller than –11, and other is greater than –11 but their difference is –11.

(k) Write two integers whose product is smaller than both the integers.

(l) Write two integers whose product is greater than both the integers.

120. What's the Error? Ramu evaluated the expression –7 – (–3) and came up with the answer –10. What did Ramu do wrong?

121. What's the Error? Reeta evaluated $-4 + d$ for $d = -6$ and gave an answer of 2. What might Reeta have done wrong?

122. The table given below shows the elevations relative to sea level of four locations.

Taking sea level as zero, answer the following questions:

Location	Elevation (in m)
A	–180
B	1600
C	–55
D	3200

15-04-2018

(a) Which location is closest to sea level?

(b) Which location is farthest from sea level?

(c) Arrange the locations from the least to the greatest elevation.

123. You are at an elevation 380 m above sea level as you start a motor ride. During the ride, your elevation changes by the following metres: 540 m, –268 m, 116 m, –152 m, 490 m, –844 m, 94 m. What is your elevation relative to the sea level at the end of the ride?

124. Evaluate the following, using distributive property.

(i) -39×99 (ii) $(-85) \times 43 + 43 \times (-15)$

(iii) $53 \times (-9) - (-109) \times 53$ (iv) $68 \times (-17) + (-68) \times 3$

125. If $*$ is an operation such that for integers a and b we have

$$a * b = a \times b + (a \times a + b \times b)$$

then find (i) $(-3) * (-5)$

(ii) $(-6) * 2$

126. If Δ is an operation such that for integers a and b we have

$$a \ \Delta \ b = a \times b - 2 \times a \times b + b \times b \ (-a) \times b + b \times b$$

then find (i) $4 \ \Delta \ (-3)$

(ii) $(-7) \ \Delta \ (-1)$

Also show that $4 \ \Delta \ (-3) \neq (-3) \ \Delta \ 4$

and $(-7) \ \Delta \ (-1) \neq (-1) \ \Delta \ (-7)$

127. Below u, v, w and x represent different integers, where $u = -4$ and $x \neq 1$. By using following equations, find each of the values:

$$u \times v = u$$
$$x \times w = w$$
$$u + x = w$$

(a) v

(b) w

(c) x

Explain your reasoning using the properties of integers.

128. Height of a place A is 1800 m above sea level. Another place B is 700 m below sea level. What is the difference between the levels of these two places?

129. The given table shows the freezing points in ^{0}F of different gases at sea level. Convert each of these into ^{0}C to the nearest integral value using the relation and complete the table,

$$C = \frac{5}{9}(F - 32)$$

Gas	Freezing Point at Sea Level (°F)	Freezing Point at Sea Level (°C)
Hydrogen	–435	
Krypton	–251	
Oxygen	–369	
Helium	–458	
Argon	–309	

130. Sana and Fatima participated in an apple race. The race was conducted in 6 parts. In the first part, Sana won by 10 seconds. In the second part she lost by 1 minute, then won by 20 seconds in the third part and lost by 25 seconds in the fourth part, she lost by 37 seconds in the fifth part and won by 12 seconds in the last part. Who won the race finally?

131. A green grocer had a profit of ₹ 47 on Monday, a loss of ₹ 12 on Tuesday and loss of ₹ 8 on Wednesday. Find his net profit or loss in 3 days.

132. In a test, +3 marks are given for every correct answer and –1 mark are given for every incorrect answer. Sona attempted all the questions and scored +20 marks though she got 10 correct answers.

(i) How many incorrect answers has she attempted?

(ii) How many questions were given in the test?

133. In a true-false test containing 50 questions, a student is to be awarded 2 marks for every correct answer and –2 for every incorrect answer and 0 for not supplying any answer. If Yash secured 94 marks in a test, what are the possibilities of his marking correct or wrong answer?

134. A multistorey building has 25 floors above the ground level each of height 5m. It also has 3 floors in the basement each of height 5m. A lift in building moves at a rate of 1m/s. If a man starts from 50m above the ground, how long will it take him to reach at 2nd floor of basement?

135. Taking today as zero on the number line, if the day before yesterday is 17 January, what is the date 3 days after tomorrow?

Think and Discuss

1. **Explain** how integers are used in real life to manage a bank account.
2. **Explain** whether –1, –4, and 5 are additive inverses.

15-04-2018

136. The highest point measured above sea level is the summit of Mt. Everest which is 8,848m above sea level and the lowest point is challenger Deep at the bottom of Mariana Trench which is 10911m below sea level. What is the vertical distance between these two points?

(D) Application

Puzzle 1

Fill in the blank space of the following magic square so that the sum of the numbers in each row, each column and each of the diagonals is – 6.

(i)

–1		
3	– 2	
	5	

(ii) In this magic square, sum of the numbers in every row, column and each of the diagonals is – 2. Fill in the blank:

7		– 6	
		1	
0	– 2		
– 5		6	– 8

Puzzle 2

If $a * b$ means $a \times b + 2$ and

$a \# b$ means $-a + b - (-3)$,

then find the value of the following:

(i) $-4 * 3$ (ii) $(-3) * (-2)$

(iii) $(-7) \# (-3)$ (iv) $2 \# (-4)$

(v) $7 * (-5)$ (vi) $(-7 * 2) \# 3$

Next, match these answers with suitable letters by looking at the table below and arrange them in increasing order of integers to decode the name of the mathematician:

Integers	–9	14	–3	4	–10	8	–33	–21	7	18
Letters	P	Y	C	T	U	I	E	G	L	D

Puzzle 3

'Equinoxes' are the two days of the year when the sun is directly above the earth's equator, due to which the days and nights are of nearly equal length everywhere on the earth.

Find the name of the month of autumn equinox using suitable properties of integers by solving the following questions. Match your answer with the letter given in the table and fill it in the box provided in each question.

(a) $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$ ☐

(b) $18946 \times 99 - (-18946)$ ☐

(c) $-1 + (-2) + (-3) + (-9) + (-8)$ ☐

(d) $15 \times (-99)$ ☐

(e) $-143 + 600 - 257 + 400$ ☐

(f) $0 \div (-12)$ ☐

(g) $-125 \times 9 - 125$ ☐

(h) $\underbrace{(-1) \times (-1) \times \ldots\ldots\ldots\ldots \times (-1)}_{20 \text{ times}}$ ☐

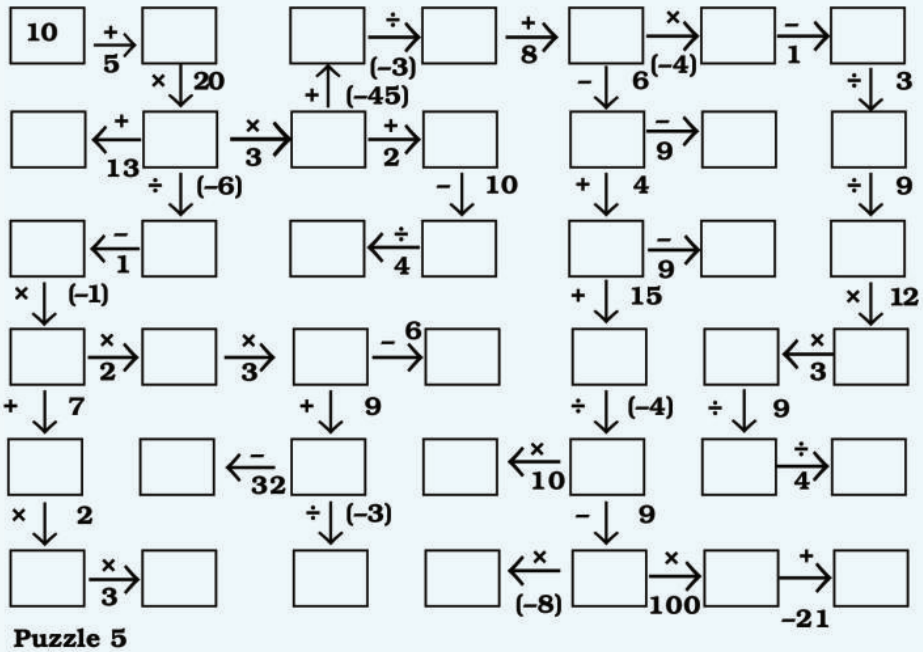
(i) $\underbrace{-4 + 4 - 4 + 4 - \ldots\ldots - 4}_{21 \text{ times}}$ ☐

1	E
–1485	T
–120	S
–30	P
–4	R
–1250	B
1894600	E
600	E
0	M

15-04-2018

Puzzle 4

Complete the number grids by following the direction of arrows.



Puzzle 5

Solve the following riddles.

(a) Minus of minus six
Minus minus-minus-seven
What do you get if this is **added** to
minus-minus-seven again?

(b) Now **add** the value in riddle (a) to minus four and then minus two you **take away**
Divide this by minus two
What is this value can you say?

(c) Take the result of riddle (b) and **subtract** from it minus six
Multiply this by minus two
What will the answer be?

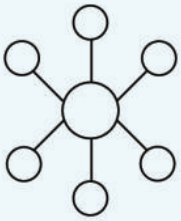


Fig. 1.4

Puzzle 6

Use the integers –2, 4, –5, –12, 20, –25 and 50 just one each in the wheel shown in Fig. 1.4 to make the product 1200 along each line.

15-04-2018