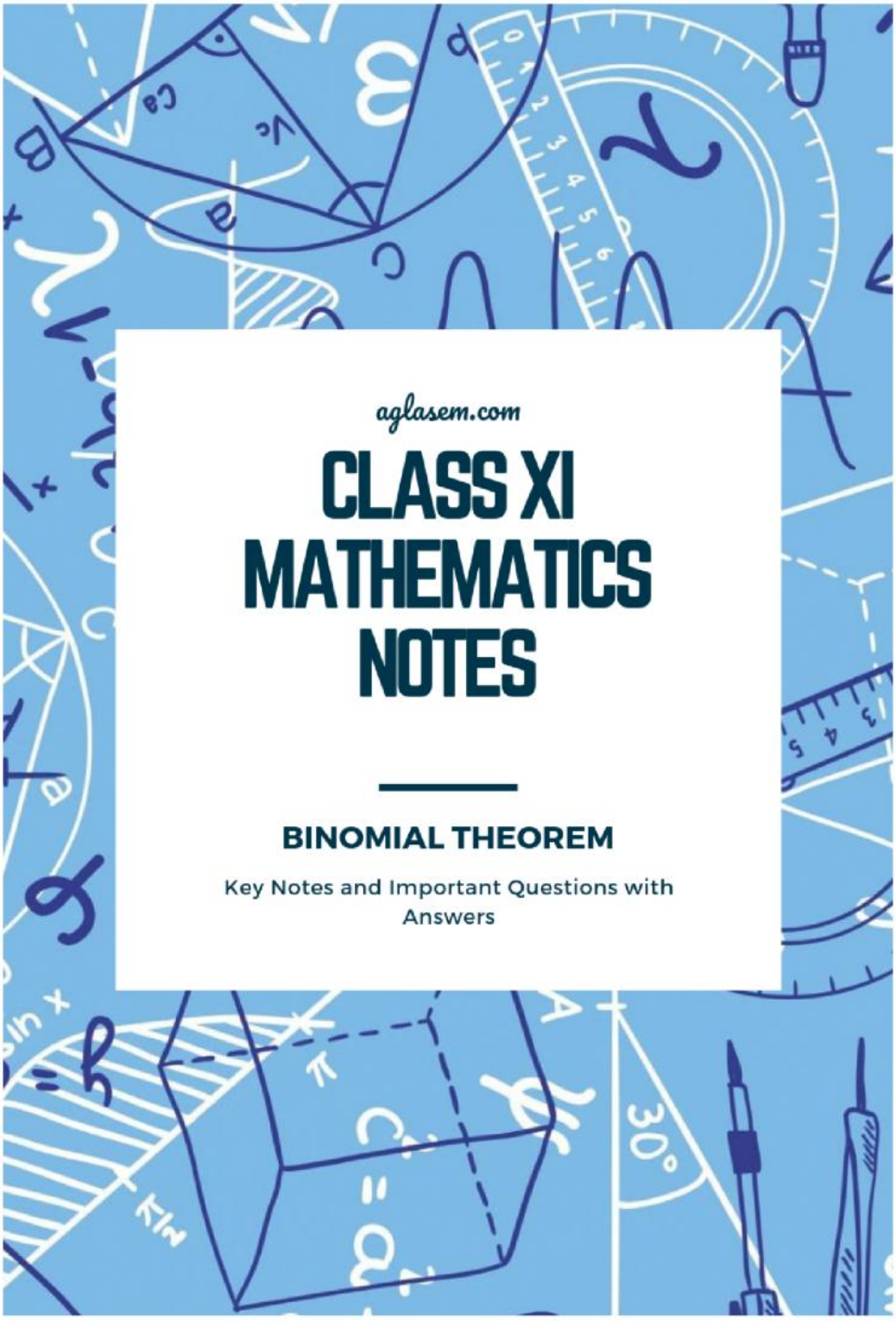


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CLASS XI MATHEMATICS NOTES

BINOMIAL THEOREM

Key Notes and Important Questions with Answers

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

- If n is a natural number and a, b are any numbers.

$$(a+b)^n = {}^nc_0a^n + {}^nc_1a^{n-1}b + {}^nc_2a^{n-2}b^2 + \ldots\ldots\ldots\ldots + {}^nc_nb^n$$

$$= \sum_{r=0}^{n} {}^nc_r a^{n-r}b^r, \quad n \in N$$

- T_{r+1} = General term

$$= {}^nc_r a^{n-r}b^r \qquad 0 \le r \le n$$

- Total number of terms in $(a + b)^n$ is $(n + 1)$.

- If n is even, then in the expansion of $(a+b)^n$, middle term is $\left(\frac{n}{2}+1\right)^{th}$ term i,e $\left(\frac{n+2}{2}\right)^{th}$ terms.

- If n is odd, then in the expansion of $(a + b)^n$, middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$

- In $(a + b)^n$, r^{th} term from the end is same as $(n - r + 2)^{th}$ term from the beginning.

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- r^{th} term from the end in $(b + a)^n$

  = r^{th} term from the beginning in $(a + b)^n$

- In $(1 + x)^n$, coefficient of x^r is ${}^nC_r$

- Some particular cases:

$$(1+x)^n = \sum_{r=0}^{n} {}^nc_r x^r$$

$$(1-x)^n = \sum_{r=0}^{n} (-1)^r {}^nc_r x^r$$

- Some properties of Binomial coefficients:

  ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

  ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^n {}^nC_n = 0$

  ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$

Important Questions with Solutions

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write number of terms in the expansion of $\left\{\left(2x+y^3\right)^4\right\}^7$.

2. Expand $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.

3. Write value of

$${}^{2n-1}c_5+{}^{2n-1}c_6+{}^{2n}c_7 \qquad use\left[{}^{n}c_r+{}^{n}c_{r-1}={}^{n+1}c_r\right]$$

4. Which term is greater $(1.2)^{4000}$ *or* 800?

5. Find the coefficient of x^{-17},in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

[**Hint :** Put x = 1, y = 1]

7. If ${}^nC_{n-3} = 720$, find n.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. How many term are free from radical signs in the expansion of

$$\left(x^{\frac{1}{5}}+y^{\frac{1}{10}}\right)^{55}$$

9. Find the constant term in expansion of $\left(x-\frac{1}{x}\right)^{10}$.

10. Find the value of

$$\frac{{}^8C_0}{6}-{}^8C_1+{}^8C_2\cdot 6-{}^8C_3\cdot 6^2+...+{}^8C_8 6^7$$

11. Find 4th term from end in the expansion of find the value of

$$\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$$

12. Write the last two digits of the number 3^{400}.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Section - C

13. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

14. In $\left(3x^2 - \frac{1}{x}\right)^{18}$ which term contains x^{12}?

15. In $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

16. Evaluate $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5$ using binomial theorem.

17. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

18. Find the coefficients of x^4 in $\left(1-x\right)^2\left(2+x\right)^5$ using binomial theorem.

19. show that$3^{2n+2} - 8n - 9$ is divisible by 8.

20. If the term free from x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

21. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1024}$

22. If for positive integers r > 1, n > 2 the coefficients of the $(3r)^{th}$ term and $(r + 2)^{th}$ powers of x in the expansion of $(1 +)^{2n}$ are equal, then prove that n = 2r + 1.

23. If a, b, c and d in any binomial expansion be the 6th , 7th , 8th, and 9th terms respectively, then prove that $\frac{b^2-ac}{c^2-bd}=\frac{4a}{3c}$.

24. If in the expansion of $(1+x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

25. Show that $2^{4n+4}-15n-16$ where $n \in N$ is divisible by 225.

26. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

27. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

28. Show that the coefficient of x^5 in the expansion of product

$(1 + 2x)^6 (1 – x)^7$ is 171.

29. If the 3rd, 4th and 5th terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n

30. If the coefficients of x^7 in $\left[ax^2+\frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax-\frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1

31. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7th term from the beginning to the 7th term from the end is 1:6 , find n.

32. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$ prove that

$$\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}.$$

33. Using binomial theorem, find the remainder when 5^{103} is divided by 13.

HOTS

34. Find the greatest value of the term independent of x in the expansion of $\left(x\sin\alpha+\frac{\cos\alpha}{x}\right)^{10}$; $where \quad \alpha\in R$.

35. Find the number of terms in the expansion of $(x + y + z)^n$

36. Find the coefficient of the term independent of x in the expansion of $\left\{\frac{x+1}{x^{213}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right\}^{10}$

37. For what value of x is the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{-\frac{1}{8}\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ equal to 180.

38. Find the remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9.

39. Find the coefficient of x^n in expansions of $(1+x)\,(1-x)^n$.

40. Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

41. Find the term independent of x in the expansion of

$(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3}x\right)^9$

42. If the coefficients of r^{th}, (r+1)th and $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in A.P find the value of r.

43. If ${}^{n-1}c_r = \left(k^2 - 3\right){}^n c_{r+1}$ then find the value of k.

44. Find the remainder of 51^{51} when divided by 25.

45. If the coefficient of $x^k \left(0 \le k \le 15\right)$ in the expansion of the expression $1+(1+x)+(1+x)^2+.....+(1+x)^{15}$ is the greatest, then find the value of k.

ANSWERS

1. 29

2. $\frac{x^3}{a^3}-\frac{6x^2}{a^2}+15\frac{x}{a}-20+15\frac{a}{x}-\frac{6a^2}{x^2}+\frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. 6 terms (0, 10, 20, 30, 40, 40, 50)

9. $-252 = -{}^{10}C_5$

10. ${}^{31}C_6 - {}^{21}C_6 / \frac{5^8}{6}$

11. $\frac{672}{x^3}$

12. 01

13. a = 3, b = 2, n = 3

14. 9th term

15. $T_3 = \frac{5}{6}$

16. 82

17. 28

18. 10

20. $k = \pm 3$

21. 129 integral terms

22. $x = \frac{1}{\sqrt{10}}$ *or* 100

24. n = 8, 4th ,5th ,and 6th

29. a = 2, x = 1, n = 7

31. 9

33. 8

34. ${}^{10}c_5.2^{-5}$ *or* $\dfrac{\lfloor\underline{10}}{\left(\lfloor\underline{5}\right)^2 2^5}$

35. $\dfrac{(n+1)(n+2)}{2}$

36. 210

37. $x=\log 5^{15}$ *or* $x=1$

38. 2

39. $(-1)^n[1-n]$

40. Zero

41. $\dfrac{17}{54}$

42. 5

43. [–2, 2]

44. 1 (one)

45. 7