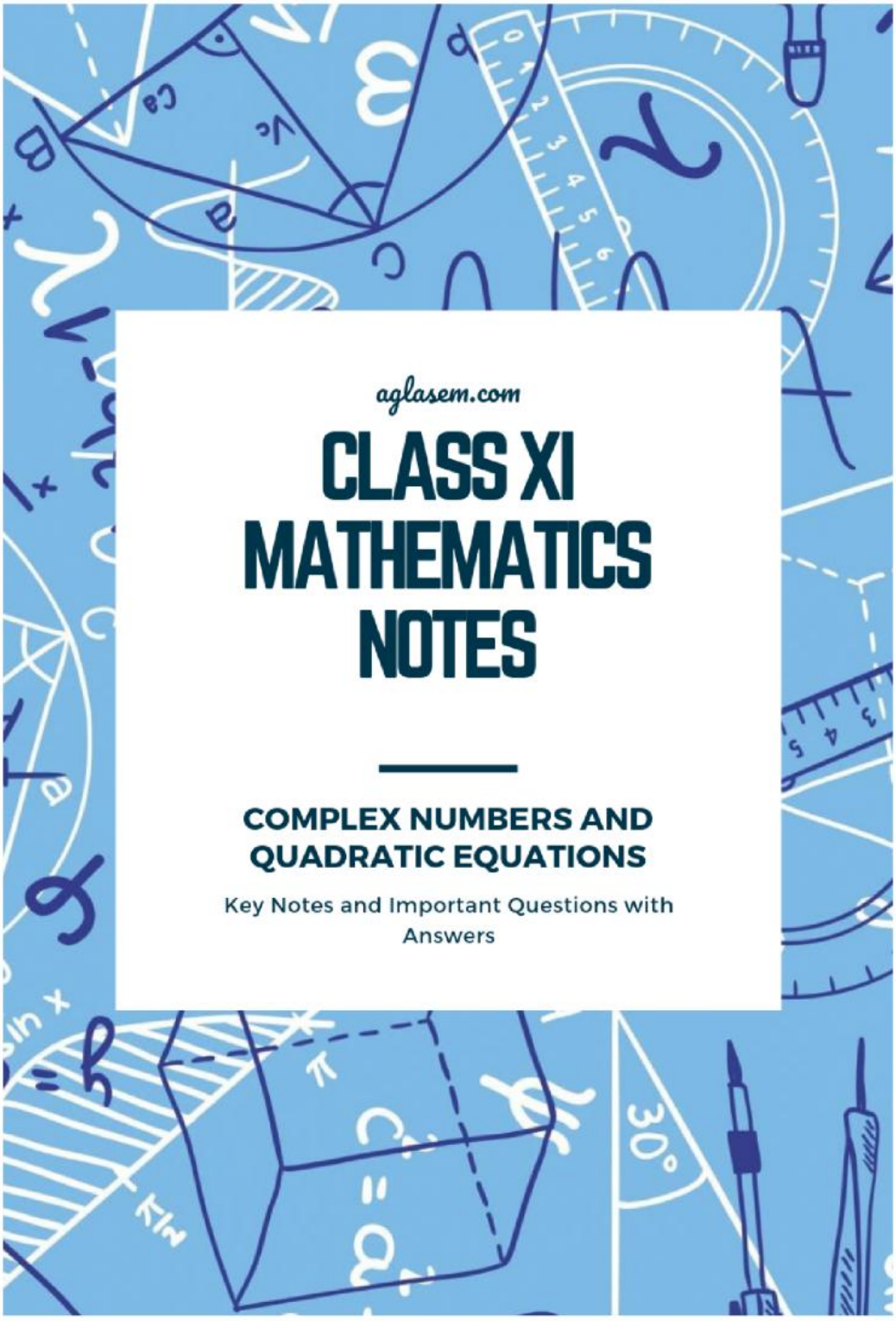

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CLASS XI
MATHEMATICS
NOTES
COMPLEX NUMBERS AND
QUADRATIC EQUATIONS
Key Notes and Important Questions with
Answers

CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form z = a + ib, where a, b $\in$ R is called a complexnumber.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id if a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- 0 + *i* 0 is additive identity of a complex number.
- –z = –a –ib is called the Additive Inverse or negative ofz = a + ib

- 1 + *i* 0 is multiplicative identity of complex number.
- $\bar{z} = a - ib$ is called the conjugate of z = a + ib
- $i^0 = 1$
- $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

  z = a + ib ($a \neq 0, b \neq 0$)
- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane
- Polar form of z = a + ib is,

  z = r ($\cos\theta$ + i $\sin\theta$) where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z, θ is called the argument or amplitude of z.
- The value of θ such that, $-\pi < \theta < \pi$ is called the principle argument of z.
- Z = x + iy, x > 0 and y > 0 the argument of z is acute angle given by $\tan\alpha = \frac{y}{x}$

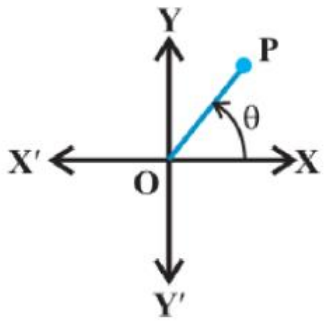
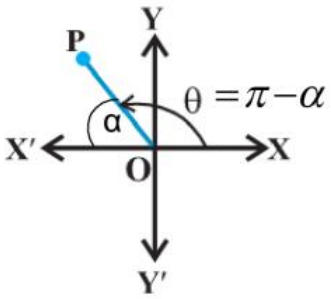


figure (i)

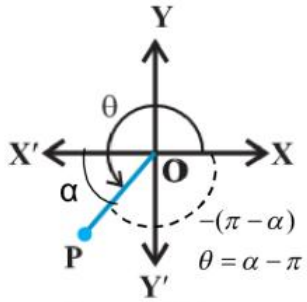
- Z = x + iy, x < 0 and y > 0 the argument of z is $\pi-\alpha$,where α is acute angle given by $\tan\alpha=\left|\frac{y}{x}\right|$



$\theta=\pi-\alpha$

figure (ii)

- Z = x + iy, x < 0 and y < 0 the argument of z is $\alpha-\pi$,where π is acute angle given by $\tan\alpha=\left|\frac{y}{x}\right|$



$\theta=\alpha-\pi$

figure (iii)

- Z = x + iy, x > 0 and y < 0 the argument of z is $-\alpha$,where α is acute angle given by $\tan\alpha=\left|\frac{y}{x}\right|$

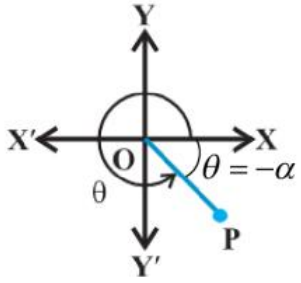


figure (iv)

- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 z_2| = |z_1|.\ |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|; |z^n| = |z|^n; |z| = |\bar{z}| = |-z| = |-\bar{z}|; z\,\bar{z} = |z|^2$
- $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$
- If $z_1 = r_1 (\cos\theta_1 + i\sin\theta_1)$

  $z_2 = r_2 (\cos\theta_2 + i\sin\theta_2)$

  then $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

  $$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

- For the quadratic equation $ax^2 + bx + c = 0$,

  $a, b, c \in R$, $a \neq 0$, if $b^2 - 4ac < 0$

  then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$



W. R. Hamilton
(1805-1865)

- $\sqrt{a+ib}$ is called square root of $z = a + ib$, $\therefore\ \sqrt{a+ib} = x + iy$

  squaring both sides we get $a + ib = x^2 - y^2 + 2i(xy)$

  $x^2 - y^2 = a$, $2xy = b$. Solving these we get x and y.

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the value of $i + i^{10} + i^{20} + i^{30}$

2. Write the additive Inverse of $6i - i\sqrt{-49}$

3. Write the multiplicative Inverse of $2 \quad i\sqrt{3}^{\,2}$

4. Write the conjugate of $\dfrac{2 \quad i}{(1 \quad 2i)^2}$

5. Write the amplitude of $\dfrac{1}{i}$

6. Write the Argument of $(1+\sqrt{3}i)(\cos\theta + i\sin\theta)$

7. Write in the form of a + *ib* $\dfrac{1}{-2+\sqrt{-3}}$

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. Evaluate :

   (i) $\sqrt{16} \quad 3\sqrt{25} \quad \sqrt{36} \quad \sqrt{625}$

   (ii) $i\sqrt{-16} + i\sqrt{-25} + \sqrt{49} - i\sqrt{-49} + 14$

   (iii) $\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3$

   (iv) $\dfrac{(3+\sqrt{5i})(3-\sqrt{5i})}{(\sqrt{3}+\sqrt{2i})-(\sqrt{3}-\sqrt{2i})}$

9. Find x and y if (x + iy) (2 – 3i) = 4 + i.

10. If n is any positi8ve integer, write value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$

11. If $z_1 = \sqrt{2}(\cos 30° + i \sin 30°)$, $z_2 = \sqrt{3}(\cos 60 + i \sin 30°)$ Find

$R_e(z_1 z_2)$

12. If $|z+4| \le 3$ then find the greatest and least values of $|z+1|$.

13. Find the real value of a for which $3i^3 - 2ai^2 + (1-a)i + 5$ is real.

14. If $\arg(z-1) = \arg(z+3i)$ where $z = x + iy$ find $x-1 : y$.

15. If z = x + iy and the amplitude of $(z - 2 - 3i)$ is $\frac{\pi}{4}$. Find the relation between x and y.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. If $x + iy = \sqrt{\frac{1+i}{1-i}}$ prove that $x^2 + y^2 = 1$

17. Find real value of θ such that, $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is a real number.

18. If $\left|\frac{z-5i}{z+5i}\right| = 1$ show that z is a real number.

19. If $x_n = \cos\frac{\pi}{2^n} + i\sin\frac{\pi}{2^n}$ Prove that $x_1 x_2 \ldots\ldots x_\infty = -1$

20. Find real value of x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$.

21. If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x + iy$. Show,

$$2.5.10\ldots\ldots(1+n^2) = x^2 + y^2$$

22. If z = 2 – 3i show that $z^2 - 4z + 13 = 0$, hence find the value of $4z^3 - 3z^2 + 169$.

23. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a + ib$, find a and b.

24. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$.

25. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$, find the least positive integral value of n

26. If $(x+iy)^{\frac{1}{3}} = a+ib$ prove $\left(\frac{x}{a}+\frac{y}{b}\right) = 4\left(a^2-b^2\right)$.

27. Convert the following in polar form:

(i) $-3\sqrt{2} + 3\sqrt{2}i$ (ii) $\frac{\left(\sqrt{3}-1\right)-\left(\sqrt{3}+1\right)i}{2\sqrt{2}}$

(iii) $i(1 + i)$ (iv) $\frac{5-i}{2-3i}$

28. Solve

(i) $x^2 - \left(3\sqrt{2} - 2i\right)x - 6\sqrt{2}i = 0$

(ii) $x^2 - (7-i)x + (18-i) = 0$

29. Find the square root of $7 - 30\sqrt{-2}$.

30. Prove that $x^2 + 4 = (x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)$

31. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represent a circle find its centre and radius.

32. Find all non-zero complex number z satisfying $\bar{z} = iz^2$.

33. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$.

34. If z_1, z_2 are complex numbers such that, $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

35. If z_1 and z_2 are complex numbers such that,

$\left|1-\overline{z}_1 z_2\right|^2-\left|z_1-z_{2^2}\right|^2=k\left(1-\left|z_1\right|^2\right)\left(1-\left|z_2\right|^2\right)$ find value of k.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

36. Find number of solutions of $z^2+\left|z\right|^2=0$.

37. If z_1, z_2 are complex numbers such that $\left|\frac{z_1-3z_2}{3-z_1.\overline{z_2}}\right|=1 \; and \; \left|z_2\right|\neq 1$ then find $\left|z_1\right|$.

38. Evaluate $x^4 - 4x^3 + 4x^2 + 8x + 44$, When $x = 3 + 2i$

39. If z_1, z_2 are complex numbers, both satisfy $z+\overline{z}=2\left|z-1\right|$

$\arg\left|z_1-z_2\right|=\frac{\pi}{4}$ then find $\mathrm{Im}\left(z_1+z_2\right)$.

40. Solve $2x^2 - (3 + 7i)x - (3 - 9i) = 0$

41. What is the locus of z if amplitude of $z - 2 - 3i$ is $\frac{\pi}{4}$

42. If $z = x + iy$ and $w=\frac{1-iz}{z-i}$ show that if $|w| = 1$ then z is purely real.

43. Express the complex number in the form $r(\cos\theta+i\sin\theta)$

(i) $1+i\tan\alpha$

(ii) $1-\sin\alpha+i\cos\alpha$

44. If $\left(\frac{1+i}{1+2^2i}\right)\times\left(\frac{1+3^2i}{1+4^2i}\right)\times.....\times\left(\frac{1+(2n-1)^2i}{1+(2n)^2i}\right)=\frac{a+ib}{c+id}$ then show that $\frac{2}{17}\times\frac{82}{257}\times.....\times\frac{1+(2n-1)^4}{1+(2n)^4}=\frac{a^2+b^2}{c^2+d^2}$.

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45. Find the values of x and y for which complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate to each other.

46. The complex number z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a equilateral triangle.

47. If $f(z) = \frac{7 - z}{1 - z^2}$ where $z = 1 + 2i$ then show that $|f(z)| = \frac{|z|}{2}$.

48. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then find the value of $|z_1 + z_2 + z_3|$

ANSWERS

1. $-1 + i$ 2. $-7 - 6i$ 3. $\frac{1}{49}$ $\frac{4\sqrt{3}i}{49}$

4. $\frac{-2}{25} - \frac{11i}{25}$ 5. $\frac{-\pi}{2}$ 6. $\frac{}{3}$

7. $\frac{-2}{7} - \frac{i\sqrt{3}}{7}$

8. (i) 0 (ii) 19 (iii) –8

(iv) $\frac{-7}{\sqrt{2}}i$

9. $x = \frac{5}{13}$ $y = \frac{14}{13}$

10. I 11. 0 (zero) 12. 6 and zero

13. a = –2 14. 1:3

15 locus of z is straight line I,e x – y + 1 = 0

17. $\theta = (2n+1)\frac{\pi}{2}$

20. X = 3, y = –1

22. zero

23. a =0, b = –2

24. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

25. n = 4

27. (i) $6\left(\cos\frac{3\pi}{4} + i\sin\frac{\pi}{4}\right)$

(ii) $1\left[\cos\left(\frac{-5\pi}{12}\right) + i\sin\left(\frac{-5\pi}{12}\right)\right]$

(iii) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(iv) $\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

28. (i) $3\sqrt{2} \ \ and \ \ -2i$ (ii) $4-3i \ \ and \ \ 3+2i$

29. $\pm\left(5-3\sqrt{2}\,i\right)$

31. Centre $\left(\frac{10}{3}, 0\right)$ and radius= $\frac{2}{3}$

32. $z = 0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

34. 1

35. K=1

36. Infinitely many solutions of the form $z = 0 \pm iy; y \in R$

37. $|z_1| = \sqrt{x^2 + y^2}$

38. 5

39. 2

40. $\frac{3}{2} + \frac{1}{2}i \quad and \quad 3i$

41. x – y + 1 = 0 straight line

43. (i) $\sec\alpha\left(\cos\alpha + i\sin\alpha\right)\ ,\ 0 \le \alpha < \frac{\pi}{2}$

$$-\sec\alpha\left[\cos\left(\alpha-\pi\right)+i\sin\left(\alpha-\pi\right)\right],\ \frac{\pi}{2} < \alpha \le \pi$$

(ii) $\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right]\ if\ 0 \le \alpha < \frac{\pi}{2}$

$-\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)+i\sin\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)\right]\ if \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$-\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)+i\sin\left(\frac{\alpha}{2}-\frac{3\pi}{4}\right)\right]\ if \frac{3\pi}{2} < \alpha < 2\pi$

45. When x = 1, y = –4 or x = –1, y = –4

48. 1 (one)