## Exercise 5.4

Q.1 (i) AP = 1, 4, 7, 10 ....

$a = 1$

$d = 4 - 1 = 3$

$a_n = a + (n-1)d$

$a_{10} = 1 + (10-1)\,3 = 1 + 27$

$= \boxed{28}$

(ii) AP = $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$ ...

$a = \sqrt{2}$

$d = 3\sqrt{2} - \sqrt{3} = 2\sqrt{2}$

$a_{18} = a + (n-1)d$

$= a\sqrt{2} + (18-1)2\sqrt{2} = \sqrt{2} + 34\sqrt{2}$

$= \boxed{35\sqrt{2}}$

(iii) AP = 13, 8, 3, −2

$a = 13$

$d = 8 - 13 = -5$

$a_n = a + (n-1)d = 13 + (n-1)(-5)$

$= 13 + (-5n) + 5 = \boxed{-5n + 18}$

(iv) AP = −40, −15, 10, 35, ...

$a = -40$

$d = -15 + 40 = 25$

$a_{10} = a + (n-1)d = -40 + (10-1)25$

$= -40 + 225 = \boxed{185}$

(v) AP = 117, 104, 91, 78, ...

$a = 117$

$d = 104 - 117 = -13$

$a_8 = a + (n-1)d = 117 + (8-1)(-13)$

$= 117 - 91 = \boxed{26}$

(vi) AP = 10, 10.5, 11, 11.5

$a = 10$

$d = 10.5 - 10 = 0.5$

$a_{11} = a + (n-1)d = 10 + (11-10)0.5$

$= 10 + 5 = \boxed{15}$

(vii) AP = $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$

$a = \frac{3}{4}$, $d = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

$a_9 = a + (n-1)d = \frac{3}{4} + (9-1)\frac{1}{2}$

$= \frac{3}{4} + 4 = \frac{3+16}{4} = \boxed{\frac{19}{4}}$

Q.2(i) AP = 3, 8, 13 ... , 248

$a = 3$

$d = 8 - 3 = 5$

$a_n = a + (n-1)d$

$248 = 3 + (n-1)5$

$248 = 3 + 5n - 5$

$248 + 2 = 5n$

$n = \frac{250}{5}\ 50$

$\boxed{n = 50}$

(ii) AP = 84, 80, 76, ... 0

$a = 84$

$d = 80 - 84 = -4$

$a_n = a + (n-1)d$

$0 = 84 + (n-1)(-4)$

$0 = 84 - 4n + 4$

$4n = 88$

$n = \frac{88}{4}\ 22$

$\boxed{n = 22}$

(iii) AP = 4, 9, 14, ..... 254

$a = 4$

$d = 9 - 4 = 5$

$a_n = a + (n-1)d$

$254 = 4 + (n-1)5$

$254 = 4 + 5n - 5$

$254 + 1 = 5n$

$n = \frac{255}{5}\ 51$

$\boxed{n = 51}$

(iv) AP = 21, 42, 63, 84, ..., 420

$a = 21$

$d = 42 - 21 = 21$

$a_n = a + (n-1)d$

$420 = 21 + (n-1)21$

$420 = 21 + 21n - 21$

$n = \frac{420}{21}\ 20$

$\boxed{n = 20}$

(v) AP = 121, 117, 113...

$a = 121$

$d = 117 - 121 = -4$

$a_n < 0$

$a + (n-1)d < 0$

$121 + (n-1)(-4) < 0$

$121 - 4n + 4 < 0$

$125 - 4n < 0$

$125 < 4n$

$4n > 125$

$n > \frac{125}{4}\ 31.25$

$n > 31.25$

$\therefore \boxed{n = 32}$

(vi) AP = $-7, -12, -17, -22, \ldots -82$

$a = -7$

$d = -12 + 7 = -5$

$a_n = a + (n-1)\,d$

$-82 = -7 + (n-1)\,-5$

$-82 = -7 - 5n + 5$

$-82 + 2 = -5n$

$-80 = -5n$

$n = \frac{80}{5}\ 16$

$\boxed{n = 16}$

$a_n = -100$

$a_n = a + (n-1)\,d$

$-100 = -7 + (n-1)\,-5$

$-100 = -7 + (-5n) + 5$

$5n = 100 - 2$

$n = \frac{98}{5}\ 19.6$

$\boxed{n = 19.6}$

As, it is not completely divisible,

$\therefore$ $-100$ is not a term of the AP.

Q.3 (i) A = $7, 10, 13, \ldots, 68$

$a = 7$

$d = 10 - 7 = 3$

$a_n = a + (n-1)\,d$

$68 = 7 + (n-1)\,3$

$68 = 7 + 3n - 3$

$68 - 4 = 3n$

$n = \frac{64}{3}$

$\boxed{n = 21.33}$

No, 68 is not a part of the AP.

(ii) AP = 3, 8, 13, ..., 302

$a = 3$

$d = 8 - 3 = 5$

$a_n = a + (n-1)d$

$302 = 3 + (n-1)5$

$302 = 3 + 5n - 5$

$302 + 2 = 5n$

$n = \frac{304}{5}$

$\boxed{n = 60.8}$

No, 302 is not a part of the AP.

(iii) AP = 11, 8, 5, 2 ..., -150

$a = 11$

$d = 8 - 11 = -3$

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)-3$

$-150 = 11 - 3n + 3$

$3n = 150 + 14$

$n = \frac{164}{3}$

$\boxed{n = 54.66}$

No, -150 is not a part of the AP.

Q.4 (i) $7, 10, 13 \ldots 43$

$a = 7$

$d = 10 - 7 = 3$

$a_n = a + (n-1)d$

$43 = 7 + (n-1)3$

$43 = 7 + 3n - 3$

$43 - 4 = 3n$

$n = \frac{39}{3}$ $13$

$\boxed{n = 13}$

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \ldots, \frac{10}{3}$

$a = -1$

$d = \frac{-5}{6} + 1 = \frac{-5+6}{6} = \frac{1}{6}$

$a_n = a + (n-1)d$

$\frac{10}{3} = -1 + (n-1)\frac{1}{6}$

$\frac{10}{3} = -1 + \frac{1}{6}n - \frac{1}{6}$

$\frac{1}{6}n = \frac{10}{3} + 1 + \frac{1}{6}$

$\frac{1}{6}n = \frac{20+6+1}{6}$

$n = \frac{27}{6} \times \frac{6}{1}$

$\boxed{n = 27}$

(iii) AP = 7, 13, 19, ..., 205

$a = 7$

$d = 13 - 7 = 6$

$a_n = a + (n-1)d$

$205 = 7 + (n-1)\,6$

$205 = 7 + 6n - 6$

$6n = 205 - 1$

$n = \frac{204}{6}\ 34$

$\boxed{n = 34}$

(iv) AP = $18, 15\frac{1}{2}, 13 \ldots, -47$

$a = 18$

$d = 15\frac{1}{2} - 18 = -2\frac{1}{2} = \frac{-5}{2}$

$a_n = a + (n-1)d$

$-47 = 18 + (n-1)\left(\frac{-5}{2}\right)$

$-47 = 18 - \frac{5n}{2} + \frac{5}{2}$

$\frac{5n}{2} = 47 + 18 + \frac{5}{2}$

$\frac{5n}{2} = \frac{94 + 36 + 5}{2}$

$n = \frac{135}{5} \times \frac{27}{1}$

$\boxed{n = 27}$

Q.5 $a = 5$

$d = 3$

$a_n = 80$

$a_n = a + (n-1)d$

$80 = 5 + (n-1)3$

$80 = 5 + 3n - 3$

$80 - 2 = 3n$

$n = \frac{78}{3}$ 26

$\boxed{n = 26}$

$\therefore$ There are 26 terms.

Q.6 $a_6 = a + 5d = 19$ ——— (1)

$a_{17} = a + 16d = 41$ ——— (2)

Subtract (1) from (2)

$a + 16d = 41$

$a + 5d = 19$

(−) (−) (−)

$11d = 22$

$d = \frac{22}{11}$

$\boxed{d = 2}$

Putting value of $d$ in (1)

$a + 5d = 19$

$a = 19 - 5(2)$

$a = 19 - 10$

$\boxed{a = 9}$

~~Q.9~~

$a_{40} = a + (40-1)d$

$= 9 + (39)2 = 9 + 78$

$= \boxed{87}$

Q.10 $a_9 = a + 8d = 0$

$\therefore \quad a = -8d$ —— (1)

$a_{29} = a + 28d$

$= -8d + 28d$

$= 20d$ —— (2)

$a_{19} = a + 18d$

$= -8 + 18d$

$= 10d$ —— (3)

Using (1)

From (2) & (3),

$\boxed{a_{29} = 2\,a_{19}}$

Hence, proved

Q.11 $a_{10} = a + 9d$ —— (1)

$a_{15} = a + 14d$ —— (2)

ATQ,

$10\,(a_{10}) = 15\,(a_{15})$

$10\,(a + 9d) = 15\,(a + 14d)$

$10a + 90d = 15a + 210d$

$15a - 10a = 90d - 210d$

$5a = -120d$

$$a = \frac{-120}{5} d$$

$$\boxed{a = -24d} \quad \text{---} (3)$$

$$a_{25} = a + (n-1)d$$
$$= a + (25-1)d$$
$$= -24d + 24d \quad (\text{From } (3))$$
$$= \boxed{0}$$

Hence, proved

Q.7 $a_{10} = a + 9d = 41$ ---- (1)

$a_{18} = a + 17d = 73$ ---- (2)

Subtract (2) from (1)

$$a + 17d = 73$$
$$a + 9d = 41$$
$$(-) \quad (-) \quad (-)$$
$$8d = 32$$
$$d = \frac{32}{8} = 4$$
$$\boxed{d = 4}$$

Putting value of d in (1)

$$a + 9d = 41$$
$$a + 9(4) = 41$$
$$a = 41 - 36$$
$$\boxed{a = 5}$$

$$a_{26} = a + 25d$$
$$= 5 + 24(4)$$
$$= 5 + 100 = \boxed{105}$$

Q.8

$AP_1 = 9, 7, 5 \ldots$

$a_1 = 9$

$d_1 = 7 - 9 = -2$

$AP_2 = 15, 12, 9 \ldots$

$a_2 = 15$

$d_2 = 12 - 5 = -3$

$a_{n_1} = a + (n-1)d$

$= 9 + (n-1)(-2)$

$= 9 - 2n + 2$

$= 11 - 2n$ ——— (1)

$a_{n_2} = a + (n-1)d$

$= 15 + (n-1)(-3)$

$= 15 - 3n + 3$

$= 18 - 3n$ ——— (2)

ATQ,

$11 - 2n = 18 - 3n$

$3n - 2n = 18 - 11$

$\boxed{n = 7}$

Q.9 (i) 3, 5, 7, 9, ..., 201

$a = 3, \quad l = 201, \quad d = 5 - 3 = 2$

$a_{12}$ from last $= l - (n-1)d$

$= 201 - (12-1)2$

$= 201 - 22$

$= \underline{\boxed{179}}$

(iii) 1, 4, 7, 10, ...., 88

$a = 1, \quad l = 88, \quad d = 4 - 1 = 3$

$a_{12}$ from last $= l - (n-1)d$

$= 88 - (12-1)3$

$= 88 - 33$

$= \boxed{55}$

Q.102 $a_{10} = a + 9d$

$a_{24} = a + 23d$

ATQ,

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$23d - 18d = 2a - a$

$a = 5d$ ——— (1)

$a_{72} = a + 71d = 5d + 71d = 76d$ ——— (2)

~~$= 5d + 71d$~~

$a_{34} = a + 33d = 5d + 33d = 38d$ ——— (3)

~~ATQ,~~

$a + 71d = 5d + 71d$

$a = 76d - 71d$

$a = 5d$

$a_{72} = a +$

ATQ, From (2) & (3)

$2(38d) = 76d$

$[a_{72} = 2(a_{34})]$

Hence, proved

Q.103 $a_{26} = a + 25d = 0$ ——— (1)

$a_{11} = a + 10d = 3$ ——— (2)

Subtract (2) from (1)

$a + 25d = 0$

$(-)\ a + 10d = 3$

$15d = -3$

$$d = \frac{-\cancel{5}1}{\cancel{5}5}$$

$$\boxed{d = \frac{-1}{5}}$$

Putting value of d in (1)

$$a + 25d = 0$$

$$a + \cancel{5}25\left(\frac{-1}{\cancel{5}}\right) = 0$$

$$\boxed{a = 5}$$

$$l = \frac{-1}{5} \text{ (given)}$$

$$a_n = a + (n-1)d$$

$$\frac{-1}{5} = 5 + (n-1)\frac{-1}{5}$$

$$\frac{-1}{\cancel{5}} = \frac{25 - n + 1}{\cancel{5}}$$

$$n = 25 + 1 + 1$$

$$\boxed{n = 27}$$

$$\therefore \boxed{d = \frac{-1}{5}} \quad \& \quad \boxed{n = 27}$$

Q.14 $a_4 = a + 3d$

$a_1 = a$

ATQ,

$$a_4 = 3(a_1)$$

$$a + 3d = 3a$$

$$2a = 3d$$
$$a = \frac{3d}{2}$$

$a_7 = a + 6d$

$a_3 = a + 2d$

ATQ,

$$a_7 = 2(a_3) + 1$$
$$a + 6d = 2(a + 2d) + 1$$
$$a + 6d = 2a + 4d + 1$$
$$6d - 4d = 2a - a + 1$$
$$2d = a + 1 \quad \text{——— (1)}$$

Putting value of a in (1),

$$2d = a + 1$$
$$2d = \frac{3d}{2} + 1$$
$$4d = 3d + 2$$
$$4d - 3d = 2$$
$$\boxed{d = 2}$$

$$a = \frac{3d}{2}$$
$$a = \frac{3(2)}{2}$$
$$\boxed{a = 3}$$

# Ex 5.4

Q.15 $a_6 = a + 5d = 12$ } given ——— (1)
$a_8 = a + 7d = 22$ } ——— (2)

Subtract (1) from (2)

$$\begin{array}{r} a + 7d = 22 \\ a + 5d = 12 \\ (-) \quad (-) \quad (-) \\ \hline 2d = 10 \end{array}$$

$$\boxed{d = 5}$$

Putting value of d in (1)

$$a + 5d = 12$$
$$a + 5(5) = 12$$
$$a = 12 - 25$$
$$\boxed{a = -13}$$

$$a_2 = a + d = -13 + 5$$
$$= \boxed{-8}$$

$$a_n = a + (n-1)d = -13 + (n-1)5$$
$$= -13 + 5n - 5 = \boxed{5n - 18}$$

Q.16 Let the total no. be n

$a = 12$ | $l = 99$
$d = 3$ |

$$a_n = a + (n-1)d$$
$$99 = 12 + (n-1)3$$
$$99 = 12 + 3n - 3$$

$3n = 99 - 9$

$n = \frac{90}{3}30$

$\boxed{n = 30}$

Q.17 $n = 60$

$a = 7$

$l = 125$

$a_{60} = a + 59d$

$125 = 7 + 59d$

$d = \frac{118}{59}2$

$\boxed{d = 2}$

$a_{32} = a + 31d$

$= 7 + 31(2)$

$= 7 + 62 = \boxed{69}$

Q.19 $a = 5$ ——— (1)

$a_{100} = -292$

ATQ,

$a + 99d = -292$

$5 + 99d = -292$ (From (1))

$99d = -292 - 5$

$d = \frac{-297}{99}3$

$\boxed{d = -3}$

$a_{50} = a + 49d = 5 + 49(-3) = 5 - 147 = \boxed{-142}$

Q.20 (i) AP = $-9, -14, -19, -24$

$a = -9$

$d = -14 + 9 = -5$

$a_{20} = a + 19d$

$= -9 + 19(-5)$

$= -9 - 95 = -104$

$a_{30} = a + 29d$

$= -9 + 29(-5)$

$= -9 - 145 = -154$

$a_{30} - a_{20} = -154 + 104$

$= \boxed{-50}$

(ii) AP = $a, a+d, a+2d, a+3d$

$a = a$

$d = a + d - a = d$

$a_{20} = a + 19d$

$a_{30} = a + 29d$

$a_{30} - a_{20} = a + 29d - a + 19d$

$= \boxed{10d}$

Q.21 AP = $a, a+d, a+2d, \ldots$

$a_n = a + (n-1)d$

$a_k = a + (k-1)d$

$a_n - a_k = [a + (n-1)d] - [a + (k-1)d]$

$= a + nd - d - a - kd + d$

$= d(n-k)$

(i) $a_{11} = 15$, $a_{13} = 79$

$a_{13} - a_{11} = (13-11)d$

$79 - 15 = 13d - 11d$

$2d = 74$

$d = \frac{74}{2}\ 37$

$\boxed{d = 37}$

(ii) $a_{10} - a_5 = 200$

$a_{10} - a_5 = (10-5)d$

$200 = 10d - 5d$

$d = \frac{200}{5}\ 40$

$\boxed{d = 40}$

(iii) 20th term is 10 more than the 18th term.

$a_{20} - a_{18} = 10$

$10 = (20-18)d$

$10 = 20d - 18d$

$d = \frac{10}{2}\ 5$

$\boxed{d = 5}$

Q-22(i) $25, 50, 75, 100, \ldots ; x = 1000$

$a = 25$

$d = 50 - 25 = 25$

$a_n = 1000$

$a_n = a + (n-1)d$

$1000 = 25 + (n-1)25$

$1000 = 25 + 25n - 25$

$n = \frac{1000}{25}\ 40$

$\boxed{n = 40}$

(ii) $-1, -3, -5, -7, \ldots ; x = -151$

$a = -1$

$d = -3 + 1 = -2$

$a_n = -151$

$a_n = a + (n-1)d$

$-151 = -1 (n-1) - 2$

$-151 = -1 - 2n + 2$

$-151 - 1 = -2n$

$-152 = -2n$

$n = \frac{152}{2}\ 76$

$\boxed{n = 76}$

(iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots ; x = 550$

$a = 5\frac{1}{2} = \frac{11}{2}$

$d = 11 - \frac{11}{2} = \frac{11}{2}$

$a_n = 550$

$a_n = a + (n-1)d$

$550 = \frac{11}{2} + (n-1)\frac{11}{2}$

$550 = \frac{11}{2} + \frac{11n}{2} - \frac{11}{2}$

$550 \times 2 = 11n$

$n = \frac{1100}{11}\ 100$

$\boxed{n = 100}$

(iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots ; x = \frac{171}{11}$

$a = 1$

$d = \frac{21}{11} - 1 = \frac{21-11}{11} = \frac{10}{11}$

$a_n = \frac{171}{11}$

$a_n = a + (n-1)d$

$\frac{171}{11} = 1 + (n-1)\frac{10}{11}$

$\frac{171}{11} = 1 + \frac{10n}{11} - \frac{10}{11}$

$\frac{171}{11} = \frac{11-10}{11} + \frac{10n}{11}$

$\frac{171}{11} = \frac{1}{11} + \frac{10n}{11}$

$\frac{171}{11} - \frac{1}{11} = \frac{10n}{11}$

$n = \frac{170}{11} \times \frac{11}{10}$

$\boxed{n = 17}$

Q.23 $a_8 = \frac{1}{2} a_2$

ATQ,

$a + 7d = \frac{1}{2}(a + d)$

$2a + 14d = a + d$

$2a - a = d - 14d$

$a + 13d = 0 \longrightarrow (1)$

$$a_{11} = \frac{1}{3} a_4 + 1$$

ATQ,

$$a + 10d = \frac{a+3d}{3} + 1$$

$$3a + 30d = a + 3d + 3$$

$$3a - a + 30d - 3d = 3$$

$$2a + 27d = 3 \quad \text{——— (1)}$$

From (1) & (2)

$$2a + 27d = 3$$

$$2(-13d) + 27d = 3$$

$$-26d + 27d = 3$$

$$\boxed{d = 3}$$

Putting value of d in (1)

$$a + 13d = 0$$

$$a + 13(3) = 0$$

$$\boxed{a = -39}$$

$$a_{15} = a + 14d = -39 + 14(3)$$

$$= -39 + 42 = \boxed{3}$$

Q.25 $a_7 = a + 6d = 32 \quad \text{——— (1)}$

$a_{13} = a + 12d = 62 \quad \text{——— (2)}$

Subtract (1) from (2)

$a + 12d = 62$
$a + 6d = 32$
$(-) \quad (-) \quad (-)$
$6d = 30$
$d = \frac{30}{6} 5$
$\boxed{d = 5}$

Putting value of $d$ in (1)

$a + 6d = 32$
$a + 6(5) = 32$
$a = 32 - 30$
$\boxed{a = 2}$

AP = $\boxed{2, 7, 12, 17, \ldots}$

Q. 26 AP = 3, 10, 17, ...
$a = 3$
$d = 10 - 3 = 7$

Let the term be $n$.
$a_{13} = a + 12d$
$a_n = a + (n-1)d$

ATQ,
$a_n - a_{13} = 84$
$a + (n-1)d - (a + 12d) = 84$
$a + nd - d - a - 12d = 84$
$nd - 13d = 84$
$d(n - 13) = 84$

$$7(n-13) = 84$$

$$n-13 = \frac{84}{7}\ 12$$

$$n = 12 + 13$$

$$\boxed{n = 25}$$

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# Ch - 5

## exercise 5.4

Q.28 $AP_1 = 63, 65, 67, \ldots$

$a = 63, \; d = 65 - 63 = 2$

$a_n = a + (n-1)d$

$= 63 + (n-1)\,2$

$= 63 + 2n - 2$

$= 61 + 2n$

$AP_2 = 3, 10, 17, \ldots$

$a = 3, \; d = 10 - 3 = 7$

$a_n = a + (n-1)d$

$= 3 + (n-1)\,7$

$= 3 + 7n - 7$

$= 7n - 4$

ATQ,

$61 + 2n = 7n - 4$

$61 + 4 = 7n - 2n$

$5n = 65$

$n = \frac{65}{5}\,13$

$\boxed{n = 13}$

Q.31 $AP = 8, 14, 20, 26, \ldots$

$a = 8, \; d = 14 - 8 = 6$

$a_{41} = a + 40d = 8 + 40(6)$

$= 8 + 240 = 248$

Let the term be $n$

$a_n = 8 + (n-1)6$

$= 8 + 6n - 6 = 6n + 2$

ATQ,

$a_n - a_{41} = 72$

$6n + 2 - 248 = 72$

$6n = 72 + 246$

$n = \frac{318}{6}$ $53$

$\boxed{n = 53}$

Q.32 AP = 9, 12, 15, 18, ...

$a = 9$, $d = 12 - 9 = 3$

$a_{36} = a + 35d = 9 + 35(3)$

$= 9 + 105 = 114$

Let the term be $n$.

$a_n = a + (n-1)d = 9 + (n-1)3$

$= 9 + 3n - 3 = 6 + 3n$

ATQ,

$a_n - a_{36} = 39$

$6 + 3n - 114 = 39$

$3n = 39 + 108$

$n = \frac{147}{3}$ $49$

$\boxed{n = 49}$

Q.33 AP = 7, 10, 13, ..., 184

$a = 7$

$d = 10 - 7 = 3$

$l = 184$

8 from last $= a_n + (n-1)d$

$= 184 + (n-1)3$

~~$= 184 + 3n$~~

$= 184 + (8-1)3$

$= 184 + 21$

8 from last $= l - (n-1)d$

$= 184 - (n-1)3$

~~$= 184 - 3n + 1$~~

$= 184 - 21$

$= \boxed{163}$

Q.34 AP = 8, 10, 12, ..., 126

$a = 8$

$d = 10 - 8 = 2$

$l = 126$

$a_{10}$ from last $= l - (n-1)d$

$= 126 - (10-1)2$

$= 126 - 18 = \boxed{108}$

Q.35 $a_4 + a_8 = 24$ ——— (1)

$a_6 + a_{10} = 44$ ——— (2)

ATQ,

$a + 3d + a + 7d = 24$

$$2a + 10d = 24 \quad \text{——— (3)}$$

$$a + 5d + a + 9d = 44$$
$$2a + 14d = 44 \quad \text{——— (4)}$$

Subtract (3) from (4)

$$2a + 14d = 44$$
$$2a + 10d = 24$$
$$(-) \quad (-) \quad (-)$$
$$4d = 20$$
$$\boxed{d = 5}$$

Putting value of d in (3)

$$2a + 10d = 24$$
$$a + 5d = 12$$
$$a + 5(5) = 12$$
$$a = 12 - 25$$
$$\boxed{a = -13}$$

$$AP = \boxed{-13, -8, -3, 2, \ldots}$$

Q. 36 $AP = 3, 15, 27, 39, \ldots$

$a = 3$

$d = 15 - 3 = 12$

$$a_{21} = a + 20d = 3 + 20(12)$$
$$= 3 + 240 = 243$$

Let the term be $n$

$$a_n = a + (n-1)d = 3 + (n-1)12$$
$$= 3 + 12n - 12 = 12n - 9$$

ATQ,

$$a_n - a_{21} = 120$$
$$12n - 9 - 243 = 120$$
$$12n = 120 + 252$$
$$n = \frac{372}{12}\ 31$$
$$\boxed{n = 31}$$

Q.37 $a_{17} = 5 + 2(a_8) - (1)$, given
$a_{11} = 43 \quad (2)$

ATQ,

~~$a_{11} = 5$~~

$$a_{11} = 43$$
$$a + 10d = 43$$
$$a = 43 - 10d \quad (3)$$

$$a_{17} = 5 + 2(a_8)$$
$$a + 16d = 5 + 2(a + 7d)$$
$$a + 16d = 5 + 2a + 14d$$
$$16d - 14d = 5 + 2a - a$$
$$2d = 5 + a$$
$$2d = 5 + (43 - 10d) \quad (\text{from } (3))$$
$$2d = 5 + 43 - 10d$$
$$12d = 48$$
$$d = \frac{48}{12}\ 4$$
$$\boxed{d = 4}$$

Putting value of d in (3)

$$a = 43 - 10d$$
$$a = 43 - 10(4)$$
$$a = 43 - 40$$
$$\boxed{a = 3}$$

$$a_n = a + (n-1)d$$
$$= 3 + 4n - 4$$
$$= \boxed{4n - 1}$$

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# Ch-5
## AP
## Exercise 5.4

Q.38 First 3-digit no. divisible by 9 = 108
Last 3-digit no. = 999
$d = 9$

ATQ,

$$a_n = a + (n-1)d$$
$$999 = 108 + (n-1)9$$
$$999 - 108 = 9n - 9$$
$$891 + 9 = 9n$$
$$n = \frac{900}{9}100$$

$$\boxed{n = 100}$$

Q.39 $a_{19} = a + 18d$
$a_6 = a + 5d$

$a_9 = a + 8d = 19$

ATQ,

$$a + 18d = 3(a + 5d)$$
$$a + 18d = 3a + 15d$$
$$18d - 15d = 3a - a$$
$$3d = 2a$$
$$a = \frac{3d}{2} \quad \text{——— (1)}$$

$a + 8d = 19$ (Putting value of a)

$$\frac{3d}{2} + 8d = 19$$

$$3d + 16d = 38$$

$$19d = 38$$

$$d = \frac{38}{19}$$

$$\boxed{d = 2} \quad \text{——} \quad (2)$$

Putting value of d in (1)

$$a = \frac{3d}{2}$$

$$a = \frac{3(2)}{2}$$

$$\boxed{a = 3}$$

AP = $\boxed{3, 5, 7, 9, 11, \ldots}$

Q.40 $a_9 = a + 8d$

$a_2 = a + d$

$a_5 = a + 4d = 22$

ATQ,

$$a + 8d = 6(a + d)$$

$$a + 8d = 6a + 6d$$

$$8d - 6d = 6a - a$$

$$2d = 5a$$

$$a = \frac{2d}{5} \quad \text{——} \quad (1)$$

$a + 4d = 22$ (Putting value of a)

$\frac{2d}{5} + 4d = 22$

$2d + 20d = 22 \times 5$

$d = \frac{22 \times 5}{22}$

$\boxed{d = 5}$

Putting value of d in ①

$a = \frac{2d}{5}$

$a = \frac{2(5)}{5}$

$\boxed{a = 2}$

AP = $\boxed{2, 7, 12, 17, \ldots}$

Q.4) $a_{24} = a + 23d$

$a_{10} = a + 9d$

ATQ,

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$23d - 18d = 2a - a$

$5d = a$ ——— ①

$a$

$a_{72} = a + 71d = 5d + 71d$

$= 76d$

$a_{15} = a + 14d = 5d + 14d$

$= 19d$

} Putting value of a

$76d = 4(19d)$

$\therefore \boxed{a_{72} = 4(a_{15})}$

Hence, proved

Q.42 $a = 110$

$d = 120 - 110 = 10$

$a_n = 990$

$a_n = a + (n-1)d$

$990 = 110 + (n-1)10$

$990 - 110 = (n-1)10$

$\frac{880}{10} = n - 1$

$n = 88 + 1$

$\boxed{n = 89}$

Q.43 $a_7 = a + 6d = \frac{1}{9}$ ——— (1)

$a_9 = a + 8d = \frac{1}{7}$ ——— (2)

Subtract (1) from (2)

$a + 8d = \frac{1}{7}$

$a + 6d = \frac{1}{9}$

(-) (-) (-)

$2d = \frac{2}{63}$

$$\boxed{d = \frac{1}{63}}$$

Putting value of $d$ in (1)

$$a + 6d = \frac{1}{9}$$

$$a + 6\left(\frac{1}{63}\right) = \frac{1}{9}$$

$$a = \frac{1}{9} - \frac{6}{63}$$

$$a = \frac{7-6}{63}$$

$$\boxed{a = \frac{1}{63}}$$

$$a_{63} = a + 62d = \frac{1}{63} + 62\left(\frac{1}{63}\right)$$

$$= \frac{1}{63} + \frac{62}{63} = \frac{63}{63} = \boxed{1}$$

.44 $a_5 + a_9 = 30$

ATQ,

$$a + 4d + a + 8d = 30$$

$$2a + 12d = 30$$

$$a + 6d = 15$$

$$a = 15 - 6d \quad \text{—— (1)}$$

$$a_{25} = 3(a_8)$$

$$a + 24d = 3(a + 7d)$$

$a + 24d = 3a + 21d$ (Putting value of a)

$(15 - 6d) + 24d = 3(15 - 6d) + 21d$

$15 - 6d + 24d = 45 - 18d + 21d$

$15 + 18d = 45 + 3d$

$18d - 3d = 45 - 15$

$15d = 30$

$d = \frac{30}{15}^{2}$

$\boxed{d = 2}$

Putting value of d in ①,

$a = 15 - 6d$

$a = 15 - 6(2)$

$a = 15 - 12$

$\boxed{a = 3}$

AP = $\boxed{3, 5, 7, 9, \ldots}$

Q. 45 AP = 40, 37, 34, 31, ...

$a = 40$

$d = 37 - 40 = -3$

$a_n = 0$

$a_n = a + (n-1)d$

$0 = 40 + (n-1) - 3$

$0 = 40 - 3n + 3$

$3n = 43$

$\boxed{n = \frac{43}{3}}$

∴ 0 is not a term of the AP.

Q.46 AP = 213, 205, 197, ... 37

$a = 213$

$d = 205 - 213 = -8$

$l = 37$

$a_n = a + (n-1)d$

$37 = 213 + (n-1) - 8$

$37 - 213 = (n-1) - 8$

$\frac{-176}{-8} = 22 = n - 1$

$n = 22 + 1$

$n = 23$

Middle term $= \frac{n+1}{2} = \frac{23+1}{2}$

$= \frac{24}{2} 12 = 12^{th}$ term

$a_{12} = a + 11d = 213 + 11(-8)$

$= 213 - 88 = \boxed{125}$

Q.47 $a_5 = a + 4d = 31$ ——— ①

$a_{25} = 140 + a_5$

ATQ,

$a + 24d = 140 + 31$

$a_{25} = 171$ ——— ②

Subtract ① from ②,

$$a + 24d = 171$$
$$a + 4d = 31$$
$$(-) \quad (-) \quad (-)$$
$$20d = 140$$
$$d = \frac{140}{20} 7$$
$$\boxed{d = 7}$$

Putting value of d in ①

$$a + 4d = 31$$
$$a + 4(7) = 31$$
$$a = 31 - 28$$
$$\boxed{a = 3}$$

AP = $\boxed{3, 10, 17, 24, 31, \ldots}$

Q.48 AP = $-\frac{4}{3}, -1, -\frac{2}{3}, -\frac{1}{3}, \ldots, 4\frac{1}{3}$.

$$a = -\frac{4}{3}$$

$$d = -1 + \frac{4}{3} = \frac{-3+4}{3} = \frac{1}{3}$$

$$l = 4\frac{1}{3} = \frac{13}{3}$$

$$l = a + (n-1)d$$
$$\frac{13}{3} = -\frac{4}{3} + (n-1)\frac{1}{3}$$
$$\frac{13+4}{3} = (n-1)\frac{1}{3}$$

$$\frac{17}{\cancel{3}} \times \frac{\cancel{3}}{1} = n-1$$

$$n = 17+1$$

$$n = 18$$

Middle terms $= \left(\frac{n}{2}\right), \left(\frac{n}{2}+1\right)$

$= \left(\frac{18}{2}\right); \left(\frac{18}{2}+1\right)$

$= 9^{th}, 10^{th}$ term

$$a_9 = a+8d = -\frac{4}{3} + 8\left(\frac{1}{3}\right)$$

$$= \frac{-4+8}{3} = \frac{4}{3}$$

$$a_{10} = a+9d = -\frac{4}{3} + 9\left(\frac{1}{3}\right)$$

$$= \frac{-4+9}{3} = \frac{5}{3}$$

$$a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{\cancel{9}3}{\cancel{3}} = \boxed{3}$$

Q.49 $a_{m+1} = a+(n-1)d$

$= a+(m+\cancel{1}-\cancel{1})d$

$= a+md$

$a_{n+1} = a+(n-1)d$

$= a+(n+\cancel{1}-\cancel{1})d$

$= a+nd$

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ATQ,

$$a + md = 2(a + nd)$$
$$a + md = 2a + 2nd$$
$$2a - a = md - 2nd$$
$$a = d(m - 2n) \quad \text{—— (1)}$$

$$a_{3m+1} = a + (n-1)d$$
$$= a + (3m + 1 - 1)d$$
$$= a + 3md$$
$$= d(m - 2n) + 3md \quad \text{(Putting (1))}$$
$$= (m - 2n + 3m)d$$
$$= (4m - 2n)d$$
$$= (2m - n)2d \quad \text{—— (2)}$$

$$a_{m+n+1} = a + (n-1)d$$
$$= a + (m + n + 1 - 1)d$$
$$= a + (m+n)d$$
$$= (m - 2n)d + (m + n)d \quad \text{(Putting (1))}$$
$$= (m - 2n + m + n)d$$
$$= (2m - n)d \quad \text{—— (3)}$$

From (2) & (3)

$$\boxed{a_{3m+1} = 2(a_{m+n+1})}$$

Hence, proved

Q. 50 No. of terms = n
First term = a
$n^{th}$ term = l

$$a_n = a + (n-1)d$$

$a_m$ from end $= l - (m-1)d$

Sum $= a + (m-1)d + l - (m-1)d$

$= \boxed{a+l}$

Hence, proved

Q.51 $a = 11$

$d = 4$

$l = 299$

$a_n = l = a + (n-1)d$

$299 = 11 + (n-1)4$

$299 - 11 = (n-1)4$

$\frac{288}{4} = 72 = n - 1$

$n = 72 + 1$

$\boxed{n = 73}$

Q.52 AP $= -2, -4, -6, \ldots, -100$

$a = -2$

$d = -4 + 2 = -2$

$l = -100$

$a_{12}$ from end $= l - (n-1)d$

$= -100 - (n-1)-2$

$= -100 - (12-1)-2$

$= -100 + 22 = \boxed{-78}$

Q.53 Yes, we can find $a_{30} - a_{20}$ without finding $a_{30}$ & $a_{20}$

AP $= -3, -7, -11, \ldots$

$a = -3$

$d = -7 + 3 = -4$

$a_{30} - a_{20} = a + 29d - (a + 19d)$

$= a + 29d - a - 19d$

$= 10d$

$= 10(-4)$

$= \boxed{-40}$

Q. 54 $a_1 = 2$ , $a_2 = 7$

$AP_1 = 2, 2+d, 2+2d, \ldots$

$AP_2 = 7, 7+d, 7+2d, \ldots$

$a_{10}$ of $AP_1 = a + 9d$

$= 2 + 9d$

$a_{10}$ of $AP_2 = a + 9d$

$= 7 + 9d$

Their difference $= 7 + 9d - (2 + 9d)$

$= 7 + 9d - 2 - 9d$

$= 5$

$a_n$ of $AP_1 = a + (n-1)d$

$= 2 + (n-1)d$

$a_n$ of $AP_2 = a + (n-1)d$

$= 7 + (n-1)d$

Their difference $= 7 + (n-1)d - [2 + (n-1)d]$

$= 7 + (n-1)d - 2 - (n-1)d$

$= \boxed{5}$ —— ①

$a_2 - a_1 = 7 - 2 = \boxed{5}$ —— ②

∴ From ① & ②,

The difference between any 2 corresponding terms of such APs is the same as the difference between their first terms.