

AglaSem Schools

SAMPLE QUESTION PAPER-9

Section-A

1. $A = \{a, b, c\}$
i.e., $n(A) = 3$
 $\therefore$ Number of subsets $= 2^3 = 8$. 1
2. Equation of line is $3x - 4y + 9 = 0$
 $\therefore$ Equation of $\perp$ line is $4x + 3y + \lambda = 0$ 1
3. ${}^{16}C_r = {}^{16}C_{r+2}$ (given)
 $\Rightarrow r + r + 2 = 16 \Rightarrow r = 7$
 $\therefore {}^7C_4 = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35$. 1
4. Equation of circle is $x^2 + y^2 - 4x + 6y - 12 = 0$
 $\therefore C(2, -3)$
 $R = \sqrt{2^2 + (-3)^2 + 12} = 5$ 1
5. $\frac{d}{dx} \log_a x = \frac{1}{x} \cdot \log_a e$ 1
6. Parabola is $2x^2 + y = 0 \Rightarrow x^2 = -\frac{1}{2}y$
 $\therefore$ Focus $\left(0, -\frac{1}{8}\right)$, Directrix $= 8y - 1 = 0$ $\frac{1}{2} + \frac{1}{2}$

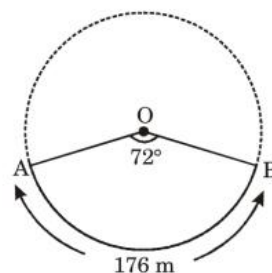
Section-B

7. $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3\}$, $C = \{5\}$.
 $\therefore (B \cap C) = \{\}$ 1
 $\Rightarrow A - (B \cap C) = \{1, 2, 3, 4, 5\} - \{\}$
 $= \{1, 2, 3, 4, 5\} = \text{L.H.S.}$ 1
 and $(A - B) = \{1, 2, 3, 4, 5\} - \{2, 3\}$
 $= \{1, 4, 5\}$
 and $(A - C) = \{1, 2, 3, 4, 5\} - \{5\}$
 $= \{1, 2, 3, 4\}$ 1
 $\Rightarrow (A - B) \cup (A - C) = \{1, 2, 3, 4, 5\} = \text{R.H.S.}$ 1
 $\therefore \text{L.H.S.} = \text{R.H.S.}$ Hence verified.

8. A horse is tied to a point at O , $OA = OB$, (Rope) horse moves along a circular path keeping rope tight. 2

$$\therefore \text{Length of rope} = \frac{\text{Arc } AB}{\angle AOB} = \frac{176 \times 180^\circ}{72 \times \pi}$$

$$\text{Length of rope} = \frac{176 \times 180 \times 7}{72 \times 22} = 180 \text{ m.}$$



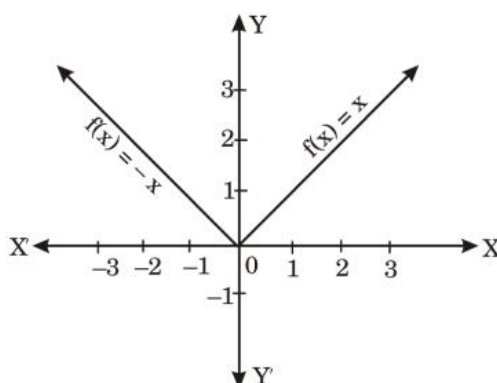
9. **Modulus function :** A function $f: R \rightarrow R$ is defined as

$$f(x) = |x| \quad \forall x \in R$$

$$\therefore f(x) = |x| = \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases} \quad 1$$

i.e., Absolute/modulus value of any function is always positive. 1

Graph :



2

OR

Given,

$$f(x) = ax + b$$

$$f(-1) = -5 \quad 1$$

$$\Rightarrow -a + b = -5 \quad \dots(i) \quad 1$$

and

$$f(3) = 3$$

$$\Rightarrow 3a + b = 3 \quad \dots(ii) \quad 1$$

On solving (i) and (ii), we get

$$\boxed{a = 2, b = -3} \quad 1$$

10. $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, (given) 1

$$\Rightarrow \sin \theta = \sqrt{2} \cos \theta - \cos \theta \quad 1$$

$$\Rightarrow \sin \theta = (\sqrt{2} - 1) \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\sqrt{2} - 1} = \cos \theta \quad 1$$

On rationalising the denominator

$$\frac{\sin \theta (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \cos \theta \quad 1$$

$$\Rightarrow \sqrt{2} \sin \theta + \sin \theta = \cos \theta$$

$$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta$$

Hence proved. 1

OR

If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$

$$\therefore \tan (\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\Rightarrow \tan (\alpha + \beta) = \frac{\left(\frac{m}{m+1} + \frac{1}{2m+1} \right)}{\left(1 - \frac{m}{m+1} \times \frac{1}{2m+1} \right)} \quad 1$$

$$= \frac{\left(\frac{m(2m+1) + (m+1)}{(m+1)(2m+1)} \right)}{\left(\frac{(m+1)(2m+1) - m}{(m+1)(2m+1)} \right)} \quad 1$$

$$= \left(\frac{2m^2 + m + m + 1}{2m^2 + 2m + m + 1 - m} \right) \quad 1$$

$$= \frac{2m^2 + m + 1}{2m^2 + 2m + 1}$$

$$\tan(\alpha + \beta) = 1 = \tan \frac{\pi}{4} \quad 1$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{4}$$

11. $P(n) : 1 + 4 + 7 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$

Put $n = 1$

$$P(1) : 1 = \frac{1}{2}(3 + 1)$$

$$\Rightarrow 1 = 1$$

So $P(1)$ is true.

Let $P(k)$ is true

$\frac{1}{2}$

$$i.e., \quad P(k) : 1 + 4 + 7 + \dots + (3k - 2) = \frac{1}{2}k(3k - 1) \quad \dots(i)$$

Now to prove $P(k + 1)$ is true.

$$i.e., \quad 1 + 4 + 7 + \dots + (3k - 2) + [3k + 1 - 2] = \frac{1}{2}(k + 1)[3(k + 1) - 1] \quad \frac{1}{2}$$

Taking L.H.S.

$$1 + 4 + 7 + \dots + (3k - 2) + (3k + 3 - 2) \quad 1$$

Using equation (i), we get

$$\frac{1}{2}k(3k - 1) + (3k + 1) = \frac{1}{2}[3k^2 - k + 6k + 2]$$

$$= \frac{1}{2}[3k^2 + 5k + 2] \quad 1$$

$$= \frac{1}{2}(3k^2 + 3k + 2k + 2)$$

$$= \frac{1}{2}[3k(k + 1) + 2(k + 1)] \quad \frac{1}{2}$$

$$= \frac{1}{2}[(3k + 2)(k + 1)]$$

$$= \frac{1}{2}(k + 1)[3(k + 1) - 1] \quad \frac{1}{2}$$

$$= \text{R.H.S.}$$

Hence proved.

12.

Class	f	x	$u_i = \frac{x_i - 45.5}{6}$	u_i^2	$f_i u_i^2$	$f_i u_i$
30.5 – 36.5	4	33.5	-2	4	16	-8
36.5 – 42.5	10	39.5	-1	1	10	-10
42.5 – 48.5	14	45.5(a)	0	0	0	0
48.5 – 54.5	27	51.5	1	1	27	27
54.5 – 60.5	45	57.5	2	4	180	90
Total	100				233	99

Here, $a = 45.5$, $h = 6$, $N = 100$

2

$$\text{Variance} = h^2 \left[\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N} \right)^2 \right]$$

$$= 36 \left[\frac{233}{100} - \left(\frac{99}{100} \right)^2 \right]$$

1

$$= 36 [2.33 - (.99)^2]$$

$$= 36 [2.33 - 0.9801]$$

$$= 36 \times 1.35$$

$$\text{Variance} = 48.6.$$

1

13. An urn contains 1 red and 1 black ball, a ball is drawn, if gets red toss a coin. If gets black throw a dice, then

$$S = \{RH, RT, B_1, B_2, B_3, B_4, B_5, B_6\}$$

$$A = \text{Event of getting an even number i.e., } (B_2, B_4, B_6)$$

1

$$\therefore P(A) = \frac{\frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times 6}$$

1

$$= \frac{\left(\frac{3}{12} \right)}{\left(\frac{1}{2} + \frac{1}{2} \right)}$$

$$P(A) = \left(\frac{1}{4} \right) = \frac{1}{4}.$$

1

14. According to definition of ellipse

$$PF_1 + PF_2 = 2a \text{ (constant)}$$

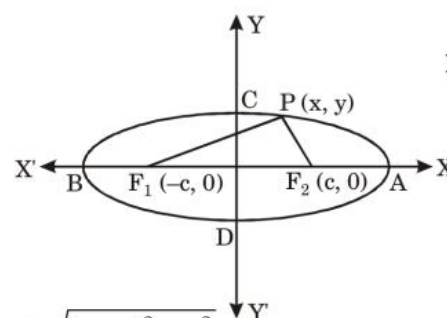
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$$\Rightarrow \sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$

$$\Rightarrow \sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

Squaring both sides, we get

$$x^2 + c^2 + 2cx + y^2 = 4a^2 + x^2 + c^2 - 2cx + y^2 - 4a\sqrt{(x-c)^2 + y^2}$$



$$\Rightarrow 4cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow (cx - a^2) = -a\sqrt{(x-c)^2 + y^2} \quad 1$$

Again squaring both sides, we get

$$c^2x^2 + a^4 - 2a^2cx = a^2(x^2 + c^2 - 2cx + y^2)$$

$$\Rightarrow c^2x^2 + a^4 - 2a^2cx = a^2x^2 + a^2c^2 - 2a^2cx + a^2y^2$$

$$\Rightarrow (a^2 - c^2)x^2 + a^2y^2 = a^4 - a^2c^2 \quad 1$$

$$\Rightarrow (a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\because a^2 - c^2 = b^2) \quad 1$$

It is standard equation of ellipse.

15. We have 600 litres of a 12% acidic solution.

$$\text{So total acid} = \frac{600 \times 12}{100} \text{ litre.}$$

$$\text{Let } x \text{ litres solution of 30\% is to be added. Total mixture} = (600 + x) \text{ litres.} \quad 1$$

So, according to question,

$$\frac{15}{100}(600 + x) < \frac{600 + 12}{100} + \frac{x \times 30}{100} < \frac{18(600 + x)}{100} \quad 1$$

$$\text{Take } \frac{15}{100}(600 + x) < \frac{7200 + 30x}{100}$$

$$\Rightarrow 9000 + 15x < 7200 + 30x$$

$$\Rightarrow 15x > 1800$$

$$\Rightarrow x > 120$$

$$\text{Now take } \frac{600 \times 12}{100} + \frac{x \times 30}{100} < \frac{18}{100}(600 + x)$$

$$\Rightarrow 7200 + 30x < 10800 + 18x \quad 1$$

$$12x < 3600$$

$$x < 300$$

$$\text{i.e., } 120 < n < 300 \quad 1$$

So we have to mix 30% acid solution between 120 litres to 300 litres.

OR

$\therefore$ Linear inequations are

$$2x + y > 4 \quad \dots(i)$$

$$x + y < 3 \quad \dots(ii)$$

$$\text{and } 2x - 3y < 6 \quad \dots(iii) \quad \frac{1}{2}$$

$$\text{Take } 2x + y = 4$$

$$\text{At } x = 0, \quad y = 4$$

$$\text{and at } y = 0, \quad x = 2$$

$$\text{At } (0, 0), \quad 0 + 0 > 4 \text{ (False)} \quad \frac{1}{2}$$

i.e., origin is not included.

Now take

$$x + y = 3$$

At $x = 0$,

$$y = 3 \text{ and}$$

At $y = 0$,

$$x = 3$$

At $(0, 0)$,

$$0 + 0 < 3 \text{ (True)}$$

So origin is included.

Now take

$$2x - 3y = 6$$

At $x = 0$,

$$y = -2$$

and at $y = 0$,

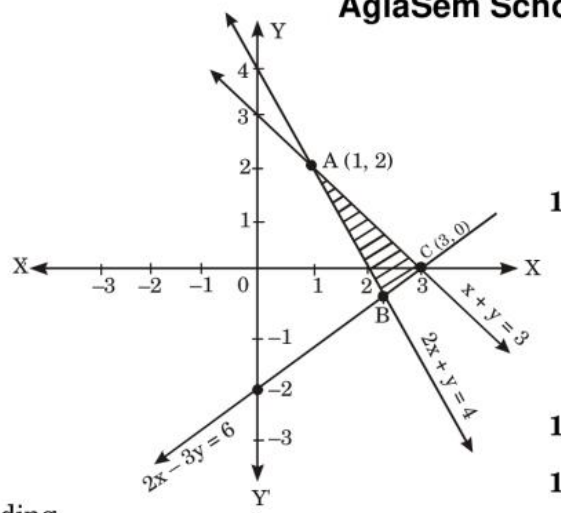
$$x = 3$$

At $(0, 0)$,

$$0 - 0 < 6 \text{ (True)}$$

So origin is included.

The shaded region ABC is the solution region excluding the lines. All the points within this region will satisfy these inequations.



16. Given $a + b + c \neq 0$

and $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P. 1

$\Rightarrow \frac{b+c}{a} + 1, \frac{c+a}{b} + 1, \frac{a+b}{c} + 1$ are also in A.P. 1

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P. 1

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. 1

- 17.

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x - 1$$

$$f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \dots + \frac{2x}{2} + 1$$

$$f'(x) = x^{99} + x^{98} + \dots + x + 1$$

$$\therefore f'(1) = 1 + 1 + 1 + \dots \text{ 100 times} = 100$$

$$\text{and } f'(1) = 0 + 0 + \dots + 0 - 1 = -1$$

$$\Rightarrow f'(1) = 100 \times f'(0)$$

Hence proved. 1

18. There is a girder (large metal beam) in the shape of parabola. 1

So equation of girder is

$$x^2 = -4a(y - 10)$$

$B(50, 0)$ lies on (i)(i)

$\therefore$

$$2500 = -4a(0 - 10)$$

$\Rightarrow$

$$4a = 250$$

$\Rightarrow$

$$x^2 = -250(y - 10)$$

Also $Q(20, y)$ lies on it

$\therefore$

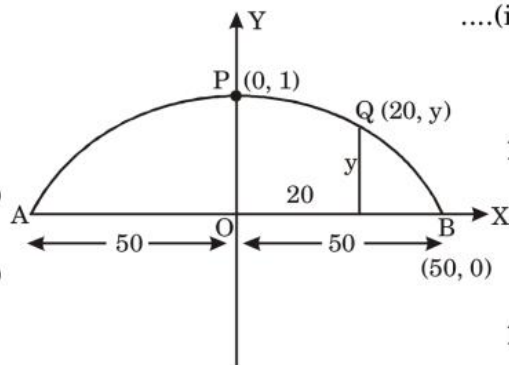
$$400 = -250(y - 10)$$

$\Rightarrow$

$$y - 10 = -\frac{400}{250}$$

$$y = 10 - 1.6 = 8.4 \text{ metre}$$

$\therefore$ Height of girder 20 m from centre is 8.4 metre. 1



Equation of ellipse

$$16x^2 + y^2 = 16$$

 $\Rightarrow$

$$\frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$a^2 = 1 \Rightarrow a = \pm 1 \text{ and } b^2 = 16 \Rightarrow b = \pm 4$$

1

 $\therefore (b > a)$ $\Rightarrow$ Y-axis is axis of ellipse. $\therefore$

$$c^2 = b^2 - a^2 = 16 - 1 = 15$$

1.

$$c = \pm\sqrt{15}$$

 $\therefore$

$$F(0, \pm\sqrt{15}), V(0, \pm 4)$$

Length of major axis = $2b = 2 \times 4 = 8$ Length of minor axis = $2a = 2 \times 1 = 2$

1

$$e = \frac{c}{b} = \frac{\sqrt{15}}{4}$$

and

$$\text{latus-racturm} = \frac{2a^2}{b} = \frac{2 \times 1 \times 1}{4} = \frac{1}{2}$$

1

19. There are 5 men and 4 women; seated in a row such that woman at even place.

So we have to place all men like

$$M_1 \square M_2 \square M_3 \square M_4 \square M_5$$

2

at $\square$ (boxes) 4 women can sit in $4!$ ways and men in $5!$ ways.

1

 $\therefore$ Total ways of arrangements = $4! \times 5!$

$$= 24 \times 120$$

$$= 2880.$$

1

Section-C

20. Quadratic equation is

$$x^2 + (2p + 1)x + p^2 + 2.$$

Let α and β be their roots such that $\beta = 2\alpha$

1

 $\therefore$

$$\alpha + 2\alpha = -(2p + 1)$$

 $\Rightarrow$

$$3\alpha = -(2p + 1)$$

....(i) 1

and

Product of roots = c/a

$$\alpha \cdot 2\alpha = (p^2 + 2)$$

 $\Rightarrow$

$$2\alpha^2 = p^2 + 2$$

....(ii) 1

On solving equations (i) and (ii),

$$2 \left[\frac{-(2p+1)}{3} \right]^2 = p^2 + 2$$

1

 $\Rightarrow$

$$2(4p^2 + 1 + 4p) = 9p^2 + 18$$

 $\Rightarrow$

$$8p^2 + 2 + 8p = 9p^2 + 18$$

1

 $\Rightarrow$

$$p^2 - 8p + 16 = 0$$

 $\Rightarrow$

$$(p - 4)^2 = 0$$

 $\Rightarrow$

$$\boxed{p = 4}$$

1

OR

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$$z = x + iy \text{ and } w = \frac{1 - iz}{z - i}$$

As

$$|w| = 1$$

$\Rightarrow$

$$\left| \frac{1 - iz}{z - i} \right| = 1$$

1

$\Rightarrow$

$$\left| \frac{1 - i(x + iy)}{(x + iy) - i} \right| = 1$$

1

$\Rightarrow$

$$\left| \frac{(1 + y) - ix}{x + i(y - 1)} \right| = 1$$

$\Rightarrow$

$$\sqrt{(1 + y)^2 + x^2} = \sqrt{x^2 + (y - 1)^2}$$

2

$\Rightarrow$

$$1 + y^2 + 2y + x^2 = x^2 + y^2 + 1 - 2y$$

$\Rightarrow$

$$4y = 0$$

1

$\Rightarrow$

$$y = 0$$

i.e.,

$$z = x + i \cdot 0.$$

$\Rightarrow z$ is purely real.

1

21.

$$\text{L.H.S.} = \cos^2 x + \cos^2 \left(x + \frac{\pi}{3} \right) + \cos^2 \left(x + \frac{\pi}{3} \right)$$

$$= \left(\frac{1 + \cos 2x}{2} \right) + \left(\frac{1 + \cos \left(2x + \frac{2\pi}{3} \right)}{2} \right) + \left(\frac{1 + \cos \left(2x - \frac{2\pi}{3} \right)}{2} \right)$$

1

$$= \frac{3}{2} + \frac{1}{2} \left[\cos 2x + \cos \left(2x + \frac{2\pi}{3} \right) + \cos \left(2x - \frac{2\pi}{3} \right) \right]$$

$$= \frac{3}{2} + \frac{1}{2} \left[\cos 2x + 2 \cos 2x \cos \left(\frac{2\pi}{3} \right) \right]$$

1

$$= \frac{3}{2} + \frac{1}{2} \left[\cos 2x + 2 \cos 2x \cos \left(\pi - \frac{\pi}{3} \right) \right]$$

1

$$= \frac{3}{2} + \frac{1}{2} \left[\cos 2x + 2 \cos 2x \left(-\cos \frac{\pi}{3} \right) \right]$$

1

$$= \frac{3}{2} + \frac{1}{2} \left[\cos 2x - 2 \times \frac{1}{2} \cos 2x \right]$$

1

$$= \frac{3}{2} + 0 = \frac{3}{2} = \text{R.H.S.}$$

1

Hence proved.

OR

In $\triangle ABC$,

$$\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = k, \text{ (say)}$$

$\Rightarrow$

$$\left. \begin{aligned} b+c &= 12k \\ c+a &= 13k \\ a+b &= 15k \end{aligned} \right\}$$

...(i) 1

and

$$\begin{aligned}
 &\Rightarrow 2(a + b + c) = 40k \\
 &\Rightarrow a + b + c = 20k \quad \dots(ii) \\
 &\therefore \text{ Using equations (i) and (ii),} \\
 &\quad a = 8k, b = 7k \text{ and } c = 5k \\
 &\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad 1 \\
 &\quad = \frac{49k^2 + 25k^2 - 64k^2}{2 \times 7k \times 5k} \\
 &\quad = \frac{10k^2}{70k^2} = \frac{1}{7} \\
 &\cos B = \frac{a^2 + c^2 - b^2}{2ac} \\
 &\quad = \frac{64k^2 + 25k^2 - 49k^2}{2 \times 8k \times 5k} \quad 1 \\
 &\quad = \frac{40k^2}{80k^2} = \frac{1}{2} \\
 \text{and} \quad \cos C = \frac{64k^2 + 49k^2 - 25k^2}{2 \times 8k \times 7k} \\
 &\quad = \frac{8k^2}{112k^2} = \frac{44}{56} \quad 1 \\
 &\therefore \cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{44}{56} \\
 &\quad = \frac{16 : 56 : 88}{112} \quad 1 \\
 &\Rightarrow \cos A : \cos B : \cos C = 16 : 56 : 88 \\
 &\Rightarrow \cos A : \cos B : \cos C = 2 : 7 : 11 \\
 &\Rightarrow \frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11} \quad 1
 \end{aligned}$$

Hence proved.

22. Total students in a hostel = 400

A = No. of students who takes tea

B = No. of students who takes milk

$\therefore$ According to question $n(A) = 250, n(B) = 200$

$$n(\bar{A} \cap \bar{B}) = 50$$

$$\Rightarrow n(\overline{A \cup B}) = 50 \Rightarrow n(A \cup B) = 400 - 50 = 350$$

$$\text{We know that } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$350 = 250 + 200 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 450 - 350$$

$$\Rightarrow n(A \cap B) = 100$$

$\therefore$ Number of students who takes both drink = 100.

It seems that mostly students like tea, it shows the traditional drink items are changing in our country.

23. General equation of circle having (h, k) centre and radius r is

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$$(x - h)^2 + (y - k)^2 = r^2 \quad \dots(i)$$

Passing $(2, -2)$ and $(3, 4)$

$$\Rightarrow (2 - h)^2 + (-2 - k)^2 = r^2 \quad \dots(ii) \quad 1$$

$$\text{and} \quad (3 - h)^2 + (4 - k)^2 = r^2 \quad \dots(iii) \quad 1$$

From (ii) and (iii),

$$(2 - h)^2 + (-2 - k)^2 = (3 - h)^2 + (4 - k)^2$$

$$\Rightarrow 4 + h^2 - 4h + 4 + k^2 + 4k = 9 + h^2 - 6h + 16 + k^2 - 8k$$

$$\Rightarrow 2h + 2k = 17 \quad \dots(iv) \quad 1$$

Also (h, k) lies on $x + y = 1$

$$\Rightarrow h + k = 1$$

$$\Rightarrow 2h + 2k = 2 \quad \dots(v) \quad 1$$

On solving equations (iv) and (v), we get

$$k = \frac{3}{2} \text{ and } y = -\frac{1}{2}$$

From equation (ii), we get

$$\left(2 + \frac{1}{2}\right)^2 + \left(-2 - \frac{3}{2}\right)^2 = r^2 \quad 1$$

$$\Rightarrow r^2 = \frac{25}{4} + \frac{49}{4} = \frac{74}{4}$$

$\therefore$ From (i), equation of circle is

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{74}{4}$$

$$\Rightarrow x^2 + y^2 + x - 3y + \frac{1}{4} + \frac{9}{4} = \frac{74}{4}$$

$$\Rightarrow \boxed{x^2 + y^2 + x - 3y - 16 = 0} \quad 1$$

It is required equation of circle.

OR

The mid points D, E and F of $\triangle ABC$ are respectively $(1, 2, -3), (3, 0, 1)$ and $(-1, 1, -4)$.

Using mid point formula,

$$\frac{x_1 + x_2}{2} = -1 \Rightarrow x_1 + x_2 = -2$$

$$\frac{x_2 + x_3}{2} = 1 \Rightarrow x_2 + x_3 = 2$$

and

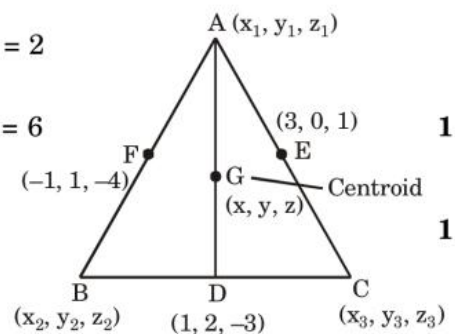
$$\frac{x_3 + x_1}{2} = 3 \Rightarrow x_3 + x_1 = 6$$

On adding all these, we get

$$2(x_1 + x_2 + x_3) = -2 + 2 + 6 = 6$$

$$\Rightarrow x_1 + x_2 + x_3 = \frac{6}{2} = 3$$

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} = 1 \quad \dots(i) \quad 1$$



i.e., $x = \frac{x_1 + x_2 + x_3}{3} = 1 = (x \text{ coordinate of centroid})$

Now, $\frac{y_1 + y_2}{2} = 1 \Rightarrow y_1 + y_2 = 2$

$$\frac{y_2 + y_3}{2} = 2 \Rightarrow y_2 + y_3 = 4$$

and $\frac{y_3 + y_1}{2} = 0 \Rightarrow y_3 + y_1 = 0$

$$\Rightarrow 2(y_1 + y_2 + y_3) = 6$$

$$\Rightarrow \frac{y_1 + y_2 + y_3}{3} = 1 \quad \dots(ii) \quad 1$$

Similarly, $\frac{z_1 + z_2}{2} = -4 \Rightarrow z_1 + z_2 = -8$

$$\frac{z_2 + z_3}{2} = -3 \Rightarrow z_2 + z_3 = -6$$

and $\frac{z_3 + z_1}{2} = 1 \Rightarrow z_3 + z_1 = 2 \quad 1$

$$\Rightarrow 2(z_1 + z_2 + z_3) = -12$$

$$\Rightarrow \frac{z_1 + z_2 + z_3}{3} = -2 \quad \dots(iii) \quad 1$$

$\therefore$ From equations (i), (ii) and (iii) we get co-ordinates of centroid.

i.e., $G(1, 1, -2)$. 1

24. Binomial expansion is $(x - 7y)^{12}$

$\therefore$ Its general term

i.e., $T_{r+1} = {}^{12}C_r \cdot (x)^{12-r} \cdot (-7y)^r$ 1

$$\begin{aligned} T_{r+1} &= (-1)^r \cdot {}^{12}C_r \cdot x^{12-r} \cdot 7^r \cdot y^r \\ &= (-1)^r \cdot {}^{12}C_r \cdot x^{12-r} \cdot y^r \cdot 7^r \end{aligned} \quad 1$$

For coeff. of x^5y^7 , we need to put $r = 7$ 2

$$\begin{aligned} \therefore \text{Coefficients of } x^5y^7 &= (-1)^7 \cdot {}^{12}C_7 \cdot (7)^7 \\ &= -\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \\ &= -65,22,46,056. \end{aligned} \quad 2$$

OR

We have $(x + a)^n$

$$T_{r+1} = {}^nC_r (x)^{n-r} a^r$$

3rd, 4th and 5th terms respectively are

$${}^nC_2 (x)^{n-2} a^2, {}^nC_3 (x)^{n-3} a^3 \text{ and } {}^nC_4 (x)^{n-4} a^4$$

$\therefore$ According to question,

$$\begin{aligned} {}^nC_2 (x)^{n-2} a^2 : {}^nC_3 (x)^{n-3} a^3 : {}^nC_4 (x)^{n-4} a^4 \\ = 84 : 280 : 560 \end{aligned}$$

Now take

$$\frac{{}^nC_3 x^{n-3} a^3}{{}^nC_2 (x)^{n-2} a^2} = \frac{280}{84} \quad 1$$

$$\frac{n(n-1)(n-2) \times 2}{6 \times n(n-1)} \cdot \frac{a}{x} = \frac{70}{21}$$

$$\Rightarrow \frac{n-2}{3} \times \frac{a}{x} = \frac{70}{21}$$

$$\Rightarrow (n-2)a = 10x \quad \dots(i) \quad 1$$

Also take

$$\frac{{}^nC_4 x^{n-4} a^4}{{}^nC_3 x^{n-3} a^3} = \frac{560}{280}$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3) \times 6}{24 \times n(n-1)(n-2)} \times \frac{a}{x} = 2$$

$$\Rightarrow \frac{n-3}{4} \times \frac{a}{x} = 2$$

$$\Rightarrow \frac{a}{x} = \frac{8}{n-3} \quad \dots(ii) \quad 1$$

On solving equations (i) and (ii),

$$\frac{10}{n-2} = \frac{8}{n-3}$$

$$\Rightarrow 10n - 30 = 8n - 16$$

$$\Rightarrow 2n = 14$$

$$\Rightarrow n = 7$$

From (i) we get

$$5a = 10x \Rightarrow a = 2x \quad \dots(iii) \quad 1$$

Now using

$${}^nC_2 (x)^{n-2} a^2 = 84$$

$$\Rightarrow {}^7C_2 (x)^{7-2} (2n)^2 = 84$$

$$\frac{7 \times 6}{2 \times 1} x^7 \times 4 = 84$$

$$\Rightarrow x^7 = \frac{84}{7 \times 6 \times 2}$$

$$\Rightarrow x^7 = 1 \Rightarrow (1)^7 \quad 1$$

$$\Rightarrow x = 1$$

$$\therefore a = 2x \Rightarrow a = 2$$

Hence $x = 1$, $n = 7$ and $a = 2$. 1

25. There are 12 questions in test it consists two parts say :

Part I consists 5 questions and part II consists 7 questions.

A student must be attend 8 questions, 3 from each.

So total ways of selecting 8, having 3 questions from each part 2

$$= \{3 \text{ in I and } 5 \text{ in II}\} \text{ or } \{4 \text{ in I and } 4 \text{ in II}\} \text{ or } \{5 \text{ in I and } 3 \text{ in II}\}$$

$$= {}^5C_3 \times {}^7C_5 + {}^7C_4 \times {}^5C_4 + {}^5C_5 \times {}^7C_3 \quad 2$$

$$\begin{aligned}
 &= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{7 \times 6}{2 \times 1} + \frac{7 \times 6 \times 5}{3 \times 2 \times 1} + \frac{5 \times 1}{1} + \frac{5}{5} \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \\
 &= 210 + 35 \times 5 + 35 \\
 &= 210 + 175 + 35 \\
 &= 420
 \end{aligned}$$

1

Teacher can judge the consistency of students through this test, because it is already mentioned that students are unique. 1

26. We have two A.P.'s say

A.P. I. First term = a_1
 and common diff. = d_1
 and A.P. II. First term = a_2
 and common diff. = d_2

$\frac{1}{2}$

$\therefore$ According to question,

$$\frac{\text{Sum of } n \text{ terms of I A.P.}}{\text{Sum of } n \text{ terms of II A.P.}} = \frac{14 - 5n}{3n + 5}$$

$\frac{1}{2}$

$$\Rightarrow \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{14 - 5n}{3n + 5}$$

1

$$\Rightarrow \frac{[2a_1 + (n-1)d_1]}{[2a_2 + (n-1)d_2]} = \frac{14 - 5n}{3n + 5}$$

1

For ratio of 8th term of two A.P.'s we have to put

$$n - 1 = 14$$

$$\Rightarrow n = 15$$

$$\therefore \frac{[2a_1 + 14d_1]}{[2a_2 + 14d_2]} = \frac{14 - 5 \times 15}{3 \times 15 + 5}$$

1

$$\Rightarrow \frac{2[a_1 + 7d_1]}{2[a_2 + 7d_2]} = \frac{14 - 75}{45 + 5}$$

1

$$\Rightarrow \frac{8^{\text{th}} \text{ term of I A.P.}}{8^{\text{th}} \text{ term of II A.P.}} = \frac{-61}{50}$$

1

□□