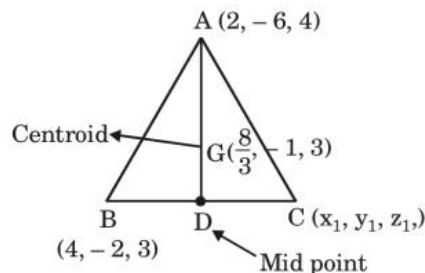


SAMPLE QUESTION PAPER-8

Section-A

1. Given that $A = \{x : x^2 < 70, x \in N\}$
 $\therefore$ Roster form = $\{1, 2, 3, 4, 5, 6, 7, 8\}$ 1
2. $R = \{(x, y) \in A \times B, x \in A, y \in B, x > y\}$
 $\therefore R = \{(3, 2), (5, 2), (5, 4)\}$ 1
3. $\frac{d}{dx} \sin^2 x = 2 \sin x \cos x = \sin 2x$ 1
4. Foci of ellipse $(\pm 5, 0)$ and $e = \frac{1}{2}$
 $i.e., c = 5 \Rightarrow e = \frac{c}{a}$
 $\Rightarrow \frac{1}{2} = \frac{5}{a} \Rightarrow a = 10 \Rightarrow a^2 = 100$
 and $b^2 = 10^2 - 5^2 = 75$
 $\therefore$ Equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{75} = 1.$ 1
5. AD is median and G is centroid
 $\therefore \frac{2 + 4 + x_1}{3} = \frac{8}{3}$
 $\Rightarrow x_1 = 2$
 Similarity $y_1 = 5$ and $z_1 = 2$.
 $\therefore$ Co-ordinates of centroid = $(2, 5, 2).$ 1
6. It is not the case that 2 is greater than 7. 1



Section - B

7. L.H.S. = $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$
 $= \frac{1}{2} [2 \cos 10^\circ \cos 50^\circ] \cos 70^\circ \cos 30^\circ$ 1
 $= \frac{1}{2} \times \frac{\sqrt{3}}{2} [(\cos 60^\circ + \cos 40^\circ) \cos 70^\circ],$
 $\therefore 2 \cos A \cos B = \cos (A + B) + \cos (A - B)$
 $= \frac{\sqrt{3}}{2 \times 4} \left[2 \times \frac{1}{2} \cos 70^\circ + 2 \cos 40^\circ \cos 70^\circ \right]$ 1
 $= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos 110^\circ + \cos 30^\circ]$
 $= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos (180 - 70)^\circ + \cos 30^\circ]$ 1
 $= \frac{\sqrt{3}}{8} [\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2}]$
 $= \frac{3}{16} = \text{R.H.S.}$ Hence proved. 1

8. A = Number of students who take tea
 B = Number of students who take coffee

$\therefore$ According to question,

$$n(A) = 26, n(B) = 18$$

and

$$n(\bar{A} \cap \bar{B}) = 8$$

But,

$$n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B}) = 8$$

So,

$$\begin{aligned} n(A \cup B) &= n(\cup) - n(\overline{A \cup B}) \\ &= 40 - 8 = 32 \end{aligned}$$

Using

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$\therefore$

$$32 = 26 + 18 - n(A \cap B)$$

$\Rightarrow$

$$n(A \cap B) = 44 - 32 = 12$$

9. $\tan^2 \theta + (1 - \sqrt{3})\tan \theta - \sqrt{3} = 0$

It is quadratic equation in $\tan \theta$, so,

$$\begin{aligned} D &= (1 - \sqrt{3})^2 + 4 \times 1 \times \sqrt{3} \\ &= 1 + 3 - 2\sqrt{3} + 4\sqrt{3} \end{aligned}$$

$$D = (1 + \sqrt{3})^2$$

$\therefore$

$$\tan \theta = \frac{-(1 - \sqrt{3}) \pm \sqrt{(1 + \sqrt{3})^2}}{2 \times 1}$$

$$\tan \theta = \frac{(\sqrt{3} - 1) \pm (\sqrt{3} + 1)}{2}$$

Taking + ve sign, we get

$$\tan \theta = \sqrt{3} = \tan \frac{\pi}{3}$$

$\Rightarrow$

$$\theta = n\pi + \frac{\pi}{3}, n \in I$$

On taking - ve sign, we get

$$\tan \theta = -1 = -\tan \frac{\pi}{4} = \tan \left(-\frac{\pi}{4}\right)$$

$\Rightarrow$

$$\theta = n\pi - \frac{\pi}{4}, n \in I.$$

10. We have a word *EQUATION* consists 8 letters with A, E, I, O, U vowels.

We can chose one vowel out of 5 in 5 ways and remaining letters can arrange in 7 ! ways

$\therefore$ Total words starting with vowel = $5 \times 7 !$

$$= 5 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 25200.$$

OR

Given that,

$$P(n, 4) : P(n - 1, 3) = 9 : 1$$

$\Rightarrow$

$$\frac{n!}{(n-4)!} \times \frac{(n-4)!}{(n-1)!} = \frac{9}{1}$$

$\Rightarrow$

$$\frac{n}{1} = 9$$

$\Rightarrow$

$$\boxed{n = 9}$$

11. An Urn consists 5 blue and x red balls.

$$\therefore \text{Total balls} = 5 + x$$

Probability of drawing two blue balls is $\frac{5}{14}$

$$\therefore \frac{5}{14} = \frac{{}^5C_2}{{}^{5+x}C_2}$$

$$\Rightarrow \frac{5}{14} = \frac{5 \times 4}{(5+x)(4+x)}$$

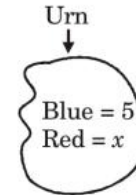
$$\Rightarrow x^2 + 9x + 20 = 56$$

$$\Rightarrow x^2 + 9x - 36 = 0$$

$$\Rightarrow (x + 12)(x - 3) = 0$$

$$\Rightarrow x = 3, x = -12 \text{ (not possible)}$$

$$\therefore \boxed{x = 3}$$



12.

$$\lim_{x \rightarrow 1} \frac{\left(1 - \frac{1}{x}\right)}{\sin[\pi(x-1)]} = \lim_{x \rightarrow 1} \frac{(x-1) \times \pi}{\pi \times x \sin \pi(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{1}{\pi x} \times \lim_{x \rightarrow 1} \left(\frac{\pi(x-1)}{\sin \pi(x-1)} \right)$$

$$= \frac{1}{\pi} \times 1 = \frac{1}{\pi} \quad \left[\because \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right) = 1 \right]$$

OR

$$\lim_{x \rightarrow 0} \frac{\tan 8x}{\sin 3x} = \lim_{x \rightarrow 0} \left[\frac{\left(\frac{\tan 8x}{8x} \right) \times 8x}{\left(\frac{\sin 3x}{3x} \right) \times 3x} \right]$$

$$= \frac{8}{3} \left[\frac{\lim_{x \rightarrow 0} \left(\frac{\tan 8x}{8x} \right)}{\lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \right)} \right]$$

$$= \frac{8/3 \times 1}{1} = \frac{8}{3} \quad \left(\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

13. Given that,

$$x + iy = \sqrt{\frac{a + ib}{p + iq}}$$

$$\Rightarrow (x - iy) = \sqrt{\frac{a - ib}{p - iq}}$$

$$\therefore (x + iy)(x - iy) = \sqrt{\frac{a + ib}{p + iq}} \times \sqrt{\frac{a - ib}{p - iq}}$$

$$x^2 + y^2 = \sqrt{\left(\frac{a^2 + b^2}{p^2 + q^2}\right)}$$

$\Rightarrow$

$$(x^2 + y^2)^2 = \left(\frac{a^2 + b^2}{p^2 + q^2}\right)$$

1

Hence proved.

OR

Let

$$x + iy = \sqrt{-16 + 30i}$$

Squaring both sides,

$$x^2 - y^2 + 2i xy = -16 + 30i$$

$\Rightarrow$

$$x^2 - y^2 = -16 \text{ and } 2xy = 30 \quad \dots(i)$$

$\therefore$

$$\begin{aligned} (x^2 + y^2)^2 &= (x^2 - y^2)^2 + (2xy)^2 \\ &= 256 + 900 \\ &= 1156 \end{aligned}$$

$\Rightarrow$

$$x^2 + y^2 = 34 \quad \dots(ii) \quad 1$$

On solving equations (i) and (ii), we get

$$x^2 = 9 \quad \text{and} \quad y^2 = 25$$

$\Rightarrow$

$$x = \pm 3 \quad \text{and} \quad y = \pm 5$$

As $2xy$ is positive. So,

$$\sqrt{-16 + 30i} = \pm (3 + 5i) \quad 1$$

Similarly take

$$u + iv = \sqrt{-16 - 30i}$$

$\Rightarrow$

$$u^2 - v^2 + 2iuv = -16 - 30i$$

$\Rightarrow$

$$u^2 - v^2 = -16, 2uv = -30 \quad \dots(iii)$$

$\Rightarrow$

$$\begin{aligned} (u^2 + v^2)^2 &= (-16)^2 + (-30)^2 \\ &= 1156 \\ u^2 + v^2 &= 34 \quad \dots(iv) \quad 1 \end{aligned}$$

On solving (iii) and (iv),

$$u^2 = 9 \quad \text{and} \quad v^2 = 25$$

$\Rightarrow$

$$u = \pm 3 \quad \text{and} \quad v = \pm 5$$

As $2uv$ is negative.

$\therefore$

$$\sqrt{-16 - 30i} = \pm (3 - 5i)$$

Hence

$$\begin{aligned} \sqrt{-16 + 30i} + \sqrt{-16 - 30i} &= \pm (3 + 5i) \pm (3 - 5i) \\ &= \pm 6. \end{aligned} \quad 1$$

14. We have two equations,

$$x \sec \theta + y \operatorname{cosec} \theta = a$$

....(i)

and

$$x \cos \theta - y \sin \theta = a \cos 2\theta$$

....(ii)

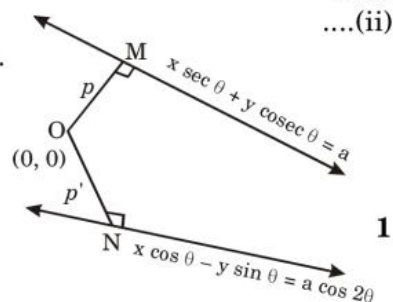
p and p' be $\perp$ distances from $O(0, 0)$ to line (i) and (ii) respectively.

So,

$$OM = \left| \frac{0 \cdot \sec \theta + 0 \cdot \operatorname{cosec} \theta - a}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \right|$$

$\therefore$

$$p = \left| \frac{-a}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}} \right|$$



$$\Rightarrow p^2 = \frac{a^2 \cdot \sin^2 \theta \cdot \cos^2 \theta}{(\sin^2 \theta + \cos^2 \theta)}$$

$$\Rightarrow p^2 = a^2 \sin^2 \theta \cos^2 \theta \quad \dots(i)$$

Now $ON = p' = \left| \frac{0 - 0 - a \cos 2\theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}} \right|$

$$\Rightarrow (p')^2 = a^2 \cos^2 2\theta \quad \dots(ii) \quad 1$$

From (i) and (ii),

$$\begin{aligned} 4p^2 + p'^2 &= 4a^2 \sin^2 \theta \cos^2 \theta + a^2 \cos^2 2\theta \\ &= a^2 \sin^2 2\theta + a^2 \cos^2 2\theta \\ &= a^2 (\sin^2 2\theta + \cos^2 2\theta) \\ &= a^2 \end{aligned} \quad 2$$

$$\Rightarrow \boxed{4p^2 + p'^2 = a^2} \quad \text{Hence proved.}$$

15. There are 52 playing cards.

Total ways of drawing two cards = ${}^{52}C_2$

A = Two cards drawn are red.

B = Two cards drawn are kings

$$\therefore P(A) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26 \times 25}{52 \times 51} = \frac{25 \times 26}{52 \times 51} \quad 1$$

$$P(B) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{12}{52 \times 51} \quad 1$$

and $P(A \cap B) = \frac{{}^2C_2}{{}^{52}C_2} = \frac{2 \times 1}{52 \times 51} = \frac{2}{52 \times 51} \quad 1$

$$\begin{aligned} \therefore P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{25 \times 26}{52 \times 51} + \frac{12}{52 \times 51} - \frac{2}{52 \times 51} \\ &= \frac{2[325 + 6 - 1]}{52 \times 51} \\ &= \frac{2 \times 330}{52 \times 51} = \frac{55}{221} \end{aligned}$$

$$\therefore P(A \cup B) = \frac{55}{221} \quad 1$$

16.

C.I.	f	C.F.	C.I.	x_i	$ x_i - \text{Med} $	$f_i x_i - \text{Med} $
16-20	5	5	15.5-20.5	18	20	100
21-25	6	11	20.5-25.5	23	15	90
26-30	12	23	25.5-30.5	28	10	120
31-35	14	37	30.5-35.5	33	5	70
36-40	26	63	35.5-40.5	38	0	0
41-45	12	75	40.5-45.5	43	5	60
46-50	16	91	45.5-50.5	48	10	160
51-55	9	100	50.5-55.5	53	15	135
Total	100					735

$$N = 100, \frac{N}{2} = 50$$

$$l = 35.5, f = 26, cf = 37$$

$$\therefore \text{Median} = 35.5 + \left(\frac{50 - 37}{26} \right) \times 5$$

$$= 35.5 + \frac{5}{2}$$

$$\text{Median} = 38$$

1

$\therefore$ Mean deviation from median

$$= \frac{\sum f_i |x_i - \text{Med}|}{\sum f_i} = \frac{735}{100} = 7.35$$

$$\boxed{\text{M.D. (from median)} = 7.35}$$

2

17. We have a greatest integer function $f(x) = [x]$, is less than or equal to x

$$f(0) = [0] = 0$$

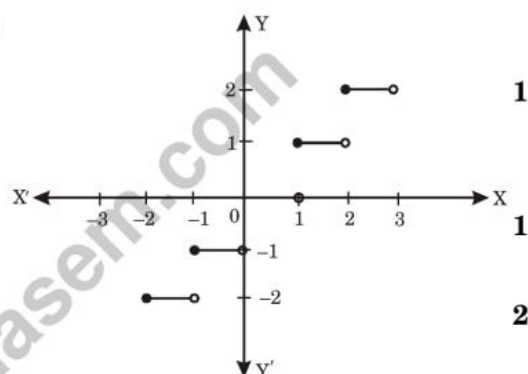
$$f(.5) = [.5] = 0$$

$$f(1) = [1] = 1$$

$$f[-.5] = [-.5] = -1$$

$$f[-1] = [-1] = -1$$

$$f[-1.7] = [-1.7] = -2$$



1

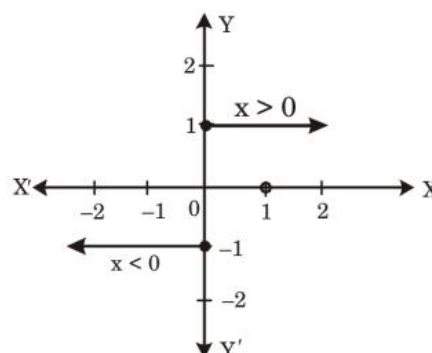
1

2

OR

Signum function $f: R \rightarrow R$ is

$$f(x) = \begin{cases} 1; & x > 0 \\ 0; & x = 0 \\ -1; & x < 0 \end{cases}$$



18. It is given that

$$PA = PB$$

$\Rightarrow$

$$PA^2 = PB^2$$

$$\Rightarrow (x - 3)^2 + (y - 4)^2 + (z + 5)^2 = (x + 2)^2 + (y - 1)^2 + (z - 4)^2$$

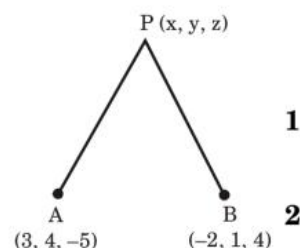
$$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z$$

$$= x^2 + 4 + 4x + y^2 + 1 - 2y + z^2 + 16 - 8z$$

$$\Rightarrow -6x - 4x - 8y + 2y + 10z + 8z + 50 - 21 = 0$$

$$\Rightarrow -10x - 6y + 18z + 29 = 0$$

$$\Rightarrow 10x + 6y - 18z - 29 = 0$$



1

2

1

- 19.

$$\text{L.H.S.} = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\theta}}}$$

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 + (1 + \cos 8\theta)}}}$$

1

But

$$1 + \cos 2A = 2 \cos^2 A$$

$$\begin{aligned}\therefore \sqrt{2 + \sqrt{2 + \sqrt{4 \cos^2 4\theta}}} &= \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} \\ &= \sqrt{2 + \sqrt{4 \cos^2 2\theta}} = \sqrt{2(1 + \cos 2\theta)} \\ &= 2 \cos \theta = \text{R.H.S.}\end{aligned}$$

1

Section - C

20. Four lines (given) forms a parallelogram.

$$\sqrt{3}x + y = 0 \quad \dots(i)$$

$$\sqrt{3}y + x = 0 \quad \dots(ii)$$

$$\sqrt{3}x + y = 1 \quad \dots(iii)$$

$$\sqrt{3}y + x = 1 \quad \dots(iv)$$

On solving (i) and (ii),

$$x = 0 \text{ and } y = 0, \text{ i.e., } A(0, 0)$$

1

Solving (ii) and (iii), we get

$$-\sqrt{3}(\sqrt{3}y) + y = 1$$

$$\Rightarrow -2y = 1 \Rightarrow y = -\frac{1}{2}$$

and

$$x = -\sqrt{3} \times \left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\therefore B\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

1

Solving equations (iii) and (iv), we get

$$\sqrt{3}(1 - \sqrt{3}x) + x = 1$$

$$\Rightarrow \sqrt{3} - 2x = 1$$

$$x = \frac{\sqrt{3} - 1}{2}$$

and

$$\begin{aligned}y &= \sqrt{3}\left(\frac{\sqrt{3} - 1}{2}\right) + 1 \\ &= \frac{-3 + 2 + \sqrt{3}}{2} = \frac{\sqrt{3} - 1}{2}\end{aligned}$$

$$\text{i.e., } C\left(\frac{\sqrt{3} - 1}{2}, \frac{\sqrt{3} - 1}{2}\right)$$

1

Now, and on solving (i) and (iv),

$$\sqrt{3}(-\sqrt{3}x) + x = 1$$

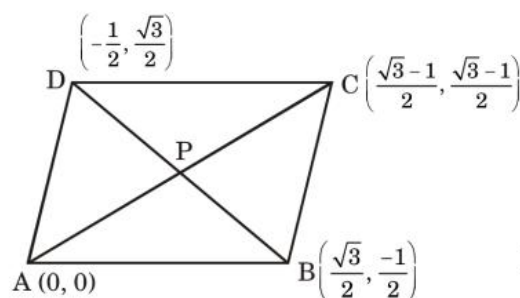
$$-2x = 1$$

$$\Rightarrow x = -\frac{1}{2}$$

$$\Rightarrow y = -\sqrt{3} \times \left(-\frac{1}{2}\right)$$

$$\text{Hence, } \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\therefore \text{Slope of AC} = \left(\frac{\sqrt{3} - 1}{2}\right) \div \left(\frac{\sqrt{3} - 1}{2}\right) = 1$$



1

and $\text{slope of } BD = \frac{\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}{\left(\frac{13}{2} + \frac{1}{2}\right)} = -1$ 1

$\therefore$ Product of slopes of AC and BD

$$= 1 \times (-1) = -1$$

Hence, diagonals of $ABCD$ are $\perp$ to each other. 1

21. $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$$= (\sqrt{2})^6 + {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 + {}^6C_3(\sqrt{2})^3$$

$$+ {}^6C_4(\sqrt{2})^2 + {}^6C_5(\sqrt{2})^1 + {}^6C_6 + (\sqrt{2})^6 - {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4$$

$$- {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 - {}^6C_5(\sqrt{2}) + {}^6C_6$$
 2

$$= 2[({\sqrt{2}})^6 + {}^6C_2({\sqrt{2}})^4 + {}^6C_4({\sqrt{2}})^2 + {}^6C_6]$$
 1

$$= 2\left[8 + \frac{6 \times 5}{2 \times 1} \times 4 + \frac{6 \times 5}{2 \times 1} \times 2 + 1\right]$$
 1

$$= 2[8 + 60 + 30 + 1]$$
 1

$$= 2 \times 99 = 198.$$
 1

OR

We have an expansion $\left(2x^2 - \frac{3}{x}\right)^{11}$

$\therefore T_{r+1} = (-1)^r {}^{11}C_r (2x^2)^{11-r} \left(\frac{3}{x}\right)^r$ 2

$$= (-1)^r {}^{11}C_r 2^{11-r} x^{22-2r} \cdot 3^r x^{-r}$$

$$= (-1)^r {}^{11}C_r \cdot 2^{11-r} \cdot 3^r \cdot x^{22-3r}$$
 2

For term involving x^5 $22 - 3r = 5$

$\Rightarrow r = \frac{17}{3}$ (which is not an integer) 2

$\therefore$ This expansion does not include any term consisting x^5 .

22. (i) $\text{R.H.S.} = \frac{b^2 - c^2}{a^2}$

By sine rule, we know that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ (say)}$$
 1

$$\therefore a = k \sin A, b = k \sin B, c = k \sin C$$

$\Rightarrow \frac{\sin^2 B - \sin^2 C}{\sin^2 A}$ 1

But $A + B + C = \pi \Rightarrow A = \pi - (B + C)$

$\Rightarrow \frac{(\sin B + \sin C)(\sin B - \sin C)}{\sin^2 [\pi - (B + C)]}$

$$\begin{aligned}
 &= \frac{2\sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \cdot 2\cos\left(\frac{B+C}{2}\right) \sin\left(\frac{B-C}{2}\right)}{\sin^2(B+C)} & 2 \\
 &= \frac{\left[2\sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B+C}{2}\right)\right] \cdot \left[2\sin\left(\frac{B-C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right)\right]}{\sin^2(B+C)} & 1 \\
 &= \frac{\sin(B+C) \cdot \sin(B-C)}{\sin^2(B+C)} \\
 &= \frac{\sin(B-C)}{\sin(B+C)} = \text{R.H.S.} & \text{Hence proved}
 \end{aligned}$$

(ii) L.H.S. = $b \cos B + c \cos C$

Using sine rule, we get

$$\begin{aligned}
 b &= k \sin B \text{ and } c = k \sin C & 1 \\
 \therefore k \sin B \cos B + k \sin C \cos C & & 1 \\
 &= \frac{k}{2} (2 \sin B \cos B + 2 \sin C \cos C) & 1 \\
 &= \frac{k}{2} [\sin 2B + \sin 2C] \\
 &= \frac{k}{2} [2 \sin(B+C) \cdot \cos(B-C)] & 1 \\
 &= k \sin(B+C) \cos(B-C) \\
 &= k \sin(\pi - A) \cos(B-C) \quad (\because B+C = \pi - A) & 1 \\
 &= k \sin A \cos(B-C) \\
 &= a \cos(B-C) = \text{R.H.S.} & \text{Hence proved. } 1
 \end{aligned}$$

23. An aged person writes letters to 4 of his friends and ask to copy of letter mailed to 4 people and with same instruction. 1

So, sequence of letters (numbers) is

$$4, 4^2, 4^3, 4^4, \dots \quad 1$$

which is G.P. with $a = 4$ and $r = 4$.

So, number of mailed letter up to 8th mail.

$$\begin{aligned}
 &= \text{sum of 8 terms of this G.P.} \\
 &= \frac{4(4^8 - 1)}{(4 - 1)} & 2 \\
 &= \frac{4}{3} \times (256 \times 256 - 1) \\
 &= \frac{4}{3} \times (65536 - 1) \\
 &= \frac{4}{3} \times 65535 = 87380
 \end{aligned}$$

Cost of mailing to each letter = Rs. (0.5).

So,

$$\text{total cost} = 87380 \times 0.5$$

$$= \text{Rs. } 43690. \quad 1$$

Importances :

(i) The ancient means was so popular, effective and people wrote and share all emotional feelings and wait for the response.

(ii) Modern means works activity and we don't feel distances among relationship. 1

24.

$$2x + y \geq -3$$

$$2x + y \leq 6, x \geq 0, y \geq 0$$

Take

$$2x + y = -3$$

At $x = 0$,

$$y = -3$$

and at $y = 0$,

$$x = -3/2$$

At $(0, 0)$,

$$0 + 0 > -3 \text{ (True)}$$

So origin is included.

Now take

$$2x + y = 6$$

At $x = 0$,

$$y = 6$$

and at $y = 0$,

$$x = 3$$

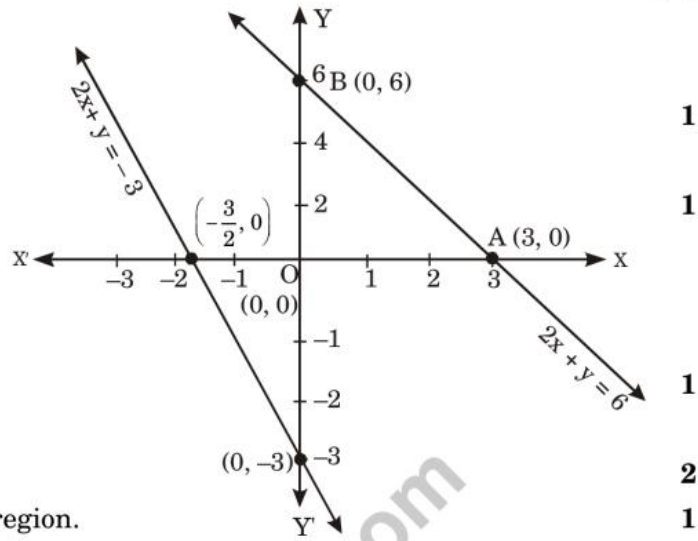
At $(0, 0)$,

$$0 + 0 < 6 \text{ (True)}$$

So here origin is also included

$x \geq 0, y \geq 0$ mean first quadrant region.

The shaded region OAB is the solution region.



25. $P(n) = 5^{2n} - 1$ is divisible by 24.

Put $n = 1$.

$P(1) : 5^2 - 1 = 24$, it is divisible by 24

i.e., $P(1)$ is true.

Let $P(k)$ is true.

∴

$$P(k) = 5^{2k} - 1 = 24\lambda; (\lambda \in N)$$

Now to prove that $P(k + 1)$ is true.

i.e., $5^{2(k+1)} - 1$ is divisible by 24.

Take

$$\text{L.H.S.} = 5^{2(k+1)} - 1$$

$$= 5^{2k} \cdot 5^2 - 1$$

$$= 25 \cdot (24\lambda + 1) - 1 \quad [\text{using equation (i)}]$$

$$= 25 \times 24\lambda + 25 - 1$$

$$= 24 (25\lambda + 1)$$

$$= 24\mu; (\mu = 25\lambda + 1 \in N)$$

i.e., $P(k + 1)$ is true.

Hence, $P(n)$ is true when $P(k)$ is true for all $n \in N$

26. There are 3 Mathematics, 4 Hindi and 5 physics books.

So,

$$\text{Total books} = 12$$

∴ Total arrangement of these books in a shelf

$$= 12!$$

When all books of same subject together then these can be arranged in $3!$, $4!$ and $5!$ ways respectively.

∴ Total ways of such arrangements = $3! \times 3! \times 4! \times 5!$

$$= 6 \times 6 \times 24 \times 120$$

$$= 1,03,680.$$

The students, who have such habits of well and proper arrangements of things can be good in behaviour, and disciplined alos.

