Institute of Actuaries of India

ACET December 2023

Mathematics

1. The solution(s) of the equation $|2x + 23| = |x + 11|$ is (are)

   A. -14.

   B. $-\frac{34}{3}$ and 4.

   C. -12 and $-\frac{34}{3}$.

   D. 6 and $-\frac{34}{3}$.

   1 mark

2. If $f(x) = 4(x - 3)$ and $g(x) = x^2 - 7x + 12$, then the value of x for which the ratio $\frac{g(x)}{f(x)}$ is undefined is

   A. 1.

   B. 2.

   C. 3.

   D. 4.

   1 mark

3. For any two sets A and B if $A \cap B = A \cup B$, then

   A. $A = B$.

   B. $A = \Omega$ (the universal set).

   C. $A = \emptyset$ (the null set).

   D. $A = B^c$.

   1 mark

4. If $f(x) = 2x^2 + 5x - 24$ and $g(x) = 3x + 2$ (x real), then the roots of $f(x) + g(x)$ are

   A. real and equal.

   B. real and distinct.

   C. complex conjugate pair of non-real roots.

   D. one is real and the other is non- real.

   1 mark

5. The value of $(\sqrt{3} + 1)^4 - (\sqrt{3} - 1)^4$ is

   A. $32\sqrt{3}$.

   B. $16\sqrt{3}$.

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C. $8\sqrt{3}$.

D. 96.

1 mark

6. The value of $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{1}{7}$ is (assuming all the angles to be in the first quadrant)

A. $\frac{\pi}{2}$.

B. $\frac{\pi}{3}$.

C. $\frac{\pi}{4}$.

D. $\frac{\pi}{6}$.

1 mark

7. The value of $\sum_{j=1}^{n}(3j-1)^2$ is

A. $\frac{n}{2}[6n^2-3n+1]$.

B. $\frac{n}{2}[6n^2+3n+1]$.

C. $\frac{n}{2}[6n^2-3n-1]$.

D. $\frac{n}{2}[6n^2+3n-1]$.

2 marks

8. The conjugate of $\frac{(1+2i)(2-i)}{(3-2i)(2+3i)}$ is

A. $\frac{63}{169}-i\frac{16}{169}$.

B. $\frac{63}{169}+i\frac{16}{169}$.

C. $-\frac{63}{169}-i\frac{16}{169}$.

D. $-\frac{63}{169}+i\frac{16}{169}$.

2 marks

9. Let

$$f(x)=\begin{cases}7 & \text{if } x\leq 3,\\ mx+n & \text{if } 3<x<12,\\ 18 & \text{if } x\geq 12,\end{cases}$$

m,n real. The function $f(x)$ is continuous everywhere if (m,n) is

A. $\left(\frac{11}{10},\frac{10}{3}\right)$.

B. $\left(\frac{11}{9},\frac{10}{3}\right)$.

C. $\left(\frac{11}{9},\frac{1}{3}\right)$.

D. $\left(\frac{11}{9},-\frac{10}{3}\right)$.

1 mark

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10. If $x^3 + y^3 = \cos x + y$, then $\frac{dy}{dx}$ is equal to

A. $\frac{3x^2 - \sin x}{1 - 3y^2}$.

B. $\frac{3x^2 - \sin x}{1 + 3y^2}$.

C. $\frac{3x^2 + \sin x}{1 + 3y^2}$.

D. $\frac{3x^2 + \sin x}{1 - 3y^2}$.

2 marks

11. Let $f(x) = -2x^3 - 9x^2 - 12x + 1$ be a real function. The interval in which $f(x)$ is increasing in x is

A. $(-\infty, -2)$.

B. $(-1, \infty)$.

C. $(-2, -1)$.

D. $(-2,0)$.

3 marks

12. The value of $\int_{-\pi/5}^{\pi/5} \frac{7 \sin x}{5 + \cos x} dx$ is

A. 0.

B. 5.

C. $\log_e 2$.

D. π.

1 mark

13. The area bounded by the curve $y = \frac{x^2}{2}$, the lines $y = 0$, $x = 1$ and $x = 3$ is

A. 13.

B. $4\frac{1}{3}$.

C. $18\frac{2}{3}$.

D. 4.

1 mark

14. The value of the integral $\int x \log 3x \, dx$ is

A. $\frac{x^2}{2} \log 3x - \frac{x^2}{2} + C$.

B. $\frac{x^2}{2} \log 3x + \frac{x^2}{4} + C$.

C. $x^2 \log 3x - \frac{x^2}{4} + C$.

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D. $\frac{x^2}{2}\log 3x - \frac{x^2}{4} + C$.

2 marks

15. If

$$M = \begin{bmatrix} 1 & \cos\theta & \sin\theta \\ -\cos\theta & -1 & 1 \\ \sin\theta & 1 & 1 \end{bmatrix},$$

then $|M|$ is equal to

A. 1.

B. -2.

C. -1.

D. 0.

1 mark

16. If

$$\begin{bmatrix} xy & 6 & 1 \\ z+4 & x+y & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 16 & a & 1 \\ -8 & 12 & b \\ c & 1 & d \end{bmatrix},$$

then $x + y + z$ is equal to

A. -16.

B. 0.

C. 16.

D. 24.

1 mark

17. The approximate positive root of $x^2 - 2 = 0$, obtained after three iterations by Newton-Raphson method starting from the initial value $x_0 = 1$, is

A. 1.41426.

B. 1.92326.

C. 1.81526.

D. 1.41667.

2 marks

18. The following table gives values of $f(x)$ for some value of x.

x	50	60	70	80
$f(x)$	205	225	248	274

The value of $f(54)$ by the Newton's forward interpolation formula is

A. 202.23.

B. 227.78.

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C. 207.12.

D. 212.64.

3 marks

19. Let $\vec{A} = \vec{x} + \vec{y} + \vec{z}$, $\vec{B} = -\vec{x} + \vec{y} - \vec{z}$ and $\vec{C} = \vec{y} - \vec{z}$. The area of the triangle ΔABC (in sq.units) is

A. 3.

B. 2.

C. 1.

D. 0.

2 marks

20. Let $\vec{A} = 2\vec{x} + 4\vec{y} + \vec{z}$, $\vec{B} = 3\vec{x} - \vec{y} + 2\vec{z}$. The angle between $\vec{A}$ and $\vec{B}$ is

A. $\cos^{-1}\frac{4}{7\sqrt{3}}$.

B. $\cos^{-1}\frac{4}{7\sqrt{6}}$.

C. $\cos^{-1}\frac{4}{\sqrt{6}}$.

D. $\cos^{-1}\frac{4}{7\sqrt{5}}$.

1 mark

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21. The mean of 20 observations is 25. On checking it was found that two observations are wrongly recorded as 15 and 17, while the correct observations were 25 and 27. Then the correct mean is

A. 30.

B. 27.

C. 26.

D. 25.5.

1 mark

22. The weighted arithmetic mean of 12, 16 and 20 with weights w_1, w_2 and w_3, respectively, is known to be 18. If the weights are changed to $3w_1, 3w_2$ and $3w_3$, then the weighted arithmetic mean will be

A. 6.

B. 18.

C. 24.

D. 54.

1 mark

23. The standard deviation of 20 observations is 2.1. Suppose each observation is first multiplied by -0.5 and then 1.5 is added after multiplication. The standard deviation of the new observations is

A. 0.45.

B. 0.525.

C. 1.05.

D. 2.55.

1 mark

24. If the relationship between two variables x and y is $3x - 5y = 4$ and mode of x is 13, then the mode of y is

A. 7.

B. 10.

C. 13.

D. -2.

1 mark

25. If for a symmetrical distribution first quartile and median are 30 and 45, respectively, then third quartile of the distribution is

A. 15.

B. 52.5.

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C. 60.

D. 75.

1 mark

26. If the letters of the word "ENGINEERING" are written down at random, what is the probability that all the Es occur together?

A. $\frac{1}{110}$.

B. $\frac{1}{495}$.

C. $\frac{9}{11}$.

D. $\frac{3}{55}$.

2 marks

27. Probabilities of solving a specific problem independently by students S_1 and S_2 are $\frac{1}{3}$ and $\frac{1}{2}$, respectively. If both try to solve the problem independently, the probability that exactly one of them solve the problem is

A. $\frac{1}{2}$.

B. $\frac{5}{6}$.

C. $\frac{1}{6}$.

D. $\frac{2}{3}$.

1 mark

28. A bag contains 5 white, 6 red and 4 black balls. Three balls are drawn one by one with replacement. What is the probability that at least one ball is red?

A. $\frac{53}{65}$.

B. $\frac{117}{125}$.

C. $\frac{27}{125}$.

D. $\frac{98}{125}$.

2 marks

29. A box of chips contains 15 chips of which 4 are defective. If 3 of the chips are selected at random and removed from the box in succession without replacement what is the probability that all three chips are defective?

A. $\frac{4}{455}$.

B. $\frac{8}{1125}$.

C. $\frac{25}{546}$.

D. $\frac{1}{3375}$.

2 marks

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30. A construction company employs three sales engineers. Engineers 1, 2, and 3 estimates the costs of 30%, 20%, and 50%, respectively, of all jobs bid by the company. Let E_i be the event that a job is estimated by engineer i, $i = 1, 2, 3$. The probabilities that engineers make error in estimating the cost are $P(\text{error}|E_1) = 0.01, P(\text{error}|E_2) = 0.03$ and $P(\text{error}|E_3) = 0.02$. If a particular bid results in error in estimating job cost, what is the probability that the error was made by engineer 1?

A. $\frac{1}{19}$.

B. $\frac{3}{19}$.

C. $\frac{6}{19}$.

D. $\frac{3}{190}$.

2 marks

31. Suppose X uniformly distributed on $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. Then $E(|X|)$ is

A. $\frac{15}{11}$.

B. 3.

C. $\frac{30}{11}$.

D. 0.

1 mark

32. The number of accidents per month at a busy intersection follows Poisson distribution with average $\lambda = 5$. Suppose each accident costs local government Rs. 15,000 for clean-up. On average, the accidents costs to the local government over a year time is

A. Rs. 36,000.

B. Rs. 75,000.

C. Rs. 180,000.

D. Rs. 900,000.

1 mark

33. Suppose X follows Poisson distribution with mean 8. Find the value of k such that $P(X = k) = P(X = k + 1)$.

A. 6.

B. 7.

C. 8.

D. 9.

1 mark

34. Suppose the research department of a steel manufacturer believes that one of the company's rolling machine is producing sheets of steel of varying thickness. The

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thickness X is a Uniform random variable with values between 150 and 200 millimeters. Any sheets less than 160 milimeters thick must be scrapped, since they are unacceptable to buyers. The probability that steel sheets produced by this machine to be scrapped is

A. 0.10.

B. 0.15.

C. 0.20.

D. 0.25.

1 mark

35. Let X be a random variable with probability density function

$$f(x) = \begin{cases} e^{-x}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

The expected value of $e^{0.75X}$ is

A. 4.

B. 0.75.

C. $e^{0.75}$.

D. $e^{0.25}$.

1 mark

36. Suppose $X \sim N(\mu, \sigma^2)$. Then which one of the following is true?

A. $P(X \geq \mu) = 0.25$.

B. $P(\mu - \sigma \leq X \leq \mu + \sigma) = \frac{1}{\sqrt{2\,\pi}} \int_{-1}^{1} e^{-\frac{z^2}{2}}\, dz$.

C. $P(X \leq \mu + \sigma) = 0.50$.

D. $P(X \geq \mu - \sigma) < 0.50$.

2 marks

37. Let X be a random variable with probability density function

$$f(x) = \theta e^{-\theta x},\ x > 0, \theta > 0.$$

Suppose the median of the distribution is M. Then the mean of the distribution is

A. M.

B. $2M$.

C. $-\frac{\ln(0.5)}{M}$.

D. $-\frac{M}{\ln(0.5)}$.

1 mark

38. Suppose $X_1 \sim Binomial\ (n_1, p_1)$ independent of $X_2 \sim Binomial\ (n_2, p_2)$. Then $P(X_1 + X_2 = 1)$ equals to

A. $p_1(1 - p_2) + p_2(1 - p_1)$.

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B. $p_1(1-p_2)^{n_2-1}+p_2(1-p_1)^{n_1-1}$.

C. $[n_1p_1(1-p_2)+n_2p_2(1-p_1)](1-p_1)^{n_1-1}(1-p_2)^{n_2-1}$.

D. $n_1n_2p_1p_2(1-p_1)^{n_1-1}(1-p_2)^{n_2-1}$.

2 marks

39. Suppose $X \sim N(5, 2^2)$ is independent of $Y \sim N(0, 1)$. Then correlation between Y and XY is equal to

A. 0.

B. 5.

C. $\frac{5}{29}$.

D. $\frac{5}{\sqrt{29}}$.

3 marks

40. If $2u = x + 5$ and $6v = 2y - 7$ and the regression coefficient of x on y is 4, then regression coefficient of u on v is

A. 6.

B. $\frac{4}{3}$.

C. 2.8.

D. 1.

3 marks

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Data Interpretation

The following frequency distribution represents the marks in a subject of 200 students in a class. Answer questions 41-43 based on the information given in this table.

Marks	Number of students
10-19	6
20-29	12
30-39	16
40-49	40
50-59	60
60-69	30
70-79	16
80-89	14
90-100	6

41. The minimum possible value of the mean of the actual marks summarized in the above frequency distribution is

    A. 10.0.

    B. 54.5.

    C. 49.8.

    D. 50.0.

    2 marks

42. The percentage of students scoring less than 80% but not less than 60%

    A. 23.

    B. 46.

    C. 30.

    D. 16.

    1 mark

43. If the pass mark is 40%, the percentage of students who failed in the subject is

    A. 74.

    B. 37.

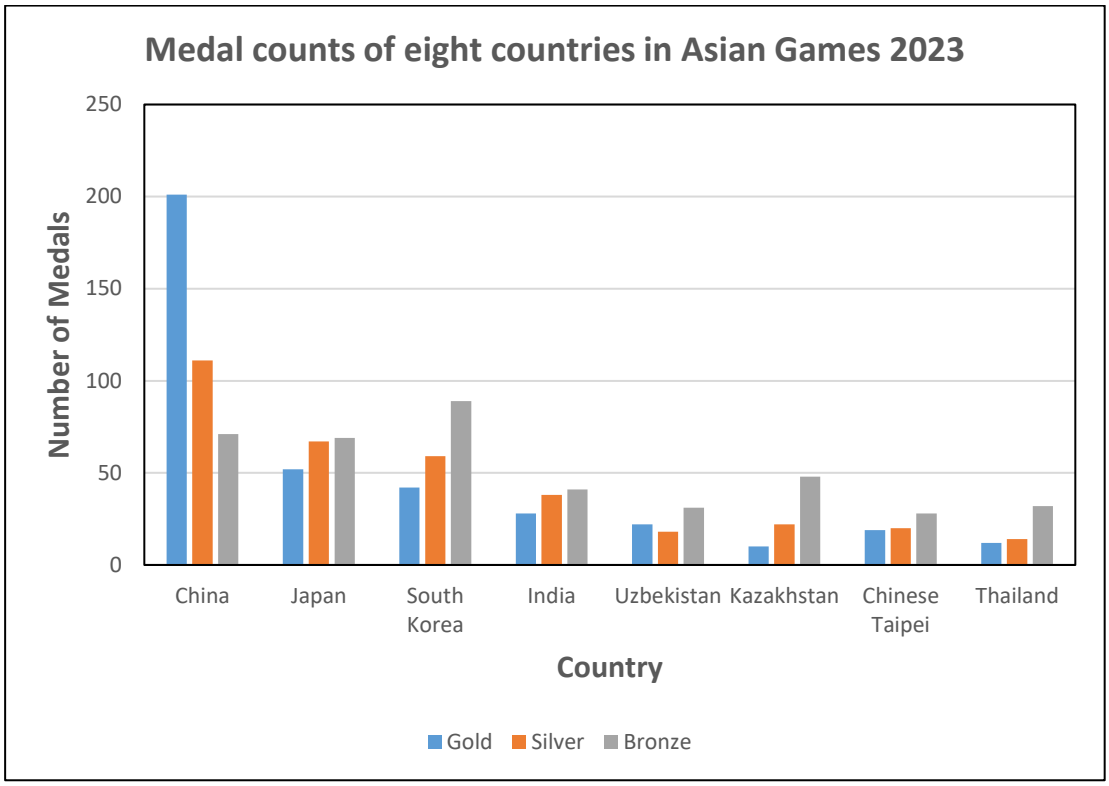
    C. 34.

    D. 17.

    1 mark

Answer questions 44-45 based on the following bar diagram on medal counts of eight countries in Asian Games 2023.

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44. Which of the eight countries won the maximum number of bronze medals?

A. China.

B. Japan.

C. South Korea.

D. Kazakhstan.

1 mark

45. Among Uzbekistan, Kazakhstan, Chinese Taipei and Thailand, which country won the maximum number of medals?

A. Uzbekistan.

B. Kazakhstan.

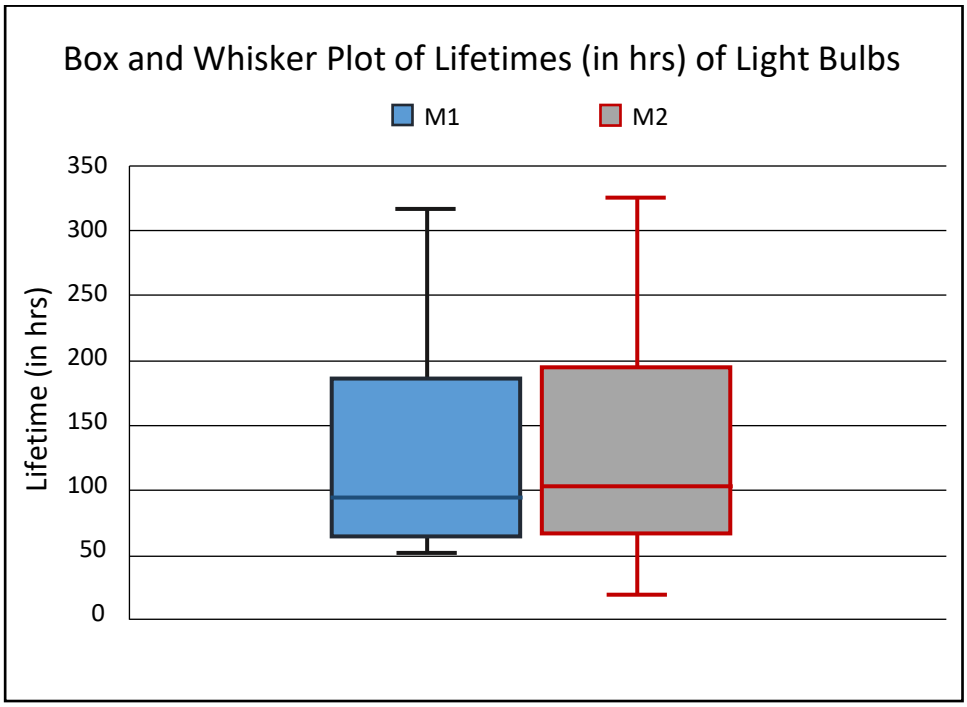
C. Chinese Taipei.

D. Thailand.

2 marks

Answer questions 46-47 based on the following box and whisker plot of lifetime (in hours) of light bulbs of two manufacturers M1 and M2.

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46. The lifetime distribution of lifetime of light bulbs of M1 is

A. positively skewed.

B. negatively Skewed.

C. symmetric but not uniform.

D. uniform.

1 mark

47. Which of the following statements is true?

A. The median lifetime of light bulbs of M1 is greater than that of M2.

B. The median lifetime of M2 is greater than 100 hrs.

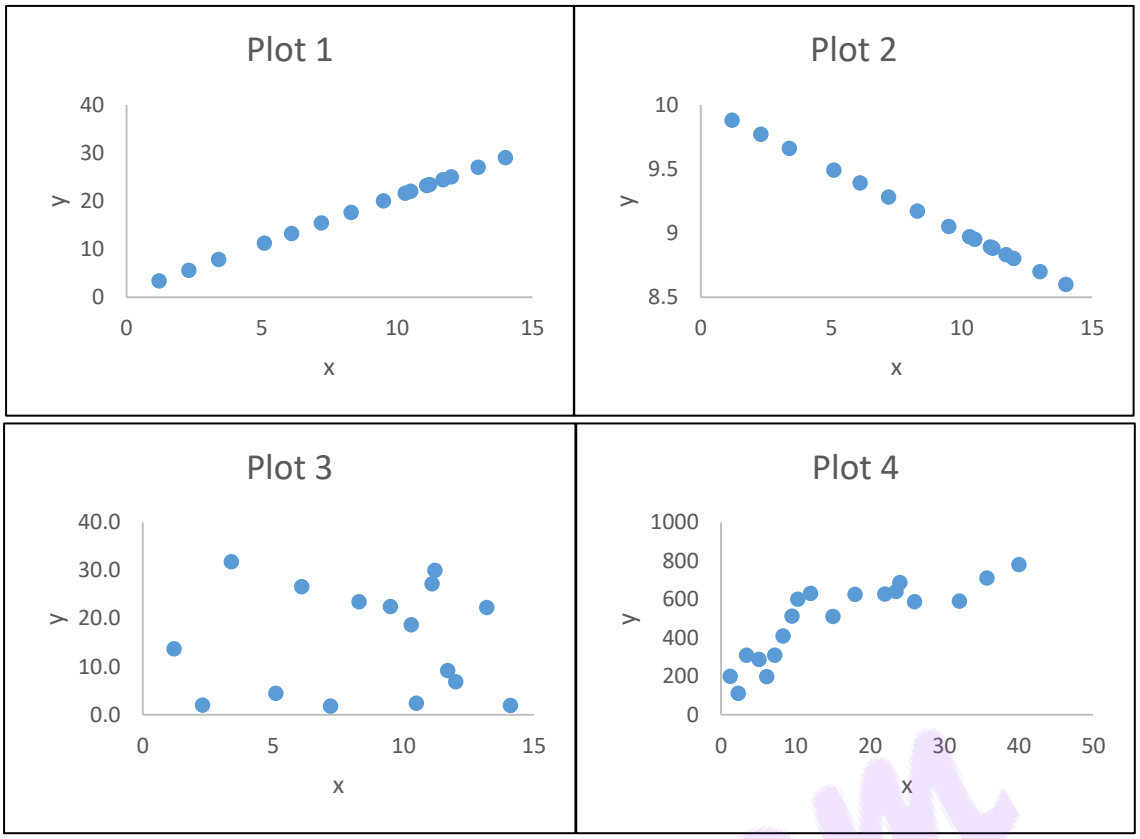
C. The lifetime distribution of light bulbs of M2 is negatively skewed.

D. The mean lifetime of lightbulbs of M1 is less than their median lifetime.

2 marks

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Answer questions 48-49 based on the following scatter plots.



48. The correlation coefficient between x and y in Plot 3 is

A. 0.75.

B. -1.

C. 1.

D. nearly zero.

1 mark

49. Which of the following statements is true?

A. The correlation coefficient between x and y is negative in Plot 1.

B. The correlation coefficient between x and y is positive in Plot 4.

C. The correlation coefficient between x and y is nearly 1 in Plot 2.

D. The correlation coefficient between x and y is 0 in Plot 2.

2 marks

The following table gives the number of vehicles arriving in a fueling station in 200 five-minute intervals. Answer questions 50-51 based on the data in the table.

No. of vehicles	0	1	2	3	4	5	6	7	8	9	10	Total
Frequency	2	6	8	17	38	65	30	20	8	4	2	200

50. The percentage of time intervals with at least 5 arriving vehicles is

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A. 64.5.

B. 70.

C. 72.7.

D. 81.8.

1 mark

51. The median of the distribution of number of vehicles is

A. 4.

B. 4.5.

C. 5.

D. 5.5.

1 mark

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52. Which of the following is a synonym for the word "Benevolent"?

A. Malevolent.

B. Kindhearted.

C. Cruel.

D. Sinister.

1 mark

53. What is a synonym for the word "Eloquent"?

A. Fluent.

B. Inarticulate.

C. Reserved.

D. Mute.

1 mark

54. What is the antonym of the word "Humble"?

A. Arrogant.

B. Modest.

C. Meek.

D. Timid.

1 mark

55. What is the antonym of the word "Voracious"?

A. Gluttonous.

B. Ravenous.

C. Undesirous.

D. Insatiable.

1 mark

56. Which phrase means "to take responsibility for one's actions"?

A. Take up.

B. Cut down.

C. Bring up.

D. Own up.

1 mark

57. Which phrase means "to reveal a secret"?

A. Keep a stiff upper lip.

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B. Break the ice.

C. Let the cat out of the bag.

D. Hit the sack.

1 mark

58. What is the term for the act of speaking or writing in an evasive or indirect manner?

A. Eloquence.

B. Verbose.

C. Concise.

D. Circumlocution.

1 mark

59. What is the term for the quality of being easily broken or damaged?

A. Resilience.

B. Fragility.

C. Robustness.

D. Tenacity.

1 mark

60. Select the correct sentence.

A. Her dedication to work and the quality of her performance is truly remarkable.

B. Her dedications to work and the quality of her performances are truly remarkable.

C. Her dedication to work and the quality of her performance are truly remarkable.

D. Her dedications to work and the quality of her performance is truly remarkable.

2 marks

61. Select the correct sentence.

A. His acumen in deciphering complex algorithms and applying it in practical scenarios have earned him numerous accolades.

B. His acumen in deciphering complex algorithms and applying them in practical scenarios has earned him numerous accolades.

C. His acumen in deciphering complex algorithms and applying them in practical scenarios have earned his numerous accolades.

D. His acumen in deciphering complex algorithms and applying it in practical scenarios has earned his numerous accolades.

2 marks

Read the passage below and answer Question No. 62.

In the heart of the city, nestled between towering skyscrapers, lies Central Park—a verdant oasis in the midst of urban chaos. This sprawling 843-acre park is a sanctuary for both New Yorkers and visitors alike. Lush meadows, tranquil lakes, and winding paths offer respite from the hustle and bustle of city life.

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Central Park is not only a haven for nature enthusiasts, but also a cultural hub. Its iconic landmarks, such as the Bethesda Terrace and Bow Bridge, have been featured in countless films and novels. The park's extensive collection of sculptures and public art installations enriches the visitor's experience, providing a glimpse into the vibrant art scene of the city.

Throughout the year, Central Park plays host to a variety of events, from open-air concerts to Shakespearean performances. Families gather for picnics, children ride carousels, and joggers traverse the network of trails. It's a place where diversity thrives, where people from all walks of life come together to share in the simple joy of being surrounded by natural beauty.

I. What makes Central Park a unique place in the city?
   i. Its proximity to skyscrapers.
   ii. Its extensive collection of sculptures.
   iii. Its location in the heart of the city.

II. Which of the following is NOT mentioned as an activity in Central Park?
   i. Attending open-air concerts.
   ii. Visiting art galleries.
   iii. Jogging on trails.

III. What role does Central Park play in the cultural life of the city?
   i. Its collection of sculptures and public art installations.
   ii. It is a popular spot for picnics and gatherings.
   iii. It is primarily a sanctuary for nature enthusiasts.

62. The correct answers to I, II and III are

A. ii, iii, iii, respectively.

B. i, ii, iii, respectively.

C. iii, ii, i, respectively.

D. iii, ii, ii, respectively.

3 marks

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63. How many days will be there between January 15, 2024 and March 05, 2024 (both the days are inclusive)?

A. 48.

B. 49.

C. 50.

D. 51.

1 mark

64. A cube of side n is painted red on all its sides. It is then cut into n^3 identical cubes. How many of the smaller cubes will have three sides painted?

A. 4.

B. 8.

C. 6.

D. $6n^2$.

1 mark

65. Given below are two statements numbered 1 and 2:

Statement 1: The depositors in the bank lost their money.

Statement 2: The bank was declared bankrupt.

A. Statement 1 is the cause and Statement 2 is the effect.

B. Statement 2 is the cause and Statement 1 is the effect.

C. Either of the two can cause the other.

D. Neither of the two can cause the other.

1 mark

66. Hardik introduces Axar as the son of the only brother of his father's wife. How is Axar related to Hardik?

A. Cousin.

B. Son.

C. Uncle.

D. Son-in-law.

1 mark

67. A watch gains 5 seconds in 3 minutes and was set right at 8 AM. What time will it show at 10 PM on the same day?

A. 10 hours 23 minutes 20 seconds.

B. 10 hours 22 minutes 20 seconds.

C. 10 hours 25 minutes 20 seconds.

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D. 10 hours 26 minutes 20 seconds.

1 mark

68. Choose the set of three statements where the third statement can be logically derived from the preceding two:

1. All Turns are Good.
2. All Cans are Good.
3. Some Turns are Cans.
4. No Vans are Cans.
5. Some Vans are not Good.
6. Some Cans are Good.

A. 2, 1, 3.

B. 1, 3, 6.

C. 3, 4, 2.

D. 4, 2, 5.

2 marks

69. Nine persons A, B, C, D, E, F, G, H, I are watching a movie in a theatre sitting in a row which has nine seats. B is at one end of the row. H is seated adjacent to both F and G. C is to the immediate right of D and at the third place to the right of E. A is immediate left of F. F is at the third place to the left of B.

What is the position of I from the left?

A. 1.

B. 3.

C. 8.

D. 2.

2 marks

70. In the parlance of reliability theory, two components of a system are said to be in series if the system fails whenever any of the components fails, and are said to be in parallel if both of them have to fail for the system to fail. Going by these definitions, identify the correct statement.

A. A motorbike is a system with two tires in parallel.

B. The renal system of a human being is a system with two kidneys in series.

C. Entry into an email account is a system in which the username and the password are two components in series.

D. A committee that is supposed to keep its report confidential till submission is a system with all its members in parallel.

1 mark

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