CHAPTER: Introduction to Euclid's Geometry

Grade: **9**
Board: **CBSE**
Book: **NCERT**

1.

Chapter Summary and Placement

<Read through the following to understand how this chapter is connected with the previous and next grades and also the important concepts covered in it.>

1. Though this chapter is not directly linked with any of the chapters but it shows how theorems of geometry were evolved which indirectly links to geometry of all grades
2. In this chapter Euclid’s approach to geometry and its link to present days Geometry were discussed

2.

LEARNING OUTCOMES

<On completion of the chapter, students will be able to:>

1. Acquire the knowledge of definitions of Euclid’s geometry
2. Differentiate between axioms and theorems
3. Determine Euclid’s definitions and axioms
4. Examine Euclid’s five postulates
5. State equivalent version of Euclid’s fifth postulate

3. TEACHING PLAN

Follow this section for getting ideas on teaching the content covered in the chapter. The subtopic given in the first column with the matching learning outcome can be explained using instructions and resources listed subsequently

Subtopic and Learning Objective #1

Subtopic:Introduction
LO:
Acquire the knowledge of definitions of Euclid’s geometry
1.To make the student to know about definitions

Teacher Activity

Introduce the definitions used by Euclid

Check for understanding:

a) The word geo in Geometry means_____

b) The bricks used in harappa and Mohenjo Daro civilisation is of the ratio_____

Resources Suggested

Blackboard, pen and paper
Resource 1 can be used for explanation

Subtopic and Learning Objective #2

Sub topic:Axioms, postulates and propositions
LO:
Differentiate between axioms and theorems
1. To make student to know about axioms and theorems

Teacher Activity

Introduce axioms, postulates and propositions

Check for understanding:

a) Differentiate between axiom and Postulates
b) Theorems or otherwise called as _____

Resources Suggested

Blackboard, pen and paper
Resource 2 can be used for explanation

Subtopic and Learning Objective #3

Sub topic:Euclid’s definitions and Axioms
LO:
Determine Euclid’s definitions and axioms
1. To make student to know about definitions and axioms proposed by Euclid

Teacher Activity

Introduce definitions and axioms of Euclid with explanation

Check for understanding:

a) Write any 3 axioms

b) Write any three definitions and identify the word which need predefinition

Resources Suggested

Blackboard, pen and paper
Resource 3 and 4 can be used for explanation

Subtopic and Learning Objective #4

Sub topic: Euclid’s Postulates
LO:
Examine Euclid’s five postulates
1. To make student to learn about Euclid’s postulates

Teacher Activity

Introduce Euclid’s postulates and examples of NCERT text book

Check for understanding:

a) State Euclid’s postulates

Resources Suggested

Blackboard, pen and paper
Resource 5 and 7 can be used for explanation

Subtopic and Learning Objective #5

Sub topic:Equivalent version of Fifth postulate
LO:
State equivalent version of Euclid’s fifth postulate
1. To make student to know about equivalent version of fifth postulate

Teacher Activity

Introduce equivalent version of Euclid's fifth postulate

Check for understanding:

a) What are the two equivalent versions of Euclid's fifth postulate.

Resources Suggested

Blackboard, pen and paper
Resource 6 can be used for explanation

Resource 1

Introduction

The word ‘geometry’ came from Greek words ‘geo’, meaning the ‘earth’, and ‘metrein’, meaning ‘to measure’. Geometry appears to have originated from the need for measuring land. This branch of mathematics was studied in various forms in every ancient civilization.

The people of these civilizations faced several practical problems which required the development of geometry in various forms

Egyptians: River Nile used to overflow and whashed out boundaries between adjoining fields of different land owners every year. After the flooding, these boundaries had to be redrawn. For this purpose the Egyptians developed a number of geometric Techniques and rule for calculating simple area and also for Simple construction. They used the knowledge of geometry to construct canal, granaries, pyramids etc.

Indians: Indian subcontinent excavations at Harappa and Mohenjo-Daro show that Indus Valley Civilization (3000 BC) made extensive use of Geometry. It was highly planned society. Roads were parallel to each other and had drainage system, this prove that town dwellers were skilled in Geometry and Mensuration. The bricks used for constructions were kiln fired and the ratio length : breadth : thickness, of the bricks was found to be 4 : 2 : 1.

Resource 1

The geometry was being developed and applied everywhere in the world, but it was happening in unsystematic manner. These geometry knowledge was passed from one generation to other orally or through palm leaf messages. Iraq or Babylonia, Egyptians and Indians used geometry for practical purpose and did a very little to develop it as a systematic Science.

E.g. They accepted the fact that diameter of a circle divides the circle into 2 half, but never thought of finding out why?

But, in Civilization like Greece, the emphasis was on the reasoning. Greeks were interested in establishing the truth of statements they discovered by deductive reasoning. Greek mathematician Thales is credited with giving first known theorem called Thales theorem in around 600BC which gives Proof for why Circle is bisected by its diameter. Other popular theorem by Greeks is Pythagoras Theorem in 572 BC.

Euclid compiled these entire existing theorems in a book called “The Elements”.

Euclid compiled the “The elements” in 300 BC. He divided “the elements” into 13 chapters, each chapter called a book. It is the second most printed book in the world after bible.

Resource 2

The term ‘Postulate’ was used by Euclid for an assumption in geometry, while the word ‘axiom’ on the other hand, were the assumptions used throughout mathematics and not specifically linked with geometry.
After stating Postulates and Axioms, Euclid used these to prove other results by applying deductive reasoning. These statements that were proved are called Propositions or theorems. Euclid deduced 465 Theorems.
In a nutshell

1. **Axioms:** Common to all domains. E.g.: Whole is greater than Part
2. **Postulates:** Specific to Geometry. E.g.: A circle can be drawn using a center and a radius.
3. **Propositions:** Statements that were proved or theorems.E.g: “Two distinct lines cannot have more than one point in common”

Resource 3

Euclid's Definition:

Euclid gave more than 100 definitions in the book elements. Few of them are:

1. A point is that which has no part.
2. A line is breadthless length.
3. A straight line is a line which lies evenly with the points on itself.
4. A surface is that which has length and breadth only.
5. A plane surface is a surface which lies evenly with the straight lines on itself.
6. The extremities of a surface are lines.
7. An obtuse angle is an angle greater than a right angle.
8. An acute angle is an angle less than a right angle.
9. A boundary is that which an extremity of anything is.
10. A figure is that which is contained by any boundary or boundaries.
11. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another; and the point is called the center of the circle.
12. A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the center.
13. A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

Resource 3

1. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in either direction, do not meet one another in either direction.
2. Equal circles are those the diameters of which are equal, or the radii of which are equal.
3. An angle of a segment is that contained by a straight line and a circumference of a circle.
4. An extremity of a solid is a surface.
5. Parallel planes are those which do not meet.
6. A cube is a solid figure contained by six equal squares.
7. A sector of a circle is the figure which, when an angle is constructed at the center of the circle, is contained by the straight lines containing the angle and the circumference cut off by them.
8. A solid is that which has length, breadth, and depth.

Euclid has used terms like part, length, breadth etc without defining it.
E.g A point is that which has no **part**. Here part is not defined.
Even if we define Part as something that occupied area, again we have to define new term; in this case it is area.
To define, many things have to be predefined and the chain is endless. Thus mathematicians agreed to leave some geometric terms as undefined.
Thus we take **point, line and plane** as undefined terms.

Resource 4

Axioms:

Some of the axioms are

1. “Whole is greater than part”. This statement is true in physics, chemistry, mathematics, geometry, biology, economics etc.
2. “Things which are equal to same thing are equal to each other”E.g.: if A=B and C=B. that is both A and C are equal to B, then A and C will be equal.
3. “If equals are added to Equals, whole are equal”. E.g. it is known than two glass A and A’ has same volume of water. Now we add equal quantity of water B, to both glass A and A’, then the final volume of water in the jar will be same.That is if A= A’ then A+ B = A’ + B.
4. “If equals are subtracted from Equals, then the remainders are equal”. E.g. it is known than two glass A and A’ has same volume of water. Now we remove equal quantity of water B, from both glass A and A’, then the final volume of water in the jar will be same. That is if A= A’ then A - B = A’ - B.
5. “Things which are double of same things are equal to one another”.E.g.: If A= A’ then 2A= 2A’.
6. “Things which are halves of same things are equal to one another”. E.g.: If A= A’ then ½ A= ½ A’.
7. “Things that coincide with one another are equal to one another”. E.g. If two line segments coincide with each other then they are equal.

Resource 5

Postulates:

There are 5 postulates given by Euclid in his 1st book (chapter) of the elements.

1. A unique straight line may be drawn from any point to any other point.

According to this postulate at least one straight line passes through two distinct points, but it does not say that there cannot be more than one such line. In his work Euclid frequently assumed without mentioning, that there is a unique line joining two points



In the above figure, the only line that passes through P and Q is line PQ

2 Every limited straight line can be extended indefinitely to a unique straight line.

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Resource 5

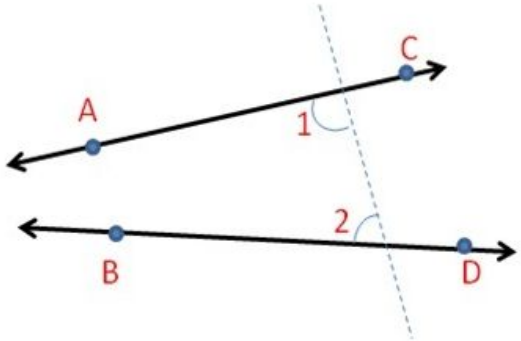


According to present day terms, this postulate tells us that a line segment can be extended on both the sides to get a line

1. A circle may be drawn with any center and any distance as radius.
2. All right angles are equal.
3. If a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, then the two lines (if extended indefinitely) will meet on that side on which the angles are less than two right angles.

In figure below if $\angle 1 + \angle 2 < 180^0$, then if AC and BD are extended, they will meet at a point on the side of A and B.

Resource 5



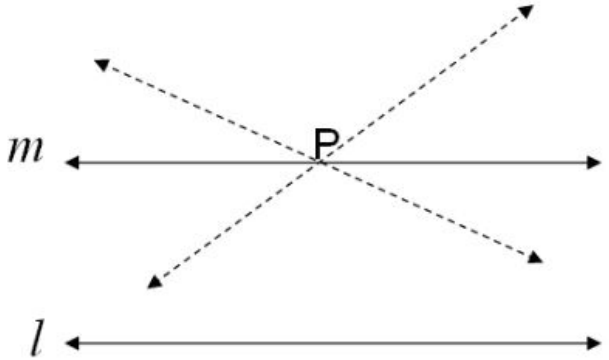
Resource 6

Equivalent version of 5th Postulate:

Two equivalent versions of Euclid's fifth postulate are:

(i) For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l'.

From the following figure, we can see that of all the lines passing through the point P, only line
m is parallel to line l.



Resource 6

This result can also be stated in the following form:
Two distinct intersecting lines cannot be parallel to the same line.

(ii) Two distinct intersecting lines cannot be parallel to the same line

Many attempts were made to prove Euclid’s fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.

Resource 7

Theorem 5.1 : Two distinct lines cannot have more than one point in common.

Proof : Here we are given two lines l and m. We need to prove that they have only one point in common.
let us suppose that the two lines intersect in two distinct points, say P and Q.
So, you have two lines passing through two distinct points P and Q.
But this assumption clashes with the axiom that only one line can pass through two distinct
points.
So, the assumption that we started with, that two lines can pass through two distinct points is wrong.
From this, we can conclude that two distinct lines cannot have more than one point in common.

4. WORKSHEETS

Print or copy the worksheet questions on the blackboard and ask the students to attempt them. They can be given as homework or a class test as well.

Worksheet 1

Use it like: test after the chapter is complete, homework

Answer the following:

1. The three steps from solids to points are :
   (A) Solids - surfaces - lines - points
   (B) Solids - lines - surfaces - points
   (C) Lines - points - surfaces - solids
   (D) Lines - surfaces - points - solids
2. The number of dimensions of solid is _____
3. The number of dimensions of surface is _____
4. A point has_____ dimension
5. Euclid divided his famous treatise “The Elements” into _____Chapters
6. The total number of propositions in the Elements are _____
7. Boundaries of solids are_____
8. Boundaries of surfaces are _____
9. In Indus Valley Civilisation (about 3000 B.C.), the bricks used for construction work were having dimensions in the ratio_____
10. It is known that if $x + y = 10$ then $x + y + z = 10 + z$.
    The Euclid’s axiom that illustrates this statement is

Worksheet 2

11.Euclid belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) India

12. Which of the following needs a proof ?

(A) Theorem (B) Axiom (C) Definition (D) Postulate

13. Euclid stated that all right angles are equal to each other in the form of

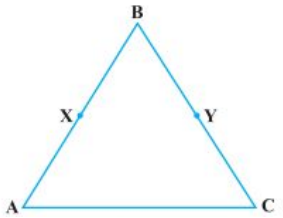
(A) an axiom (B) a definition (C) a postulate (D) a proof

Worksheet 3

Use it like: test after the chapter is complete, homework

Answer the following:

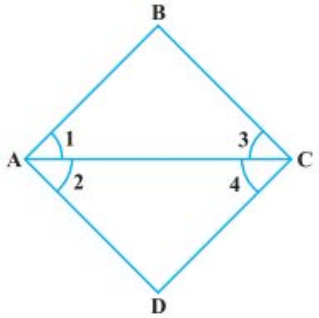
1. In the Figure, we have AB = BC, BX = BY. Show that AX = CY.



2. In the figure,

a) we have ∠1 = ∠2, ∠2 = ∠3. Show that ∠1 = ∠3.

b) we have ∠1 = ∠3 and ∠2 = ∠4. Show that ∠A = ∠C



5. SOLUTIONS

This section gives you sample answers for activities and questions listed above.

Worksheet 1

1. A
2. 3
3. 2
4. Zero or no
5. 13
6. 465
7. Surfaces
8. Lines
9. 4:2:1
10. Equals added to equals, wholes are equal
11. B
12. A
13. C

Worksheet 2

Need to prove by using following hints:

1. By using axiom ‘If equals are subtracted from Equals, then the remainders are equal’ it can be proved
2.

a) By using axiom ‘Things which are equal to same thing are equal to each other’ it can be proved

b) By using axiom“If equals are added to Equals, whole are equal”. It can be proved

6. EXTRA RESOURCES

Carry out the following suggested activities/ tasks if you have the time and resources to do them

Teacher can use these videos if time and facilities permit:

1. https://www.youtube.com/watch?v=CYQps3_1-H0
2. https://www.youtube.com/watch?v=wQAiytWGgEk