MODEL QUESTION PAPER 2027

MATHEMATICS AND STATISTICS

Std. XII (H.S.C.) · Maharashtra State Board

Time: 3 Hours **Max. Marks: 80**

General instructions:

(1) All questions are compulsory.
(2) The numbers to the right indicate full marks.
(3) Use of a logarithmic table is allowed; use of a calculator is not allowed.
(4) For each MCQ, only the first attempt will be evaluated.
(5) Start each section on a new page.

SECTION A

Q. 1. Select and write the correct answer of the following (each 2 marks): **16**

(i) The approximate value of the function $f(x) = x^3 - 3x + 5$ at $x = 1.99$ is ____. (a) 6.09 (b) 6.91 (c) 7.09 (d) 7.91

(ii) If $f(x) = \sin^{-1} x/\sqrt{(1 - x^2)}$ and $g(x) = e^{(\sin^{-1} x)}$ then integral f(x).g(x) dx = ____. (a) $e^{(\sin^{-1} x)}(1 - \sin^{-1} x) + c$ (b) $e^{(\sin^{-1} x)}(\sin^{-1} x - 1) + c$ (c) $e^{(\sin^{-1} x)}(\sin^{-1} x + 1) + c$ (d) $e^{(\cos^{-1} x)}(\cos^{-1} x - 1) + c$

(iii) The value of $\sin[\sin^{-1}(-1/\sqrt{2}) - 3\sin^{-1}(\sqrt{3}/2)]$ = ____. (a) $-1/\sqrt{2}$ (b) $1/\sqrt{2}$ (c) $\sqrt{3}/2$ (d) $-\sqrt{3}/2$

(iv) Write the dual of $p \wedge t$.

(v) Find the angle between the lines $r = (i + 2j + 3k) + \lambda(2i - j + k)$ and $r = (i + 2j + 3k) + \mu(i + 2j + k)$.

(vi) In a Binomial Distribution, E(X) = 6, Var(X) = 4.2 then the value of n is ____. (a) 22 (b) 24 (c) 20 (d) 26

(vii) If $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$, then x = ____. (a) -1 (b) 1/6 (c) 1/3 (d) 3/2

(viii) If the p.d.f. of a continuous r.v. X is $f(x) = (x + 2)/18$, for $-2 < x < 4$, = 0 otherwise then $P(|X| < 1)$ = ____. (a) 1/9 (b) 2/9 (c) 1/27 (d) 2/27

Q. 2. Answer the following (each 1 mark): **4**

(i) Evaluate: $\int e^x(1 + x)/\cos^2(x e^x)\, dx$.

(ii) Find the general solution of equation $\cos 5\theta = 1/2$.

(iii) Write the order of differential equation $(d^{2y}/dx^2)^{(5/2)}$ = cube root of (dy/dx).

(iv) Evaluate: $\int 1/(25 - 9x^2)\, dx$.

SECTION B

Q. 3 to Q. 14. Attempt any EIGHT of the following (each 2 marks): **16**

(i) Evaluate: integral from 0 to -1 of $e^{(-x)}$ dx.

(ii) Using truth table, determine whether statement pattern $[(p \vee q) \wedge \sim p] \wedge \sim q$ is a tautology or a contradiction or a contingency.

(iii) If a, b, c are the position vectors of points A, B, C respectively and 10a = 7b + 3c then find the ratio in which the point C divides the line segment AB.

(iv) Find the area bounded by curve $x^2 + y = 0$, X-axis and lines x = 1 and x = 4.

(v) Using truth table, prove that $\sim p \wedge q = (p \vee q) \wedge \sim p$.

(vi) Solve the D.E. $3e^x \tan y\, dx + (1 + e^x) \sec^2 y\, dy = 0$.

(vii) Construct the switching circuit for the statement $(\sim p \wedge q) \vee (p \wedge \sim r)$.

(viii) Test whether the function $f(x) = x - 1/x$, $x \in R$, $x \neq 0$ is increasing or decreasing.

(ix) Find dy/dx, if $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

(x) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 9$, $x \in [1, 4]$.

(xi) A stone is dropped into a quiet lake and waves in the form of circles are generated. Radius of the circular wave increases at the rate of 3 cm/sec. How fast the area enclosed is increasing when the radius is 8 cm?

(xii) If A = [[2, -4], [3, 1]], then the adjoint of matrix A is ____. (a) [[1, 3], [4, -2]] (b) [[-1, 3], [-4, 1]] (c) [[-1, -3], [-4, 2]] (d) [[1, 4], [-3, 2]]

SECTION C

Q. 15 to Q. 26. Attempt any EIGHT of the following (each 3 marks): **24**

(i) A pair of dice is thrown 4 times. If getting a doublet is considered as success, find the probability of two successes.

(ii) Solve the differential equation x dy/dx - sqrt(x^2 + y^2) = y.

(iii) Prove that: $\sin^{-1}(3/5) + \cos^{-1}(12/13) = \sin^{-1}(56/65)$.

(iv) Find the shortest distance between the lines, (x-1)/2 = (y-2)/3 = (z-3)/4 and (x-2)/3 = (y-4)/4 = (z-5)/5.

(v) If $y = \sin^{-1} x$, then show that: $(1 - x^2)\, d^{2y}/dx^2 - x\, dy/dx = 0$.

(vi) Find the vector equation of the plane passing through the points A(2, 1, 1), B(0, 2, 3) and C(4, 5, 6).

(vii) Find E(X) and V(X), where X is the number obtained on uppermost face, when a fair die is thrown.

(viii) Are the four points A(1, 2, 1), B(2, -3, 4), C(3, 4, -5), D(2, 3, -2) coplanar? Justify your answer.

(ix) In triangle ABC, prove that $a(b \cos C - c \cos B) = b^2 - c^2$.

(x) The angle between the line r = (i + 2j + k) + lambda(i + j + k) and the plane r.(2i - j + k) = 8 is ____. (a) $\sin^{-1}(\sqrt{2}/3)$ (b) $\sin^{-1}(\sqrt{3}/2)$ (c) $\sin^{-1}(1/2)$ (d) $\sin^{-1}(1/\sqrt{2})$

(xi) Verify LMVT for the function f(x) = log x, on [1, e].

(xii) The probability distribution of X is as follows: x: 0, 1, 2, 3, 4; P[X=x]: 0.1, k, 2k, 2k, k. Find (i) k, (ii) P(X < 2), (iii) P[1 <= X < 4].

SECTION D

Q. 27 to Q. 34. Attempt any FIVE of the following (each 4 marks): **20**

(i) Solve the linear programming problem (L.P.P.) by graphical method. Minimize z = 8x + 10y Subject to 2x + y >= 7, 2x + 3y >= 15, x >= 0, y >= 2.

(ii) If x = f(t) and y = g(t) are differentiable functions of t, so that y is differentiable function of x and dx/dt != 0 then prove that dy/dx = (dy/dt)/(dx/dt). Hence find dy/dx, if x = sin t, y = cos t.

(iii) Prove that the homogeneous equation of degree two in x and y, $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab >= 0$.

(iv) integral from 1 to 2 of $(1/x^2)$. e^(1/x) dx = ____. (a) sqrt(e) + 1 (b) sqrt(e) - 1 (c) sqrt(e)(sqrt(e) - 1) (d) (sqrt(e) - 1)/e

(v) Prove that: integral from a to b of f(x) dx = integral from a to b of f(a + b - x) . dx. Hence, find integral from pi/6 to pi/3 of $\sin^2 x$. dx.

(vi) A body cools according to Newton's law from 100 deg C to 60 deg C in 20 minutes. The temperature of the surrounding being 20 deg C. How long will it take to cool down to 40 deg C?

(vii) If A = [[1, 2], [3, 4]], prove that A.(adj A) = (adj A).A = |A|.I.

(viii) Evaluate: integral $(3x^2 + 4x - 5)/((x^2 - 1)(x + 2))$. dx.