Institute of Actuaries of India

ACET June 2023

Mathematics

1. The set of values of x satisfying $\frac{x}{2} \geq \frac{3x-2}{5} - \frac{x+1}{2}$ is

   A. $-\frac{9}{4} \leq x < \infty$.

   B. $-\frac{9}{4} < x < \infty$.

   C. $-\frac{9}{4} \leq x \leq 0$.

   D. $-\frac{9}{4} < x < 0$.

   2 marks

2. A class in a college has 70 students. Among them, 48 students have opted for mathematics as optional and 32 students have opted for physics as optional. Each student has opted for at least one of these two subjects. The number of students who have opted for both mathematics and physics is

   A. 60.

   B. 32.

   C. 16.

   D. 10.

   1 mark

3. If $f(x) = 2x + 1$, Then

   A. $f^{-1}(x) = \frac{x}{2} - 1$.

   B. $f^{-1}(x) = \frac{x-1}{2}$.

   C. $f^{-1}(x) = 2x - 1$.

   D. $f^{-1}(x) = \frac{1}{2x+1}$.

   1 mark

4. The principal value of $\text{cosec}^{-1}\sqrt{2}$ is

   A. $\frac{\pi}{2}$.

   B. $\frac{\pi}{3}$.

   C. $\frac{\pi}{4}$.

   D. $\frac{\pi}{6}$.

   1 mark

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5. The coefficient of x^5 in the expansion of $\left(x-\frac{1}{x}\right)^{15}$ is

A. $\binom{15}{5}$.

B. $-\binom{15}{5}$.

C. 1.

D. -1.

1 mark

6. The value of $\log_3 x + \log_9 x + \log_{27} x = \frac{11}{3}$. Then, the value of x is

A. 39.

B. 27.

C. 9.

D. 3.

2 marks

7. If x and y are real and if $(x+iy)(5+4i)$ is the conjugate of $-4+12i$, then x and y are, respectively,

A. $\frac{28}{41}, \frac{76}{41}$.

B. $\frac{76}{41}, -\frac{28}{41}$.

C. $-\frac{68}{41}, \frac{44}{41}$.

D. $-\frac{68}{41}, -\frac{44}{41}$.

3 marks

8. The first and third terms of a Harmonic Progression (HP) are $\frac{1}{3}$ and $\frac{1}{7}$, respectively. Then the sum of 15 terms formed by the reciprocals of HP is

A. 245.

B. 255.

C. $\frac{465}{2}$.

D. 265.

2 marks

9. If $x^2 - x + 3\mu = 0$ and $4x^2 - 5x + \mu = 0$, $(\mu \neq 0)$ have the common root α, then α is equal to

A. $\frac{14}{11}$.

B. $\frac{11}{14}$.

C. $-\frac{14}{11}$.

D. 0.

3 marks

10. The following table gives the value $\Phi(x)$, for three different values of x from standard

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normal table.

x	1.0	1.1	1.2
$\Phi(x)$	0.8413	0.8643	0.8849

Assuming $\Phi(x)$, to be linear in between the successive values of x given above, the value of $\Phi(1.05)$ is

A. 0.8320.

B. 0.8852.

C. 0.8528.

D. 0.8621.

1 mark

11. The function $f(x) = xe^x$; $x > 0$ has the following tabulated values.

x	0	0.25	0.5	0.75	1.00
$f(x)$	0	0.3210	0.8244	1.5878	2.7828

The approximate value of the integral $\int_0^1 f(x)dx$ by Trapezoidal rule is

A. 1.37895.

B. 0.689475.

C. 0.9421.

D. 1.03115.

2 marks

12. The value of $\lim_{x\to 5} \frac{e^x - e^5}{x-5}$ is

A. e^5.

B. e^{-5}.

C. $\log_e 5$.

D. 1.

1 mark

13. If $y = e^{x^3 - \frac{1}{2}\log_e x}$, then $\frac{dy}{dx}$ is equal to

A. $e^{3x^2 - \frac{1}{2x}}\left(3x^2 - \frac{1}{2x}\right)$.

B. $e^{x^3}\frac{1}{\sqrt{x}}\left(3x^2 - \frac{1}{2x}\right)$.

C. $e^{x^3}\sqrt{x}\left(3x^2 - \frac{1}{2x}\right)$.

D. $e^{3x^2 - \frac{1}{2x}}$.

2 marks

14. The interval in which the function $f(x) = x^3e^{-2x}$ is strictly increasing in x is

A. $\left(-\infty, -\frac{3}{2}\right)$.

B. $\left(-\frac{3}{2}, \infty\right)$.

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C. $\left(-\infty, \frac{3}{2}\right)$.

D. $\left(\frac{3}{2}, \infty\right)$.

1 mark

15. The value of the integral $\int_4^5 5^x dx$ is

A. $\frac{2500}{\log_e 5}$.

B. $2500 \log_e 5$.

C. $\frac{1}{\log_e 5}$.

D. $\log_e 5 - \log_e 4$.

2 marks

16. The value of $\int \left(3x^2 - 4\tan x + 3\sqrt{x}\right)\, dx$ is

A. $x^3 + 4\log(\cos x) - 2x\sqrt{x} + C$.

B. $x^3 - 4\log(\cos x) - 2x\sqrt{x} + C$.

C. $x^3 - 4\log(\cos x) + 2x\sqrt{x} + C$.

D. $x^3 + 4\log(\cos x) + 2x\sqrt{x} + C$.

1 mark

17. The inverse of $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ is

A. $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$.

B. $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$.

C. $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

D. $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$.

1 mark

18. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, and if $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, then θ is equal to

A. $\frac{\pi}{2}$.

B. $\frac{\pi}{3}$.

C. $\frac{\pi}{4}$.

D. $\frac{\pi}{6}$.

1 mark

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19. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$ and if $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \circ \vec{b} + \vec{b} \circ \vec{c} + \vec{c} \circ \vec{a}$ is equal to

A. -7.

B. 7.

C. 14.

D. -14.

1 mark

20. If

$$M = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix},$$

then the rank of MM^T (T denotes the transpose) is

A. 0.

B. 3.

C. 2.

D. 1.

1 mark

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21. The weighted arithmetic mean of 13, 15, 18, 20 with weights w_1, w_2, w_3 and w_4 respectively, is known to be 16. If the weights are changed to $4w_1, 4w_2, 4w_3$ and $4w_4$, then the weighted arithmetic mean is

A. 4.

B. 16.

C. 64.

D. in between 16 and 64.

1 mark

22. The mean of the numbers $1, 3, 4, 4, 5, 7$ is M. The numbers $2, 2, 2, 3, 3, 4, p$ have mean $M - 1$ and median q. Then p and q are

A. 4.5 and 3.

B. 5 and 2.5.

C. 5 and 3.

D. 4 and 3.

1 mark

23. If the random variables X and Y are such that the arithmetic mean of $7, 7, 8, X, 10, 12, Y$ is 11, then the correlation coefficient between X and Y is

A. -1.

B. 0.

C. 0.5.

D. 1.

1 mark

24. Marks obtained by 70 students in a subject are given in the following table.

Marks	20	70	50	60	75	90	40
Number of students	8	12	18	6	9	5	12

The median and mode marks are

A. $50, 90$.
B. $60, 50$.
C. $55, 50$.
D. $50, 50$.

1 mark

25. If $P(E) = \frac{2}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$, then $P(\bar{E}|\bar{F}) \times P(\bar{F}|\bar{E})$ is equal to

A. $\frac{5}{6}$.

B. $\frac{5}{7}$.

C. $\frac{25}{42}$.

D. 1.

1 mark

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26. If E and F are independent events such that $P(E) = p$, $P(F) = 2\,P(E)$ and probability that exactly one of E and F occurs is $\frac{5}{9}$, then the possible values of p are

A. $\frac{2}{5}, \frac{1}{4}$.

B. $\frac{1}{9}, \frac{5}{12}$.

C. $\frac{1}{5}, \frac{5}{12}$.

D. $\frac{1}{3}, \frac{5}{12}$.

1 mark

27. Let X be a random variable with probability density function

$$f(x) = \frac{1}{2\sqrt{2\,\pi}} e^{-\frac{1}{8}(x^2-20x+100)}, \qquad -\infty < x < \infty$$

The mean, median and mode of the distribution are

A. 8,8,8.

B. 10,10,10.

C. 8,10,12.

D. 10,8,10.

1 mark

28. Suppose X is a random variable with probability density function

$$f(x) = \frac{1}{\lambda^2} x e^{-\frac{x}{\lambda}}, \;\; x > 0, \lambda > 0.$$

If 2 is the unique mode of the distribution, then λ equals

A. 2.

B. 3.

C. 4.

D. 5.

1 mark

29. The lifetime X (in years) of a memory chip in a laptop computer has the distribution with probability density function

$$f(x) = \begin{cases} \frac{1}{8} x e^{-\frac{x^2}{16}}, & 0 \le x < \infty \\ 0, & \text{otherwise} \end{cases}$$

The probability that a memory chip will not fail before 6 years is

A. $1 - e^{-2.25}$.

B. $1 - e^{-2}$.

C. e^{-2}.

D. $e^{-2.25}$.

1 mark

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30. The number of chocolate chips in a certain type of cookie have a Poisson distribution. The probability that a cookie of this type contain at least one chocolate chip is $1 - e^{-4.2}$. The probability that a randomly selected cookie contains exactly one chocolate chip is

A. $e^{-4.2}$.
B. $1 - e^{-4.2}$.
C. $4.2e^{-4.2}$.
D. $1 - 4.2e^{-4.2}$.

1 mark

31. Let X be a random variable with probability mass function

$$p(x) = \begin{cases} \frac{1}{4}, & \text{if } x = 1, 2, \\ \frac{1}{4} + \frac{p}{4}, & \text{if } x = 3, \\ \frac{1}{4} - \frac{p}{4}, & \text{if } x = 4, \\ 0 & \text{otherwise.} \end{cases}$$

If $E(X) = 2.4$, then p equals

A. 0.10.
B. 0.40.
C. 0.48.
D. 0.84.

1 mark

32. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{\phi(x)}{\Phi(b) - \Phi(a)}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

where $\phi(x)$ and Φ are the probability density function and cumulative distribution function of standard normal distribution. Then $E(X)$ equals

A. $\frac{\phi(a) - \phi(b)}{\Phi(b) - \Phi(a)}$.
B. $\frac{\phi(b) - \phi(a)}{\Phi(b) - \Phi(a)}$.
C. $\frac{\phi(a) + \phi(b)}{2[\Phi(b) - \Phi(a)]}$.
D. 0.

1 mark

33. If the letters of the word '*ANACONDA*' are written at random, the probability that no two *A*'s occur together is

A. $\frac{3}{14}$.
B. $\frac{5}{14}$.
C. $\frac{5}{84}$.
D. $\frac{1}{12}$.

2 marks

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34. Suppose A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. The probability that exactly two of A, B and C will hit the target, when the three of them independently hit one shot each, is

A. $\frac{47}{120}$.

B. $\frac{27}{120}$.

C. $\frac{2}{3}$.

D. $\frac{1}{2}$.

2 marks

35. An insurance company insured 1000 scooters, 2000 cars and 3000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0.02, 0.03 and 0.04, respectively. One of the insured vehicles meets with an accident. The conditional probability that it is a truck is

A. 0.33.

B. 0.04.

C. 0.5.

D. 0.6.

2 marks

36. Let X have the probability mass function

$$p(x) = \frac{1}{4}\left(\frac{3}{4}\right)^x, \qquad x = 0, 1, 2, \ldots$$

Then $P(X = 3|X \geq 3)$ equals

A. $\frac{1}{4}$.

B. $\frac{27}{64}$.

C. $\frac{27}{256}$.

D. $\frac{1}{2}$.

2 marks

37. Let $X \sim Binomial\,(2, p)$ and $Y \sim Binomial(4, p)$. If $P(X \geq 1) = \frac{5}{9}$, then $P(Y \geq 1)$ is

A. $\frac{16}{81}$.

B. $\frac{25}{81}$.

C. $\frac{65}{81}$.

D. $\frac{4}{9}$.

2 marks

38. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

The correlation coefficient between X and X^4 is

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A. -1.

B. 0.

C. 0.5.

D. 1.

2 marks

39. The relationship between x and y is $y + 2x = 0$, and that between u and v is $v = 6 + 3u$. If the regression coefficient of x on u is 2.4, then the regression coefficient of y on v is

A. -0.4.

B. -1.6.

C. 0.8.

D. 1.2.

3 marks

40. Let X_1 and X_2 have the joint probability density function

$$f(x_1, x_2) = \begin{cases} k(x_1 + x_2), & 0 < x_1 < 1, \ 0 < x_2 < 1, \\ 0, & \text{otherwise,} \end{cases}$$

where k is constant. Define a random variable

$$Z = \begin{cases} 1, & \text{if } X_1 + X_2 \leq 1, \\ 0, & \text{if } X_1 + X_2 > 1. \end{cases}$$

Then $E(Z)$ equals

A. $\frac{2}{3}$.

B. $\frac{3}{5}$.

C. $\frac{1}{3}$.

D. $\frac{4}{5}$.

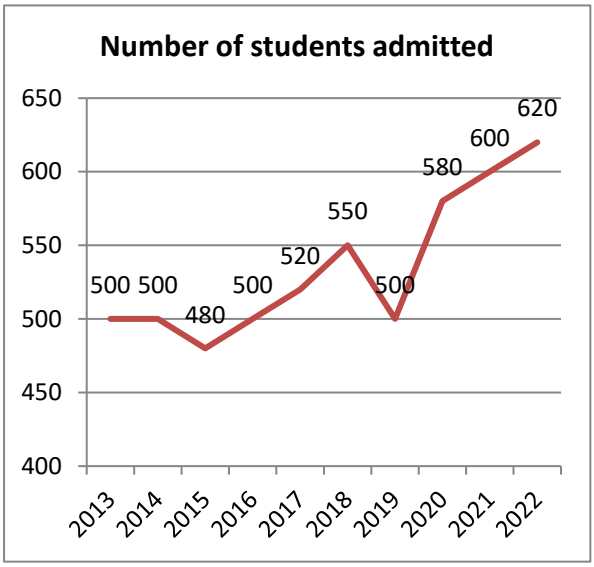
3 marks

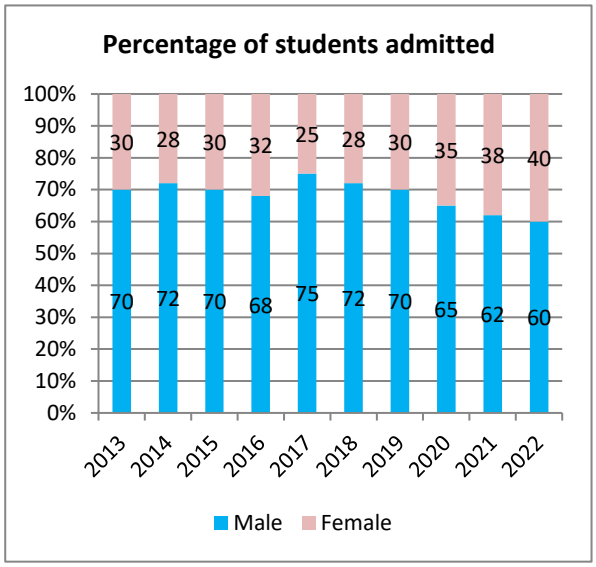
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Data Interpretation

The following graph and bar chart give the number of students admitted to different courses of a University and the percentages of male and female students for 10 years from 2013 to 2022. Answer questions 41-44 based on the information given in this table.





41. The number of years in which percentage of male students is less than twice that of female students is

    A. 0.

    B. 1.

    C. 2.

    D. 3.

1 mark

42. The number of male students admitted is highest in the year

    A. 2017.

    B. 2018.

    C. 2019.

    D. 2020.

1 mark

43. The minimum and maximum number of female students were admitted in the years

    A. 2014 and 2020.

    B. 2015 and 2021.

    C. 2017 and 2022.

    D. 2019 and 2022.

2 marks

44. The years in which the number of male students admitted is more than 2.5 times the number of female students are

    A. 2013, 2014 and 2017.

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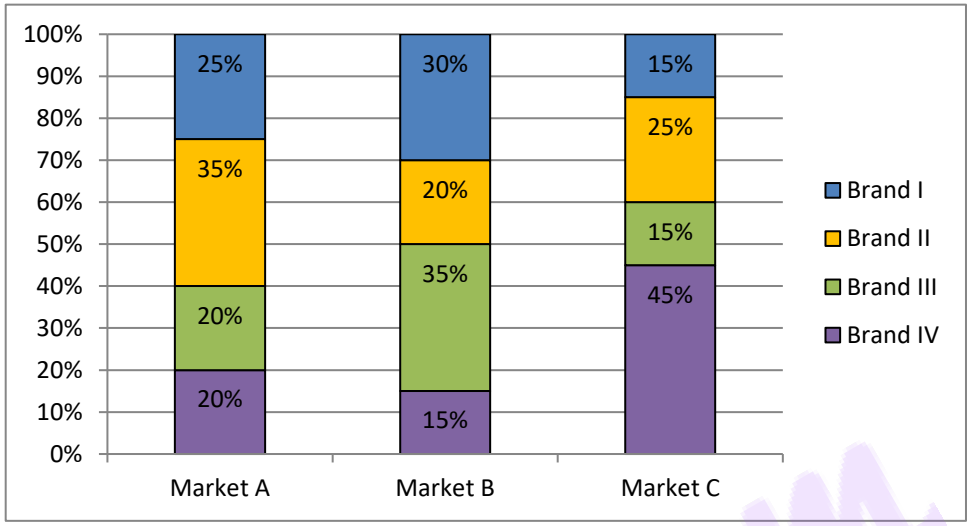
B. 2014, 2015 and 2018.

C. 2015, 2018 and 2019.

D. 2014, 2017 and 2018.

2 marks

The composite column chart given below show the market shares of four brands of a particular product in three different markets. Market A is twice as large as Market C and half as large as Market B in value. Answer questions 45-48 based on this information.



45. Which of the market shares described below is the smallest?

A. Share of Brand I in Market C.

B. Share of Brand II in Market B.

C. Share of Brand I in Market A.

D. Share of Brand IV in Market A.

1 mark

46. Identify the correct one among the statements given below.

A. Business value of Brand IV is more in Market A than in Market B.

B. Business value of Brand I is more in Market C than in Market A.

C. Business value of Brand II is more in Market B than in Market A.

D. Business value of Brand III is more in Market C than in Market A.

1 mark

47. The value share of Market B in the combined market (A, B and C) is

A. 28.6%.

B. 33.3%.

C. 14.3%.

D. 57.1%.

1 mark

48. The brand having the largest value share in the combined market (A, B and C) is

A. Brand I.
B. Brand II.
C. Brand III.
D. Brand IV.

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2 marks

The following table gives proficiency in mathematics and proficiency in reading among adolescents for 20 countries. Answer questions 49-51 based on the information in the table.

Sr. No.	Country	Proficiency in math		Proficiency in reading	
		Boys	Girls	Boys	Girls
1	Singapore	92	95	86	92
2	South Korea	84	86	81	89
3	Vietnam	79	83	81	91
4	Israel	63	69	60	77
5	Malaysia	56	60	47	61
6	UAE	45	48	46	68
7	Thailand	43	51	31	49
8	Canada	84	84	82	90
9	Finland	83	87	80	93
10	UK	82	80	79	86
11	Belgium	82	79	75	82
12	Sweden	81	82	77	86
13	Germany	79	79	76	84
14	France	79	79	75	84
15	New Zealand	79	78	77	86
16	Russia	78	78	73	83
17	Australia	78	77	76	85
18	Portugal	77	77	76	84
19	USA	74	72	77	85
20	Italy	64	61	72	81

49. The country where proficiency in reading is better for boys than girls is

A. Canada.
B. South Korea.
C. Thailand.
D. None.

2 marks

50. The number of countries where proficiency in reading is at least 75 both for boys and girls is

A. 15.
B. 16.
C. 14.
D. 18.

1 mark

51. Among the countries with math proficiency level more than 75 for boys and girls, which are the countries where boys performed better than girls?

A. UK, Belgium, New Zealand and Australia.
B. UK, Belgium, USA and Italy.
C. Germany, Finland and Italy.
D. USA, UK, Italy.

1 mark

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52. One who files a suit in a court is called a

A. suitor.
B. plaintiff.
C. accused.
D. victim.

1 mark

53. One who abandons one's religious faith is called a

A. zealot.
B. apostate.
C. fanatic.
D. adherent.

1 mark

54. One who is able to use the left and right hands equally well is called

A. ambidextrous.
B. ambivalent.
C. equivocal.
D. multifaceted.

1 mark

55. Fill the blank:

Ramesh helped the old lady to _____ the bus.

A. mount on.
B. get on.
C. get up.
D. put on.

1 mark

56. Fill the blank:

Rama went to the library to _____ a book.

A. look out.
B. look after.
C. look for.
D. look up to.

1 mark

57. Fill the blank:

The pupils _____ talking when the teacher arrives.

A. carry on.

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B. get on.

C. break out.

D. remain on.

1 mark

58. Fill the blank:

I've been taking English lessons for three months but I _____ haven't made much progress!

A. already.

B. still.

C. always.

D. yet.

1 mark

59. Fill the blank:

_____ earning a low salary, Linda gave money to her parents.

A. Although.

B. Even.

C. In spite of.

D. Though.

1 mark

60. Choose the meaning of underlined phrase in the following sentence:

His investments helped him make a killing in the stock market.

A. Lose money quickly.

B. Plan a murder.

C. Murder someone.

D. Make money quickly.

2 marks

61. Choose the meaning of underlined phrase in the following sentence:

I've got a busy day tomorrow, so I think I'll hit the sack.

A. Get prepared.

B. Go to sleep.

C. Take a bath.

D. Start packing.

2 marks

Read the passage below and answer Question No. 62.

Mathematician John von Neumann and economist Oskar Morgenstern gave a revolutionary new idea called Game Theory in the 1940s. It used the established mathematical models to develop viable strategies for the best outcome for all participants (referred to as players). In addition to mathematics, it also applies principles of social sciences and psychology in real-

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world market situations. A few classical scenarios for research include the following.

Pirate Game: There is one stockpile of money which is more than enough for all players. They have the choice to divide it equally or manipulate others to take the larger share for themselves.

Shrinking Pie Game: All players are faced with a constantly depleting resource. Only after they decide how to split it they can sustain the market.

Prisoner's Dilemma: The players are isolated and forced to confess a secret. The choice here is that if a player gives up first, they walk free while everyone else faces severe consequences. However, if no one gives it up, they collectively face a milder punishment.

I. There is abundant resource or money in
   i. Pirate Game
   ii. Shrinking Pie Game
   iii. Both

II. In Prisoner's Dilemma,
   i. All players get the same punishment.
   ii. All players continue in the game till the last.
   iii. Can't say

III. Game theory
   i. Has feasible strategies for the best outcome for all players.
   ii. Is a topic of pure mathematics.
   iii. Both

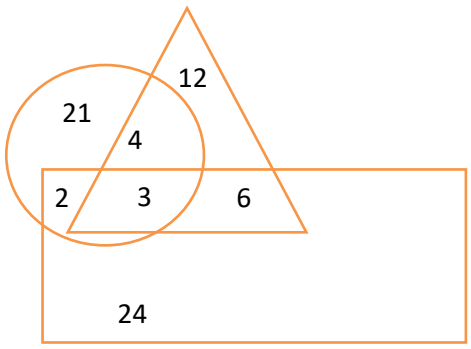
62. The correct answers to I, II and III are
   A. ii, iii, iii, respectively.
   B. i, ii, iii, respectively.
   C. i, iii, i, respectively.
   D. iii, ii, i, respectively.

3 marks

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63. In the diagram below, the circle represents students who drink orange juice, the triangle represents students who drink lemon juice and the rectangle represents students who drink mango juice.



The number of people who drink orange and lemon juice but not mango juice is

A. 4.

B. 7.

C. 3.

D. 37.

1 mark

64. At 12:00 noon, in how much time (in minutes) will the angle (when counted in the clockwise direction) between the hour and the minute hands of a clock be 275 degrees?

A. 35.

B. 40.

C. 45.

D. 50.

1 mark

65. If today is Sunday, then what was the day 52 days before today?

A. Thursday.

B. Tuesday.

C. Wednesday.

D. Friday.

1 mark

66. A cuboid of dimensions 45 cm, 75 cm and 90 cm is to be divided into identical cubes of side X cm. What is the maximum possible value of X?

A. 10.

B. 12.

C. 15.

D. 17.

1 mark

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67. Identify the sequence, among the given options, which does not follow the same logical order as the other three sequences.

A. Grass, grasshopper, frog, snake, hawk.

B. Sentence, paragraph, section, chapter, book.

C. Household, village, block, district, state.

D. Chromosome, nucleus, cell, tissue, body.

1 mark

68. C has 4 grandsons. D, son of C, has siblings X and P, but has no kids. S and M are the only nephews of P. Z and Y are grandsons of C. How are S and Z related?

A. Father and son.

B. Cousins.

C. Grandfather and grandson.

D. Uncle and nephew.

1 mark

69. Choose the set of three statements where the third statement can be logically derived from the preceding two:

a. All Turns are Good
b. All Cans are Good
c. Some Turns are Cans
d. No Vans are Cans
e. Some Vans are not Good
f. Some Cans are Good

A. b, a, c.
B. d, b, e.
C. c, d, b.
D. a, c, f.

2 marks

70. There are seven people A, B, C, D, E, F and G standing in a row. B and E are at the two ends. C is standing exactly between G and D. A is standing to the immediate left of D. G is standing exactly between C and E.

Who is standing to the immediate left of G?

A. E.

B. C.

C. D.

D. A.

2 marks

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