Exercise 7.5

Factorise each of the following expressions.

1. $16x^2 - 25y^2$

$= (4x)^2 - (5y)^2$ $\quad \{ a^2 - b^2 = (a+b)(a-b) \}$

$= (4x+5y)(4x-5y)$

2. $27x^2 - 12y^2$

$= 3(9x^2 - 4y^2)$

$= 3\{(3x)^2 - (2y)^2\}$ $\quad \{ a^2 - b^2 = (a+b)(a-b) \}$

$= 3(3x+2y)(3x-2y)$

3. $1449a^2 - 289b^2$

$= (129)^2 - (17b)^2$ $\quad \{ a^2 - b^2 = (a+b)(a-b) \}$

$= (129a+17b)(129a-17b)$

4. $12m^2 - 27$

$= 3(4m^2 - 9)$

$= 3[(2m)^2 - (3)^2]$ $\quad \{ a^2 - b^2 = (a+b)(a-b) \}$

$= 3(2m+3)(2m-3)$

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5 $125x^2 - 45y^2$

$= 5(25x^2 - 9y^2)$

$= 5[(5x)^2 - (3y)^2]$

$= 5(5x+3y)(5x-3y)$

6 $144a^2 - 169b^2$

$= (12a)^2 - (13b)^2$ $\{a^2-b^2 = (a+b)(a-b)\}$

$= (12a+13b)(12a-13b)$

7 $(2a-b)^2 - 16c^2$

$= (2a-b)^2 - (4c)^2$ $\{a^2-b^2 = (a+b)(a-b)\}$

$= \{(2a-b)+4c\}\{(2a-b)-4c\}$

$= (2a-b+4c)(2a-b-4c)$

(8) $(x+2y)^2 - 4(2x-y)^2$

$= (x+2y)^2 - \{2(2x-y)\}^2$

$= (x+2y)^2 - (4x-2y)^2$

$= \{(x+2y)+(4x-2y)\}\{(x+2y)-(4x-2y)\}$

$= (x+2y+4x-2y)(x+2y-4x+2y)$

$= (5x)(4y-3x)$

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(9) $3a^5 - 48a^3 = 3a^3(a^2 - 16)$

$= 3a^2[a^2 - 4^2]$

$= 3a^2(a+4)(a-4)$

(10) $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2$

$= \{a^2 + 4b^2\}\{a^2 - 4b^2\}$

$= (a^2+4b^2)\{(a)^2 - (2b)^2\}$

$= (a^2+4b^2)(a+2b)(a-2b)$

(11) $x^8 - 1 = (x^4)^2 - (1)^2$

$= (x^4+1)(x^4-1)$

$= (x^4+1)\{(x^2)^2 - (1)^2\}$

$= (x^4+1)(x^2+1)(x^2-1)$

$= (x^4+1)(x^2+1)(x+1)(x-1)$

(12) $64 - (a+1)^2 = (8)^2 - (a+1)^2$

$= (8+a+1)\{8-(a+1)\}$

$= (9+a)(8-a+1)$

$= (9+a)(7-a)$

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(13) $36l^2 - (m+n)^2$

$= (6l)^2 - (m+n)^2$

$= \{6l+m+n\}\{6l-m-n\} = (6l+m+n)(6l-m-n)$

(14) ~~8~~ $25x^4y^4 - 1$

$= (5x^2y^2)^2 - (1)^2$

$= (5x^2y^2+1)(5x^2y^2-1)$

15 $a^4 - \frac{1}{b^4} = (a^2)^2 - \left(\frac{1}{b^2}\right)^2$

$= \left(a^2+\frac{1}{b^2}\right)\left(a^2-\frac{1}{b^2}\right)$

$= \left(a^2+\frac{1}{b^2}\right)\left(a+\frac{1}{b}\right)\left(a-\frac{1}{b}\right)$

(16) $x^3 - 144x = x(x^2-144)$

$= x[(x)^2 - (12)^2]$

$= x(x+12)(x-12)$

17 $(x-4y)^2 - 625$

$= (x-4y)^2 - (25)^2$

$= (x-4y+25)(x-4y-25)$

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(20) $(x+y)^2 - (a-b)^2$

$= \{(x+y)+(a-b)\}\{(x+y)-(a-b)\}$

$= (x+y+a-b)(x+y-a+b)$

(21) $\frac{1}{16}x^2y^2 - \frac{4}{49}y^2z^2$

$= \left(\frac{1}{4}xy\right)^2 - \left(\frac{2}{7}yz\right)^2$

$= \left(\frac{1}{4}xy + \frac{2}{7}yz\right)\left(\frac{1}{4}xy - \frac{2}{7}yz\right)$

(22) $75a^3b^2 - 108ab^4$

$= 3ab^2(25a^2 - 36b^2)$

$= 3ab^2[(5a)^2 - (6b)^2]$

$= 3ab^2(5a+6b)(5a-6b)$

(23) $x^5 - 16x^3$

$= x^3(x^2 - 16)$

$= x^3(x^2 - 4^2) = x^3(x+4)(x-4)$

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(24) $\frac{50}{x^2} - \frac{2x^2}{81}$

$= 2\left[\frac{25}{x^2} - \frac{x^2}{81}\right] = 2\left[\left(\frac{5}{x}\right)^2 - \left(\frac{x}{9}\right)^2\right]$

$= 2\left(\frac{5}{x} + \frac{x}{9}\right)\left(\frac{5}{x} - \frac{x}{9}\right)$

(25) $256x^5 - 81x$

$= x(256x^4 - 81)$

$= x\left[(16x^2)^2 - (9)^2\right]$

$= x\left[(16x^2+9)(16x^2-9)\right] = x(16x^2+9)(16x^2-9)$

$= x(16x^2+9)\{(4x)^2 - (3)^2\}$

$= x(16x^2+9)(4x+3)(4x-3)$

(26) $a^4 - (2b+c)^4$

$= \{(a^2)^2 - \{(2b+c)^2\}^2$

$= \{a^2 + (2b+c)^2\}\{(a^2) - (2b+c)^2\}$

$= (a^2 + (2b+c)^2)(a+2b+c)(a-2b-c)$

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(27) $(3x+4y)^4 - x^4$

$= \{(3x+4y)\}^2 - (x^2)^2$

$= \{(3x+4y)^2 + x^2\}\{(3x+4y)^2 - x^2\}$

$= \{(3x+4y)^2 + x^2\}(3x+4y+x)(3x+4y-x)$

(28) $p^2q^2 - p^4q^4$

$= p^2q^2(1 - p^2q^2)$

$= p^2q^2\{1 - (pq)^2\}$

$= p^2q^2(1+pq)(1-pq)$

(29) $3x^3y - 243xy^3 = 3xy(x^2 - 81y^2)$

$= 3xy[(x)^2 - (9y)^2]$

$= 3xy(x+9y)(x-9y)$

(30) $a^4b^4 - 16c^4$

$= (a^2b^2)^2 - (4c^2)^2$

$= (a^2b^2)^2 - (4c^2)^2$

$= (a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$

$= (a^2b^2 + 4c^2)\{(ab)^2 - (2c)^2\}$

$= (a^2b^2 + 4c^2\}(ab+2c)(ab-2c)$

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(31) $x^4 - 625$

$= (x^2)^2 - (25)^2$

$= (x^2+25)(x^2-25)$

$= (x^2+25)(x^2-5^2)$

$= (x^2+25)(x+5)(x-5)$

(32) $x^4 - 1 = (x^2)^2 - (1)^2$

$= (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

(33) $49(a-b)^2 - 25(a+b)^2$

$= \{7(a-b)\}^2 - \{5(a+b)\}^2$

$= (7a-7b)^2 - (5a+5b)^2$

$= \{7a-7b+5a+5b\} \{(7a-7b)-(5a+5b)\}$

$= (12a-2b)(7a-7b-5a-5b)$

$= 2(6a-b)(2a-12b)$

$= 2(6a-b) \times 2(a-6b)$

$= 4(6a-b)(a-6b)$

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34 $x-y-x^2+y^2$

$= (x-y) - (x^2-y^2)$

$= (x-y) - \{(x+y)(x-y)\}$

$= (x-y)\{1-(x+y)\}$

$= (x-y)(1-x-y)$

(35) $16(2x-1)^2 - 25y^2$

$= \{4(2x-1)\}^2 - (5y)^2$

$\Rightarrow \{4(2x-1)+5y\}\{4(2x-1)-5y\}$

$= (8x-4+5y)(8x-4-5y)$

$= (8x+5y-4)(8x-5y-4)$

(36) $4(xy+1)^2 - 9(x+1)^2$

$\Rightarrow \{2(xy+1)\}^2 - \{3(x+1)\}^2$

$= (2xy+2)^2 - (3x+3)^2$

$\Rightarrow \{(2xy+2+3x+3)\}\{(2xy+2)-(3x+3)\}$

$= (2xy+3x+5)(2xy-3x-1)$

(37) $(2x+1)^2 - 9x^4$

$= (2x+1)^2 - (3x^2)^2$

$= (2x+1+3x^2)(2x+1-3x^2) = (3x^2+2x+1)(-3x^2+2x+1)$

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(38) $x^4 - (2y-3z)^2$

$= (x^2)^2 - (2y-3z)^2$

$\Rightarrow (x^2+(2y-3z))\{x^2-(2y-3z)\}$

$= (x^2+2y-3z)(x^2-2y+3z)$

(39) $(a^2-b^2)+a-b$

$= (a+b)(a-b)+(a-b)$

$= (a-b)(a+b+1)$

(40) $16a^4-b^4 = (4a^2)^2-(b^2)^2$

$= (4a^2+b^2)(4a^2-b^2)$

$= (4a^2+b^2)\{(2a)^2-(b)^2\}$

$= (4a^2+b^2)(2a+b)(2a-b)$

(41) $a^4-16(b-c)^4 = (a^2)^2-\{4(b-c)^2\}^2$

$= \{a^2+4(b-c)^2\}\{a^2-4(b-c)^2\}$

$= \{a^2+4(b-c)^2\}\{(a)^2-(2(b-c))^2\}$

$= \{a^2+4(b-c)^2\}\{(a+2b-2c)(a-2b+2c)\}$

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(42) $2a^5 - 32a$

$= 2a[a^4 - 16] = 2a[(a^2)^2 - (4^2)]$

$= 2a(a^2+4)(a^2-4)$

$= 2a(a^2+4)(a^2-2^2)$

$= 2a(a^2+4)(a+2)(a-2)$

(43) $a^4b^4 - 81c^4 = (a^2b^2)^2 - (9c^2)^2$

$= (a^2b^2+9c^2)(a^2b^2-9c^2)$

$= (a^2b^2+9c^2)\{(ab)^2 - (3c)^2\}$

$= (a^2b^2+9c^2)(ab+3c)(ab-3c)$

(44) ~~$2a^5 - 32a$~~

$xy^9 - 9x^9 = xy(y^8 - x^8)$

$= xy[(y^4)^2 - (x^4)^2]$

$= (xy)(y^4+x^4)(y^4-x^4)$

$= xy(y^4+x^4)\{(y^2)^2 - (x^2)^2\}$

$= xy(y^4+x^4)(y^2+x^2)(y^2-x^2)$

$= xy(y^4+x^4)(y^2+x^2)(y+x)(y-x)$

45 $x^3 - x = x(x^2-1)$

$= x(x^2-1^2) = x(x+1)(x-1)$

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(46) $18a^2x^2 - 32$

$= 2[9a^2x^2 - 16]$

$= 2[(3ax)^2 - (4)^2] = 2(3ax+4)(3ax-4)$