

252

--	--

252

**ENTRANCE EXAMINATION, 2018**

**ENTRANCE EXAMINATION, 2018**

**Ph.D. PHYSICAL SCIENCES**

**Ph.D. PHYSICAL SCIENCES**

[ Field of Study Code : PHYH (898) ]

Time Allowed : 3 hours

Maximum Marks : 100

SUBJECT .....  
(Field of Study/Language)

FIELD OF STUDY CODE .....

NAME OF THE CANDIDATE .....

**INSTRUCTIONS FOR CANDIDATES**

- (i) This Question Paper consists of two Parts, i.e., Part—A and Part—B.
- (ii) All questions are compulsory. Answers should be written in the space following each question.
- (iii) Use of calculators is permitted.
- (iv) Extra pages are attached at the end of the Question Paper for Rough Work.

REGISTRATION NO. 

--	--	--	--	--

CENTRE OF EXAMINATION. ....

DATE .....

.....  
(Signature of Candidate)

.....  
(Signature of Invigilator)

.....  
(Signature and Seal of Presiding Officer)

/252

/252

**FOR OFFICE USE ONLY**

Question No.	Marks	Question No.	Marks
A1		B1	
A2		B2	
A3		B3	
A4		B4	
A5		B5	
A6		B6	
Total [A]		B7	
		B8	
		B9	
		B10	
		Total [B]	

Total Marks [A + B] = .....

**Useful Physical Constants :**

Acceleration due to gravity,  $g = 9.81 \text{ m/s}^2$

Avogadro number,  $N_A = 6.022 \times 10^{23} / \text{mol}$

Boltzmann constant,  $k_B = 1.38 \times 10^{-23} \text{ J/K}$

Charge of electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Gravitational constant,  $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

Mean radius of the earth,  $R_e = 6.37 \times 10^6 \text{ m}$

Permittivity of vacuum,  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

Planck constant,  $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

Rest mass of electron,  $m_e = 9.11 \times 10^{-31} \text{ kg}$

Rest mass of neutron,  $m_n = 1.67 \times 10^{-27} \text{ kg}$

Speed of light in vacuum,  $c = 3 \times 10^8 \text{ m/s}$

Stefan-Boltzmann constant,  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4$

Universal gas constant,  $R = 8.31 \text{ J/mol}\cdot\text{K}$

Bohr magneton  $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$

**Conversion Factors :**

1 calorie = 4.2 J

1 eV =  $1.6 \times 10^{-19}$  J

## PART—A

**Note :** Answer **all** questions. Each question carries **10** marks.

**A1.** Consider the contour integral

$$I = \oint_C \frac{e^z dz}{i(1-2z)(z-2)}$$

where the contour  $C$  is given by  $|z| = 1$ .

- (a) Using the residue theorem or otherwise, evaluate  $I$ .  
(b) Using (a), evaluate the integral

$$I_1 = \int_0^{2\pi} \frac{e^{\cos\theta} \sin(\sin\theta)}{5 - 4\cos\theta} d\theta$$

aglasem.com

- A2.** A particle of mass  $m$  moves in one dimension, in the region  $x \geq 0$ , under the influence of the potential

$$V(x) = e^{-\alpha x} - e^{-\beta x}; \quad \alpha > \beta > 0$$

- (a) Sketch  $V(x)$  vs.  $x$ .
- (b) Show that there is only one point of equilibrium and it is a stable point.
- (c) What is the angular frequency ( $\omega$ ) of small oscillation around this point of stable equilibrium?

aglasem.com

- A3.** Consider a non-relativistic particle of mass  $m$  moving along the  $x$ -axis in a potential  $V(x)$ . If the motion is bounded between the points  $x_1$  and  $x_2$ , the Bohr-Sommerfeld quantization condition reads as  $\int_{x_1}^{x_2} p(x) dx = \frac{1}{2} \left( n + \frac{1}{2} \right) h$ , where  $n = 0, 1, 2, \dots$ . If  $V(x) = \alpha x^4$ , where  $\alpha$  is a positive constant, show that, for large  $n$ , the  $n$ th energy level  $E_n$  is proportional to  $n^{4/3}$ . [Hint : In the integral change the variable from  $x$  to  $\alpha x$  with an appropriate choice of  $\alpha$ .]

aglasem.com

- A4.** Consider a system of two identical bosons that do not interact with each other. However, due to interaction with external fields, each of them has energy eigenstates with energies  $\epsilon$ ,  $2\epsilon$  and  $3\epsilon$ . If the combined system is at a temperature  $T$ , what is the mean square fluctuation  $(\langle E^2 \rangle - \langle E \rangle^2)$  of the total energy of the two particles? How would this result be modified if the two particles are identical fermions?

(a) Calculate the electric field  $E(r)$ .

(b) Calculate the total charge  $Q(r)$  within a sphere of radius  $r$  (center of the sphere) as a function of  $r$ .

(c) What are the limiting values of  $Q(r)$  as  $r/r_0$  goes to 0 and  $(r/r_0)$  goes to  $\infty$ ?

(d) Comment on the results of part (c) physically.

aglasem.com

**A5.** The electric potential of a charge distribution is given by

$$V(r) = B \frac{e^{-\lambda r}}{r}$$

where  $B$  and  $\lambda$  are the positive constants.

- (a) Calculate the electric field  $\mathbf{E}(r)$ .
- (b) Calculate the total charge  $Q(r)$  within a sphere of radius  $r$  (centre of the sphere is at the origin).
- (c) What are the limiting values of  $Q(r)$  as (i)  $r$  goes to 0 and (ii)  $r$  goes to  $\infty$ ?
- (d) Interpret the results of part (c) physically.

aglasem.com



- A6.** Estimate the numerical value of the Fermi energy for conduction electrons in silver. The mass number of silver is 107.9 and its density is  $10.49 \text{ g/cm}^3$ . State your assumptions clearly.

Note: Answer all questions. Each question carries 4 marks.

Q1. Calculate the de Broglie wavelength of an electron of kinetic energy 100 eV.

aglasem.com

## PART—B

**Note :** Answer **all** questions. Each question carries 4 marks.

**B1.** Calculate the de Broglie wavelength of an electron of kinetic energy 100 eV.

aglasem.com

- B2.** Calculate the determinant of the matrix  $e^{i\omega A}$ , where  $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . Here  $\omega$  is a real number. [Hint : Write down the definition of  $e^{i\omega A}$  in terms of a power series in  $A$  and calculate the matrix  $A^2$ .]

aglasem.com

- B3.** The first ionization potential (derived from the formula  $E = Z^2 e^2 / (2a_0)$ ) for hydrogen is 13.6 eV. For helium, the experimentally measured value of the first ionization potential is 24.6 eV. What is the ground state energy of a neutral He atom?

aglasem.com

- B4.** A 10 V unregulated DC power source and a 6 V Zener diode are used to make a 6 V regulated power supply. The maximum current that can be passed through the diode is 100 mA. Calculate the series resistance  $R_z$  and its wattage.

aglasem.com

- B5.** A rocket starts from rest and is propelled forward by ejecting the burnt fuel. Assuming that the burnt residues are ejected at a constant speed  $v_e$  (with respect to the rocket), calculate the final speed of the rocket when all the fuel has been used up. Initially, the mass of the fuel is half the mass of the rocket (without the fuel).

aglasem.com

- B6.** The mean lifetime of a  $\pi^+$  meson in its rest frame is  $2.5 \times 10^{-8}$  s. If a burst of  $\pi^+$  mesons travels with a speed of  $0.8c$  ( $c$  being the speed of light in vacuum), how far would it travel during its mean life in the laboratory frame?

aglasem.com

- B7.** X-rays with wavelength of  $\lambda = 1 \text{ \AA}$  are scattered from a carbon block. The Compton shift observed is  $\Delta\lambda = 0.025 \text{ \AA}$ . What is the kinetic energy (in eV) imparted to the recoiling electron?

aglasem.com



- B8.** Calculate the electric polarization of helium gas with number density of  $2.6 \times 10^{25} / \text{m}^3$ , if it is subjected to an electric field of  $6 \times 10^5 \text{ V/m}$ . (The atomic polarizability of helium is  $0.23 \times 10^{-40} \text{ F}\cdot\text{m}^2$ .)

aglasem.com

- B9.** A particle of mass  $m$  is moving in one dimension in a potential given by  $V(x)$ , where  $V(x) = \infty$  for  $x < 0$  and is equal to  $\frac{1}{2}m\omega^2 x^2$  for  $x \geq 0$ . Using the results for a simple harmonic oscillator, calculate the energy levels of this system.

aglasem.com

- B10.** Consider a face-centered cubic lattice. Suppose you place identical spheres at each point of this lattice (with the centres of the spheres coinciding with the lattice points). Volume fraction is defined to be the volume of space occupied by the spheres divided by the total volume. What is its maximum possible value in the present case?

aglasem.com